Nonlinear Pricing of Telecommunications with Call and Network Externalities*

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Abstract

This paper investigates how call and network externalities affect a monopolist's optimal nonlinear tariff in a two-way telecommunication service market, where consumers' valuations for outgoing and incoming calls are positively correlated and the firm is not allowed to charge incoming calls. It is shown that, in contrast to the traditional efficiency-on-the-top result, all subscribers including those with the highest willingness-to-pay make suboptimal quantities of calls in the equilibrium. The presence of call externalities may result in the existence of no-call-making subscribers in equilibrium. Also, the firm may have incentives to sell the service below costs to some low-valuation consumers in order to take advantage of the effect of network externalities.

Key words: Telecommunications, Call and Network Externalities, Nonlinear Pricing.

JEL classification : D42, L12, L96.

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1 Introduction

It is well-known that there exist network and call externalities in two-way communications markets. The network externality, which results from adding new subscribers, is the aggregate benefit that other subscribers enjoy by being able to call and to be called by new subscribers.[1] On the other hand, the call externality is the benefit of incoming calls to a subscriber who does not have to pay for the calls, usually the party being called. With those two externalities, adding a new customer to a network increases the surplus of other subscribers who are able to call and be called by the new customer, and therefore affects not only consumers' demand for the service but also their demand for subscription.

Economists have been quite active in analysing the effect of network externalities on the firm's pricing policy in telecommunications.[2] The main focus has been on how Ramsey pricing should be adjusted in order to internalise network externalities (i.e. to induce more customers to participate). Efficient pricing with network externalities typically involves a lower price/cost mark-up for uniform pricing (e.g. Willig, 1979) and a lower fixed fee for two-part tariffs (e.g. Littlechild, 1975), compared with ordinary markets without network externalities. Oren and Smith (1981) show that using two-part tariffs a profit-maximising monopolist can mitigate the critical-mass problem. Oren-Smith-Wilson (1982) extend the analysis to a general nonlinear pricing, and additionally point out that the firm may have incentives to serve some low-demand consumers by reducing fixed fees even though it does not cover the costs. On the other hand, Einhorn (1993) emphasises the importance of the identity of the marginal customer in determining the welfare-maximising two-part tariff with network externalities.

The call externality, however, has been given little attention in the literature, which is somewhat surprising considering roughly half of benefits of using a telephone service is from receiving calls. It is probably because there has been no market mechanism by which consumers can express their preferences for incoming calls. As Acton and Vogelsang (1990) point out, it is hard to apply the Coase theorem because negotiations between a caller and a receiver to internalise call externalities require costly telephone calls. Also, Willig (1979) argues that call

[1] Here we are dealing with *direct* network externalities, where consumer utility directly depends on the market size, independently of price system. There also exist *indirect* network externalities that are realised indirectly via market mechanisms such as economies of scale. For further discussions about the classification, see Economides (1996) among others.

[2] Mitchell and Vogelsang (1991) provide a good survey on the topic.

externalities are hard to deal with if they are not easily related to other variables, e.g. outgoing calls, other purchases, or the number of subscribers. To my knowledge, the first economic analysis incorporating call externalities into telecommunication pricing is Squire (1973). He derives the first-best two-part tariff with call and network externalities, which consists of a per-call price less than the marginal cost reflecting the effect of call externalities. However, it is not implementable without subsidising the firm using public funds, indicating the need for a departure from the first-best. Focusing on the interaction between network and call externalities, Einhorn (1990) shows that the relative importance of call externalities to network externalities increases with the number of subscribers, and that the price/cost markup for outgoing calls decreases as the penetration rate increases. However, he does not provide the exact form of the optimal tariff. Furthermore, both papers assume that the firm has complete information about consumer preferences.

This paper extends the previous work on telecommunication pricing to a more general environment, where consumers benefit not only from network externalities but also from call externalities, and the firm uses general nonlinear tariffs. Consumers are heterogeneous, and have private information on their preferences for the service. Consumers' valuations for outgoing and incoming calls are assumed to be positively correlated. Also, we assume that the firm is not allowed to charge incoming calls.[3] Our analysis is focused more on the call externality, highlighting how it changes some of the previous results established in a framework with network externalities only. In order to incorporate the effect of call externalities into the pricing mechanism, we assume a simple but natural calling pattern in which the quantity of calls each subscriber receives is related to the aggregate quantity of outgoing calls made in the market (see the next section for details). With call externalities, the firm generally charges smaller per-unit prices for outgoing calls in order to induce subscribers to make more calls, and extracts the increased consumers' surpluses due to the effect of call externalities by charging larger fixed fees. In fact, we often observe that the proportion of fixed fees (rental charges in the UK) is larger than actual call charges for light telephone users.[4] The main findings of our analysis are as follows:

[3] This seems natural in the telecommunication market, although in the US and some European countries mobile phone users sometimes pay for receiving calls.

