Input price discrimination with downstream Cournot competitors

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Abstract: This paper addresses third-degree price discrimination in input markets. I decompose the upstream monopolist's profit into two parts, one that depends on average input prices, and one that depends on their distribution. This decomposition is useful in comparing the welfare properties of price discrimination against uniform pricing. Under reasonable assumptions, input price discrimination negatively affects both consumer surplus and total welfare.

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1. Introduction

This paper considers the problem of third-degree price discrimination in input markets and its welfare properties. This is an important issue since in many network industries there are examples of upstream firms that sell some of their outputs as inputs to other downstream firms, who use them in their production processes. Despite the current wave of liberalization, many network segments are bound to remain natural monopolies for technological reasons (e.g., transmission grids in electricity, backbones for the Internet, local loops in telecommunications). As a result, from a public policy perspective, whether to allow price flexibility to the owner of “bottleneck” segments is still an interesting question. From a theoretical point of view, Katz (1987) and DeGraba (1990) have provided partial answers to the problem of third-degree input price discrimination, while some of their findings have recently been extended by Yoshida (2000).

In this paper I propose a novel approach; one that allows more general results to be obtained when downstream firms compete over quantities. It is well known that output and price in a Cournot industry are independent of the distribution of marginal costs whilst the distribution of costs affects profits (this property is not unique to Cournot games; see, for example, Bergstrom and Varian, 1985). This property can be used to obtain a new perspective on the problem of price discrimination. In particular, since the input price is part of the downstream cost structure, the monopolist's problem becomes that of altering the cost structure of downstream firms. I show that the upstream monopolist profit can be decomposed into two parts - one that depends on the distribution of input prices and a second that depends on the average input price. This decomposition turns out to be extremely useful to highlight the differences between the outcomes under a discriminatory regime, as opposed to a regime of price uniformity, and to characterize equilibria and their welfare properties.[1]

[1] There is a recent related work by Long and Soubeyran (2001) that analyzes a class of two-stage games where oligopolists - competing against each other in a second stage - may jointly manipulate in a first stage their marginal costs of production. In their paper there is not a vertical structure that is central in this paper, where the upstream monopolist's interests diverge from the maximization of total downstream profits. Long and Soubeyran (1999) also employ a similar methodology in another paper that does include a vertical structure, but their focus is on different issues, such as the incentive for downstream firms to self-supply access, while they do not study the welfare properties of input price discrimination.

2. The model

An upstream market is monopolized by an incumbent firm. The monopolist produces, at a constant marginal cost, an essential input that is supplied to a downstream sector. The upstream marginal cost is normalized to zero.[2] There are n downstream firms that produce an homogeneous good and compete in quantities. Following the notation of Yoshida (2000), in order to produce a unit of the final good, downstream firm i requires $\alpha_i > 0$ units of the essential input, and some other inputs which are combined in fixed proportion and cost β_i. If the monopolist supplies the input at a unit wholesale price w_i,[3] then firm i's marginal cost is $c_i = k_i + \beta_i$ where $k_i = \alpha_i w_i$ is a "weighted" input price. If discrimination is not allowed, $w_i = w$, $i = 1, \ldots, n$. Cost parameters can have a generic distribution. Given two random variables a_i and b_i, $i = 1, \ldots, n$, let $\bar{a} = \sum_{i=1}^{n} a_i / n$, $\sigma_a^2 = \sum_{i=1}^{n} (a_i - \bar{a})^2 / n$, $\sigma_{ab} = \sum_{i=1}^{n} (a_i - \bar{a})(b_i - \bar{b}) / n$ and $\rho = \sigma_{ab} / (\sigma_a \sigma_b)$ denote respectively the average, the variance, the covariance and the degree of correlation.

Let q_i and x_i denote the quantities respectively supplied by firm i in the final market and demanded by firm i in the input market ($x_i = \alpha_i q_i$), $Q = \sum_{i=1}^{n} q_i$ and $X = \sum_{i=1}^{n} x_i$. The demand for the final good is $P(Q)$, $P' < 0$.

I consider a two-stage game. The monopolist first sets input price(s), then downstream firms set quantities. In the last stage, I restrict attention to interior equilibria. The profit of a generic downstream firm is $\pi_i^d = (P(Q) - c_i) q_i$, and the relative first-order condition is:

(1) $$\frac{\partial \pi_i^d}{\partial q_i} = P'