[4] Of course, this phenomenon is partly because the firm tries to reimburse the large fixed cost of installing networks.

i) The optimal nonlinear tariff leads to downward distortions in outgoing call consumption for all types of subscribers including the highest type. This is a contrast to the well-known efficiency-on-the-top result in the standard screening model. Under the no receiver charge rule, the firm is not able to control the quantity of incoming calls by price (the only screening instrument available for the firm is the quantity of incoming calls). Therefore, given that consumers' valuations for outgoing and incoming calls are positively correlated, it is optimal for the firm to partially sacrifice the efficient internalisation of call externalities for the sake of consumer screening. Nevertheless, it should be noted that the firm internalises some of the call externality using the fixed part of the nonlinear tariff, and also indirectly via the interdependency between the call and network externality (the effect of the call externality increases with the number of subscribers).

ii) If the effects of the two externalities are sufficiently strong, there may exist subscribers who only receive calls without making any outgoing calls in equilibrium. In other words, it may be optimal for the firm to offer potential consumers the option of subscribing to the network for a fixed fee even though they end up only receiving calls without making any outgoing calls. Offering this option to customers is sometimes a useful strategic tool for the firm in exploiting both call and network externalities. This property is mainly due to the presence of call externalities, although it is reinforced by the network externality effect. An anecdotal example is found in mobile telecommunications, where an important reason to subscribe to a network is to be able to be reached by others. Some subscribers (e.g. students) often use their mobile phone for receiving incoming calls only without making any outgoing calls. For outgoing calls, they tend to use an alternative service, say a fixed-link (public) telephone.[5] Also, shops or public institutions sometimes have a receive-only telephone in order to prevent their employees from using the phone for their personal purposes. An extreme case is pager service, which

[5] Several mobile operators in the UK have recently introduced new services encouraging this kind of calling pattern. Examples include *Pay as you Talk Smartstep* of Vodafone and *New Pay & Go* of BT Cellnet. By subscribing those services for a fixed fee, consumers get a mobile handset plus a small amount of initial call value for activation. On purchase they can receive calls free of charge, and do not need to pay any additional amount until they make the first outgoing call. Since there is *no minimum top-up period*, they do not have any obligation of making outgoing calls, while being connected to the network for receiving incoming calls. Typically, the per-call charge for this kind of service is higher than the ordinary service which requires additional payments or a certain compulsory amount of outgoing calls.

basically allows for receiving messages only. Most pager service operators charge a fixed fee for their subscribers without any volume-related charge.

iii) Finally, the presence of network externalities may induce the firm to sell the service below costs to some low-valuation consumers. A similar finding was discovered by Oren-Smith-Wilson (1982) in a framework with network externalities only. A slight difference, however, is that the presence of call externalities weakens the firm's incentive to sell the service below costs, and as a result it can be implemented by absorbing parts of either the marginal cost or the fixed cost or both, not the fixed cost only as in Oren-Smith-Wilson. In fact, it is a quite common practice in some countries that mobile companies provide handsets below costs in order to induce a larger subscription. Also, Internet access providers often offer basic connection equipments below costs or even free of charge.[6]

2 The model

Consider a monopolistic firm selling a two-way telecommunications service. The cost structure is simple: there are per-person fixed costs k (for installing a subscriber line in a fixed-link network or providing a handset in a mobile network) and constant marginal cost c for delivering a unit call from a caller to a receiver. By subscribing to the network consumers get utilities not only from making calls but also from receiving calls. We make the following assumptions for analytical simplicity:

1. *Uniform calling pattern*: Every subscriber equally distributes his/her outgoing calls across all the other subscribers in the network.[7]

2. *Indifference to communication partners*: Subscribers gain the same benefit from an outgoing (incoming) call regardless of who the recipient (caller) is.

[6] A similar phenomenon is also observed in markets for computer software with two-way functional features such as reading and writing in word-processors (just like calling and being called in telecommunications). For example, Adobe Systems Inc. offers *Acrobat Reader* (the read-only version) free of charge. The users of *Acrobat Reader* benefits from reading documents produced by the owners of *Acrobat Writer* (the read-and-writable full version). Of course, *Acrobat Writer* is sold for a certain price. The firm uses the read-only version as a market expansion device to take advantage of network externalities.

[7] This property, introduced by Squire (1973) and Rohlfs (1974), has been widely employed in analysing telecommunications pricing. For some discussion about nonuniform calling patterns, see Rohlfs (1974, section 4).

3. *No charge for incoming calls*: The firm is not allowed to charge consumers for receiving calls, and subscribers do not reject incoming calls.[8]

Under the assumption of uniform calling pattern, we can easily disaggregate a subscriber's outgoing calls into the calls he/she makes to each individual in the network. Define q as the number of calls a subscriber makes to *each* of the other subscribers in the network. Then, given a participation level or subscription rate n (the percentage of all potential customers who actually become subscribers), the subscriber's *total* number of outgoing calls is simply given by nq. Given tariff $T(\cdot)$, which specifies the amount a subscriber has to pay for his/her total quantity of outgoing calls, the corresponding payment is simply given by $T(nq)$. We further assume that the tariff can be decomposed into the multiplicative form of $T(nq) = nt(q)$. Restricting our attention to a static model, for any equilibrium tariff $\widehat{t}$ and subscription rate $\widehat{n}$, we can always find a function T satisfying the relation $T(\widehat{n}q) = \widehat{n}\widehat{t}(q)$ in the equilibrium.[9] We proceed the analysis with a subscriber's *per-person* outgoing-call quantity q instead of his/her *total* number of outgoing calls nq, and also with t rather than T. As will be shown later, this disaggregate approach greatly simplifies the analysis, allowing us to avoid the problem of multiple equilibria.

There is a continuum of potential subscribers, whose preferences for telephone calls are measured by a parameter θ with distribution function $F(\theta)$ and density $f(\theta)$ in $[\underline{\theta}, \overline{\theta}]$. The distribution of consumer types is public information, but the firm cannot distinguish different types of consumers. Given a subscription rate n, a customer of type θ when making q calls to each of the other subscribers and receiving total $\widetilde{Q}$ calls gets gross surplus of

$$U(q, \widetilde{Q}, \theta, n) \equiv \theta n u(q) + \alpha(\theta)\widetilde{Q},$$

where $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Under the assumption of uniform calling pattern, all the subscribers will receive the same number of calls in equilibrium, regardless of how many calls they make (i.e. $\widetilde{Q}$ is independent of θ). Replacing $\widetilde{Q}$ with $n\widetilde{q}$, the gross surplus can be rewritten

[8] This means that consumers take the equilibrium quantity of incoming calls as given, and the utility from receiving calls is extra benefits to subscribers. We ignore the possibility of nuisance calls which give the receivers negative utilities.

[9] As suggested by Dybvig and Spatt (1983), this kind of pricing scheme can be useful for overcoming the critical mass problem by insuring potential customers against the possibility of a small, low-valued network.

as

$$U(q, \widetilde{q}, \theta, n) \equiv n[\theta u(q) + \alpha(\theta)\widetilde{q}],$$

where $\widetilde{q}$ is just the average of the incoming calls the customer receives from a subscriber in the network.[10] Utilities from outgoing and incoming calls are additively separable. Here, n reflects the effect of network externalities, and α the effect of call externalities. We model the network externality using a simple multiplicative specification. This seems more appropriate in the context of telecommunications, rather than the additive specifications usually employed in the previous literature. The benefits of a larger network are realised by *actually* making more calls, rather than via some proxy (additive) term as in additive specifications. We assume that $\alpha > 0$, $\alpha'(\cdot) > 0$, $\alpha''(\cdot) \geq 0$ (i.e. the valuations for outgoing and incoming calls are positively correlated).[11]

3 The optimal nonlinear tariff

Following Katz and Shapiro (1985), we assume that consumers base their subscription and call consumption decisions on their expectation of network size (i.e. subscription rate), and require that their expectation be fulfilled in equilibrium (*rational expectations*). Given an expected subscription rate $n^a > 0$ and tariff $t(\cdot)$, a type-θ consumer optimally chooses his/her *per-person* outgoing-call quantity and obtains the following utility:

$$\begin{aligned} v(\theta) &\equiv \max_q : U(q, \widetilde{q}, \theta, n^a) - n^a t(q) \qquad (1) \\ &= n^a \left\{ \max_q : \theta u(q) + \alpha(\theta)\widetilde{q} - t(q) \right\}. \end{aligned}$$

Given t, the utility-maximising *per-person* outgoing-call quantity should satisfy the first-order condition:[12]

$$\theta u'(q) = t'(q). \qquad (2)$$

[10] The model can be easily extended to a general utility function for incoming calls, say $u_I(\cdot)$, where $u_I'(\cdot) > 0$ and $u_I''(\cdot) < 0$.

[11] The (weak) convexity of $\alpha(\cdot)$ implies that the relative valuation of incoming calls to outgoing calls is increasing in consumer type. It makes the problem tractable by simplifying the incentive constraint the firm faces, although it reduces the generality of the analysis by sacrificing the opposite case.

[12] Recall that consumers take the quantity of incoming calls as given.

By the envelope condition, we have

$$v'(\theta) = n^a[u(q(\theta)) + \alpha'(\theta)\widetilde{q}]. \tag{3}$$

The presence of network externalities usually leads to the multiplicity of equilibria (see Rohlfs (1974), Farrell and Saloner (1985), and Katz and Shapiro (1985) among others). However, the problem of multiple equilibria becomes much less stringent under the decomposed utility and tariff structures specified above.

Lemma 1 *Suppose that for given $t(\cdot)$ and $\widetilde{q}$ we have a unique θ satisfying $\theta u(q(\theta)) + \alpha(\theta)\widetilde{q} = t(q(\theta))$, where $q(\theta)$ satisfies (2). Then, the resulting fulfilled-expectation equilibrium is unique.*

Now we are prepared to derive the optimal nonlinear tariff. In the standard nonlinear pricing without any externalities, the firm can choose the optimal quantity schedule without any concerns about the optimal participation level.[13] Here, however, solving for the optimal nonlinear price schedule is not straightforward, because consumers are interrelated due to both network and call externalities. The firm's optimal pricing policy involves the optimal participation level as well as the optimal quantity schedule.

It is useful to solve the problem in two stages. First, for a given (expectation-fulfilling) subscription level n, the firm chooses the price schedule to maximise profits. Second, we determine the optimal subscription level, which together with the optimal price schedule obtained in the first stage fully characterises the optimal tariff. As usual in the standard nonlinear pricing, it is more convenient to solve for the quantity schedule rather than the tariff itself. Once the optimal quantity schedule is obtained, the optimal tariff can be easily found using (2). The firm's problem is to maximise profits under the incentive and participation constraints. Given that the utility function satisfies the single-crossing condition, the incentive constraint, which is necessary for consumers' truth-telling of their type, can be summarised by the following two conditions:

$$v'(\theta) = n[u(q(\theta)) + \alpha'(\theta)\widetilde{q}] \tag{4}$$

and

$$n[u'(q(\theta))q'(\theta) + \alpha''(\theta)\widetilde{q}] \geq 0, \tag{5}$$

[13] In fact, the optimal participation level is automatically determined according to the type distribution.

which are just the first and second-order conditions for the consumer utility-maximisation.[14] Under the quasi-convexity of $\alpha(\cdot)$, condition (5) is simplified to the so-called monotonicity condition:

$$q'(\theta) \geq 0. \tag{6}$$

Given n, there exists a unique marginal type $\theta = F^{-1}(1-n)$, where $F^{-1}(\cdot)$ is the inverse of the distribution function of consumer type. Since $v(\theta)$ is increasing, the participation constraint is simply given by

$$v(F^{-1}(1-n)) = 0.$$

3.1 The optimal quantity schedule

Defining $\pi(n)$ as the maximum profit the firm can obtain for a given subscription level n, we have

$$\pi(n) \equiv \max_q \int_{F^{-1}(1-n)}^{\overline{\theta}} [nt(q(\theta)) - cnq(\theta) - k] f(\theta) d\theta$$

subject to (4) and (6).

As usual in the standard nonlinear pricing, we will initially ignore the monotonicity constraint (6), and show that it is satisfied at the optimum under a certain condition for the type distribution. Substituting out t using (1) and eliminating v by integration by parts using (4), the profit function can be rewritten as[15]

$$\pi(n) = \max_q \int_{F^{-1}(1-n)}^{\overline{\theta}} \{n[(\theta - \tfrac{1-F(\theta)}{f(\theta)})u(q(\theta)) + (\alpha(\theta) - \tfrac{1-F(\theta)}{f(\theta)}\alpha'(\theta))\widetilde{q} - cq(\theta)] - k\} f(\theta) d\theta. \tag{7}$$

Here, it is misleading to directly apply the pointwise maximisation to find the optimal call-quantity schedule as in the standard nonlinear pricing, since the call-quantity allocation for a specific type influences the profits obtainable from all the other types due to the presence of call externalities. To get around this problem, we define the marginal profit the firm can get from

[14] This is now standard in adverse selection models (see for example Tirole (1988, section 3.5)).

[15] In this integration by parts, we take $F(\theta) - 1$ as the integral of $f(\theta)d\theta$. Note that this procedure incorporates condition (4) into the objective function, and that the participation constraint has been already incorporated into the problem.

allocating one more unit of per-person outgoing calls to a consumer of type $\theta \in [F^{-1}(1-n), \overline{\theta}]$ as follows:

$$s(\theta) \equiv n(\theta - \tfrac{1-F(\theta)}{f(\theta)})u'(q(\theta)) + \int_{F^{-1}(1-n)}^{\overline{\theta}} [\alpha(\theta) - \tfrac{1-F(\theta)}{f(\theta)}\alpha'(\theta)]f(\theta)d\theta - nc, \tag{8}$$

By inducing a type-θ consumer to make one more unit of outgoing calls to every other subscriber, the firm can not only earn the type-specific revenue (the first term in (8)), but also *partially* extract all the subscribers' extra surpluses resulting from receiving one more incoming call (the second term in (8)). The third term in (8) is the corresponding marginal cost. Then, the optimal allocation of individual per-person outgoing-call quantity is given by

$$\begin{cases} [\theta - \tfrac{1-F(\theta)}{f(\theta)}]u'(q^*) = c - \eta(n) \text{ for } \theta > \widetilde{\theta} \\ q^* = 0 \text{ for } \theta \leq \widetilde{\theta} \end{cases}, \tag{9}$$

where

$$\eta(n) = \frac{1}{n}\int_{F^{-1}(1-n)}^{\overline{\theta}} [\alpha(\theta) - \tfrac{1-F(\theta)}{f(\theta)}\alpha'(\theta)]f(\theta)d\theta \tag{10}$$

and $\widetilde{\theta}$ is the cut-off type such that

$$(\widetilde{\theta} - \tfrac{1-F(\widetilde{\theta})}{f(\widetilde{\theta})})u'(0) = c - \eta(n). \tag{11}$$

The presence of $\eta(n)$ here indicates that the firm internalises some of the call externality even though it cannot fully control the quantity of incoming calls under the no receiver charge rule. The firm partially extracts the benefits consumers get from receiving calls directly through the fixed part of the nonlinear tariff, and also indirectly via the relatedness of the call externality to the network externality, i.e. the effect of the call externality increases with the number of people in the network.

We assume that $0 < \eta(n) < c$ for all $n \in [0, 1]$. The monotonicity constraint ($q'(\theta) \geq 0$), which is required for incentive compatibility, is satisfied under the standard monotonic hazard rate condition:

$$\tfrac{1-F(\theta)}{f(\theta)} \text{ is decreasing.} \tag{12}$$

Given n, η is a constant. So, the cut-off type $\widetilde{\theta}$ (which is unique under condition (12)) can be easily calculated from (11), and the corresponding subscription rate $\widetilde{n}$ is given from the identity

$\widetilde{n} \equiv 1 - F(\widetilde{\theta})$. We focus on cases where the cut-off type is interior for any $n > 0$ (i.e. $\underline{\theta} < \widetilde{\theta} < \overline{\theta}$). The optimal quantity schedule induces consumers of type less than or equal to $\widetilde{\theta}$ *not* to make any outgoing calls. However, it does not immediately mean that those consumers are excluded from the market. As will be shown shortly, with call externalities the firm may wish to serve those no-call-making subscribers by allowing them to receive incoming calls for a fixed fee. The firm uses the outgoing-call quantity as a consumer screening device. The following proposition characterises the resulting quantity allocations of outgoing calls along the consumer type.

Proposition 1 (Inefficiency of outgoing-call allocation) *Given n, all types of subscribers (including the highest type) consume suboptimal quantities of outgoing calls.*

An interesting point, which diverges from the standard efficiency-on-the-top result, is that even the highest type of subscribers consume a socially suboptimal quantity of outgoing calls. This arises because i) consumers differ in their valuations for incoming calls and those valuations are positively correlated with their valuations for outgoing calls, and ii) the firm's screening capability is limited because it cannot control the quantities of incoming calls under the no receiver charge rule. With the quantity of incoming calls as the only screening instrument, the firm's optimal pricing policy involves a partial sacrifice of the efficient internalisation of call externalities for the sake of consumer screening. For example, if consumers were identical in their valuation for incoming calls (α constant) we would get the standard efficiency-on-the-top result. Also, in the absence of call externalities the standard efficiency-on-the-top result would be restored.

The following lemma shows how the optimal quantity schedule (equivalently the optimal marginal prices) changes as subscription rate n varies.

Lemma 2 *$\eta'(n) < 0$.*

The above lemma implies that the unit-price of calls increases as n increases. This is not surprising. The larger is the subscription rate, the bigger is the degree of heterogeneity among subscribers in terms of consumer valuation for incoming calls, and so the larger are the informational rents the firm should forgive to customers. So, the firm's incentive for internalising call externalities gets smaller as the subscription rate increases. A related result is that the optimal *per-person* outgoing call quantity (q^*) decreases as the number of subscribers increases. However, it is not clear whether a subscriber's total number of outgoing calls (nq^*) is also negatively correlated to the number of subscribers.

3.2 The optimal participation rate

Now we fully characterise the equilibrium by choosing the optimal subscription level. The following property will be useful in the proceeding analysis.

Lemma 3 *Provided* $\widetilde{n}(0) > 0$ *and* $\widetilde{n}(1) < 1$, *there exists a unique* $\overline{n} \in (0, 1)$ *satisfying* $\widetilde{n}(\overline{n}) = \overline{n}$.

For $n < \overline{n}$, the type of the marginal consumers who are indifferent between subscribing to the network or not is always greater than the cut-off type (i.e. $F^{-1}(1-n) > \widetilde{\theta}$), and therefore all subscribers make a strictly positive number of outgoing calls in equilibrium. Otherwise ($n > \overline{n}$), there exist no-call-making subscribers in equilibrium (i.e. the consumers whose type fall in $[F^{-1}(1-n), \widetilde{\theta}]$ subscribe to the network, but do not make any outgoing calls).

Plugging in the optimal outgoing-call quantity schedule characterised in (9) into the profit function, and rearranging using the fact that $n\widetilde{q} = \int_{F^{-1}(1-n)}^{\overline{\theta}} q^*(\theta) f(\theta) d\theta$ in equilibrium, we have

$$\pi(n) = \begin{cases} \int_{F^{-1}(1-n)}^{\overline{\theta}} \{n\varphi(\theta) - k\} f(\theta) d\theta \text{ for } n < \overline{n} \\ \int_{F^{-1}(1-\widetilde{n})}^{\overline{\theta}} \{n\varphi(\theta) - k\} f(\theta) d\theta + (n - \widetilde{n})[\alpha(F^{-1}(1-n))\overline{Q} - k] \text{ for } n \geq \overline{n} \end{cases}, \quad (13)$$

where $\varphi(\theta) = [\theta - \frac{1-F(\theta)}{f(\theta)}]u(q^*(\theta)) - [c - \eta(n)]q^*(\theta)$, and $\overline{Q} = \int_{F^{-1}(1-\widetilde{n})}^{\overline{\theta}} q^*(\theta) f(\theta) d\theta$ denotes the total number of calls a subscriber receives in case of $n \geq \overline{n}$. For $n < \overline{n}$, adding one more subscriber into the network produces both the network and call externality effect since the new subscriber makes a strictly positive number of calls in the equilibrium. For $n \geq \overline{n}$, however, serving one new subscriber has the network externality effect only because the new subscriber only receives incoming calls without making any outgoing calls. Note that the second term in the bottom of condition (13) denotes the total profit the firm can earn from the no-call-making subscribers. The firm, not being able to charge incoming calls, cannot screen no-call-making subscribers. So, for a given n the maximum subscription fee the firm can charge for no-call-making customers is simply given by $\alpha(F^{-1}(1-n))\overline{Q}$.

Note that the profit function in (13) is continuous, but non-differentiable at $n = \overline{n}$.[16] Specifically, the right-hand derivative is larger than the left-hand derivative at $n = \overline{n}$. This implies that the optimal subscription rate will never occur at $n = \overline{n}$ because we are solving a maximisation problem. Also, the profit function can be non-concave mainly because of the presence of network externalities. But, we focus on cases where it is possible to find the optimum using the first-order condition. Then, we have three different cases to consider. First, if the right-hand derivative is non-positive at $n = \overline{n}$ (i.e. $\pi'(\overline{n}+) \leq 0$), the optimal subscription level n^* (which must be less than $\overline{n}$) is determined by the following first-order condition:

$$\underbrace{\int_{F^{-1}(1-n^*)}^{\overline{\theta}} [\varphi(\theta) + n^*\eta'(n^*)q^*(\theta)]f(\theta)d\theta}_{\text{inframarginal effect}} + \underbrace{\left[n^*\varphi(F^{-1}(1-n^*)) - k\right]}_{\text{marginal effect}} = 0. \tag{14}$$

The first term represents the contribution of adding one more subscriber into the network to the profit obtained from the existing subscribers through the network and call externality effects (the inframarginal effect). The second term is the corresponding marginal profit gain or loss. Second, if the left-hand derivative is non-negative at $n = \overline{n}$ (i.e. $\pi'(\overline{n}-) \geq 0$), the optimal subscription level (which is larger than $\overline{n}$) is determined by the following first-order condition:[17]

$$\begin{aligned}\underbrace{\int_{F^{-1}(1-\widetilde{n})}^{\overline{\theta}} [\varphi(\theta) + n^*\eta'(n^*)q^*(\theta)]f(\theta)d\theta - (n^* - \widetilde{n})[\tfrac{\alpha'(F^{-1}(1-n^*))}{f(F^{-1}(1-n^*))}\overline{Q}]}_{\text{inframarginal effect}} \\ + \underbrace{\left[\alpha(F^{-1}(1-n^*))\overline{Q} - k\right]}_{\text{marginal effect}} = 0.\end{aligned} \tag{15}$$

In this case, the subscribers of type less than or equal to $\widetilde{\theta}(n^*)$ just receive incoming calls without making any outgoing calls in equilibrium. Otherwise, we have a positive right-hand derivative and a negative left-hand derivative ($\pi'(\overline{n}+) > 0$ and $\pi'(\overline{n}-) < 0$), and the optimal subscription level is the one associated with the local maximum with the largest profit.

Proposition 2 (No-call-making subscribers) *Provided $\pi'(\overline{n}-) \geq 0$, there exist subscribers who only receive incoming calls without making any outgoing calls in equilibrium.*[18]

[16] It is continuous and differentiable elsewhere though.

[17] Note that $\widetilde{n}$ is independent of n for $n \geq \overline{n}$. Also, we ignore the possibility of serving all the potential consumers (corner solution) by assuming that the fixed cost k is large enough that it is optimal for the firm to exclude at least the lowest-type consumers.

[18] Note that no-call-making subscribers can also exist in the third case where $\pi'(\overline{n}+) > 0$ and $\pi'(\overline{n}-) < 0$ if the optimal subscription rate is larger than $\overline{n}$.

This is the case where the effects of call and network externalities are sufficiently strong so that the firm wishes to serve some low-type consumers who are actually not willing to make any outgoing calls given the optimal tariff. The subscribers of type $\theta \leq \widetilde{\theta}$, even though they do not make any calls, can still get some utilities by receiving calls from other call-making subscribers. The firm partially extracts the surpluses of those no-call-making subscribers by charging fixed fee $\alpha(F^{-1}(1-n^*))\overline{Q}$. Moreover, their subscription increases the other call-making subscribers' surpluses due to the presence of network externalities, allowing the firm to extract more surpluses from the call-making high-type consumers. So, both call and network externalities are responsible for the firm serving no-call-making subscribers. However, this property is related more closely to the call externality effect rather than the network externality effect in the sense that without call externalities consumers not making any outgoing calls would get zero utility and therefore we would not observe no-call-making subscribers.

Proposition 3 (Selling the service below costs) *Provided the inframarginal effect of serving one more subscriber is positive at the optimum, the firm has the incentive to sell the service below costs to a range of low-type customers.*

Suppose that serving some consumers of low types does not cover the costs. The additional subscriptions, however, increase the surpluses of other high-type subscribers via the network and call externality effects (they call more and possibly receive more). If those externality effects are sufficiently strong so that the surplus extraction gain from the high types is more than enough to compensate the profit loss from serving the low types below costs, the firm has incentives to sell the service below costs. The presence of call externalities may reduce the firm' incentive because it is more difficult for the firm to extract consumers' surpluses from incoming calls as the subscription rate increases (lemma 2). So, this property is more strongly related to the network externality effect than the call externality effect. A similar result has been found by Oren-Smith-Wilson (1982). Not taking into account the effect of call externalities, however, they reach the conclusion that the firm sells the service below costs by absorbing parts of fixed costs only, while in the present model with call externalities it can be implemented by absorbing parts of either marginal or fixed costs or both, depending on the shape of the optimal marginal price schedule. For instance, the firm absorbs parts of the marginal cost as long as the optimal marginal prices are less than the *actual* marginal cost c for a range of low types.

Remarks: In order to highlight the role of the call externality effect in driving the results obtained above and to ease the comparison with those established in the previous work, we briefly summarise how the equilibrium result differs in the following three cases.[19]

1. *No externalities* ($\alpha = 0, n = 1$)[20]: The model is then essentially the same as the standard nonlinear pricing, leading to the standard results: the efficiency-on-the-top, the optimal participation rate determined by the condition $(\theta^\dagger - \frac{1-F(\theta^\dagger)}{f(\theta^\dagger)})u'(0) = c$, the absence of no-call-making subscribers, and no service being sold below costs.

2. *Only network externalities* ($\alpha = 0$): The optimal quantity schedule is independent of the subscription rate, and so we still get the efficiency-on-the-top result. With network externalities, however, the firm still needs to decide on the optimal subscription rate. Since consumers would get no benefits without making outgoing calls, no-call-making subscribers never exist in equilibrium. The firm may have incentive to sell the service below costs, but only through absorbing parts of fixed costs.

3. *Both call and network externalities*: The presence of call externalities leads to downward distortions of the outgoing call quantity for all types of subscribers including the highest type. There may exist no-call-making subscribers in equilibrium. This property is mainly due to the presence of call externalities, although it is intensified by the network externality effect. The presence of network externalities may induce the firm to serve some low-type consumers below costs through absorbing parts of either the marginal cost or the fixed cost or both, although the firm's incentive is generally weakened by the call externality effect.

4 Concluding remarks

This paper has investigated how a monopolist's optimal nonlinear pricing is affected by call and network externalities in two-way telecommunications markets, where consumers' valuations for outgoing and incoming calls are positively correlated and the firm is not allowed to charge

[19] The case with call externalities only seems inappropriate in the context of telecommunications. An illustrative example is provided in the working paper version of this article, Hahn (2002a), which contains the comparison between the nonlinear tariff and the two-part tariff under a certain parametric specification.

[20] In fact, replacing n with a positive constant will lead to the same result.

incoming calls. It has been shown that the presence of call externalities results in downward distortions of the outgoing call quantity for all types of subscribers including the highest type. The firm's screening capability is limited under the no receiver charge rule, i.e. the firm cannot fully control the quantity of incoming calls. Given that consumers' valuations for outgoing and incoming calls are positively correlated, with the quantity of incoming calls as the only screening instrument it is optimal for the firm to partially sacrifice the efficient internalisation of call externalities for the sake of consumer screening. The presence of call externalities may also induce the firm to serve some low-type subscribers who only receive incoming calls without making any outgoing call in equilibrium. Also, the firm may have incentives to sell the service below costs to some low-valuation consumers in order to take advantage of the effect of network externalities. Although we have considered only two positive consumption externalities closely related to telecommunications, the model and techniques can be easily applied to situations with other types of externalities, such as negative congestion externalities in roads or networks, with some modifications.

A remaining task would be to examine what happens if the firm is allowed to charge incoming calls. In this case, it is natural to assume that consumers are given the right to choose the quantity of incoming calls. Then, the receiver charge rule may have the potential to improve on the efficiency of outgoing-call allocation, although we need to examine more carefully the interaction between outgoing and incoming calls in determining the optimal tariff. Also interesting is to relax the assumption that the valuations for outgoing and incoming calls are perfectly correlated, and examine the case where consumer preferences for incoming and outgoing calls are represented by two different parameters. Then, the model corresponds to a multidimensional screening with externalities, which is technically much more involved. Here, an interesting question would be whether the optimal nonlinear tariff leads to the standard efficiency-on-the-top result. Nowadays we often observe more than one telecommunications service providers in many countries. So, it is certainly worthwhile to extend the model to a competitive setting. In this case, an important issue is the interconnection or compatibility decisions between the competing networks and related regulatory policies. Early work on the interconnection issue suggests that individual firms' profit is independent of access charges and therefore the efficient allocation of outgoing calls can be achieved, unless the tariff structure is restricted to uniform pricing (see Armstrong (1998) and Laffont-Rey-Tirole (1998), and also

Dessein (2002) and Hahn (2002b) for some extensions). If we allow for partial participation (which seems more appropriate in the presence of network externalities), however, the profit-neutrality result is likely to fail (see Poletti and Wright (2000)). It will be interesting to see whether the profit-neutrality and efficient allocation results are retained in the presence of call externalities, and whether access charges can be used as a policy instrument to internalise call externalities. For example, Jeon-Laffont-Tirole (2001) show that the efficient allocation result holds in the case of homogeneous consumers if the firms are allowed to charge for incoming calls. In another line of research, Economides-Lopomo-Woroch (1996a,b) point out that access charges can be used by a dominant firm as a strategic tool to raise rivals' costs or even to monopolise the market, and the imposition of a reciprocity rule can eliminate the strategic power. In such a context, an interesting question is how the presence of call externalities affects the strategic advantage of the dominant firm. My initial conjecture is that the call externality may reduce the price-squeeze and the monopolisation incentives of the dominant firm. Since consumers get benefits from receiving incoming calls, a small network may choose to focus on a market segment consisted of consumers with low valuations for outgoing calls, avoiding large access payments to the dominant network.

5 Appendix

5.1 Proof of lemma 1

From condition (3), note that $v(\cdot)$ is strictly increasing given that $n^a > 0$ and $\alpha(\theta) > 0$. Then, given t and n^a we can find θ_m such that $v(\theta_m) = 0$, i.e.

$$\theta_m u(q(\theta_m)) + \alpha(\theta_m)\widetilde{q} - t(q(\theta_m)) = 0, \tag{16}$$

where $q(\theta_m)$ is a type-θ_m consumer's optimal per-person outgoing-call quantity. Here θ_m is the type of the marginal consumers who are indifferent between subscribing or not. Suppose that θ_m satisfying (16) is unique. Then, consumers of type higher than θ_m will subscribe, and those with type lower than θ_m will not. Since $v(\cdot)$ is increasing, the *ex post* subscription level n^p is given by the identity:

$$n^p \equiv 1 - F(\theta_m). \tag{17}$$

Equilibria are characterised by equations (16) and (17). The fulfillment of consumer expectations requires that $n^a = n^p = n$. For a given tariff, equation (16) implicitly gives us a (unique) marginal type, independently of n^a. Then, the ex post subscription level n^p, which is obtained from (17) using the corresponding marginal type, is required to be equal to the original consumer expectation n^a.

In the (n, θ_m)-plane, equation (16) represents a horizontal line (depicted as the solid line in Figure 1) moving up and down as the tariff changes. Equation (17) represents a monotonic decreasing curve (depicted as the dashed line in Figure 1) which is fully characterised by $F(\cdot)$. The equilibrium corresponds to the intersection of the two lines, and is unique and stable as shown in Figure 1 below. Here an equilibrium is said to be stable when the market converges to the original equilibrium for a small perturbation of consumer expectation.

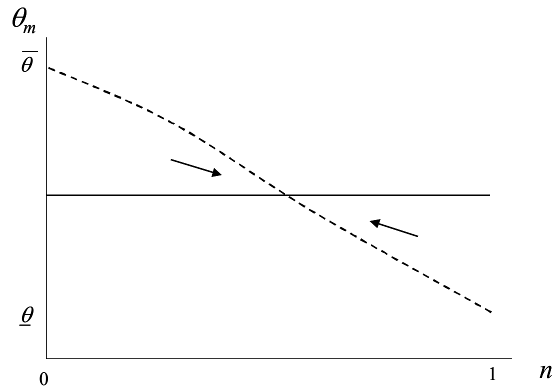

Figure 1: The fulfilled-expectation equilibrium

5.2 Proof of proposition 1

Given n, the *social* marginal cost of providing a unit of outgoing call is given by $c - \omega(n)$, where $\omega(n) = \frac{1}{n} \int_{F^{-1}(1-n)}^{\overline{\theta}} \alpha(\theta) f(\theta) d\theta$ reflects the effect of call externalities. Under the uniform calling pattern, the marginal social benefit of allowing one additional unit of incoming call in the network should be evaluated as the average valuation for the marginal unit of incoming call among the given subscribers with mass n. Comparing $\eta(n)$ and $\omega(n)$, we have

$$\omega(n) - \eta(n) = \frac{1}{n} \int_{F^{-1}(1-n)}^{\overline{\theta}} \tfrac{1-F(\theta)}{f(\theta)} \alpha'(\theta) f(\theta) d\theta$$

which is positive given $\alpha'(\theta) > 0$. So, the monopoly marginal price in (9) is always greater than the social marginal cost (i.e. $c - \eta(n) > c - \omega(n)$). Therefore, from (9) it is clear that all types of subscribers consume inefficiently low quantities of outgoing calls.

5.3 Proof of lemma 2

Since $\alpha(\cdot) - \frac{1-F(\cdot)}{f(\cdot)}\alpha'(\cdot)$ is increasing in θ given $\alpha'(\cdot) > 0$, $\alpha''(\cdot) \geq 0$, and condition (12), it must be that $\eta'(n) = \dfrac{1}{n^2}\{\, n[\alpha(F^{-1}(1-n)) - \frac{n}{f(F^{-1}(1-n))}\alpha'(F^{-1}(1-n))] - \int_{F^{-1}(1-n)}^{\overline{\theta}}[\alpha(\theta) - \frac{1-F(\theta)}{f(\theta)}\alpha'(\theta)]f(\theta)d\theta \,\} < 0$.

5.4 Proof of lemma 3

Given $\eta'(n) < 0$ and the monotonic hazard rate condition in (12), condition (11) implies that $\widetilde{\theta}'(n) > 0$. Then, by definition $\widetilde{n}(\cdot)$ must be monotonic decreasing, which is sufficient to prove that the solution to $\widetilde{n}(x) = x$ is unique and between 0 and 1 given that $\widetilde{n}(0) > 0$ and $\widetilde{n}(1) < 1$.

5.5 Proof of proposition 3

Consider the case of $n^* < \overline{n}$. The sign of the inframarginal effect is not clear. But, suppose that it is positive. Then, the profit obtainable from the marginal type of consumers, which is given by the second term of (14), must be negative in order to satisfy the first-order condition. This negative marginal effect immediately implies that serving the marginal type of consumers actually make deficits. Then, by continuity the firm sells the service below its costs to a range of low types in the equilibrium. Similarly for the case of $n^* \geq \overline{n}$.

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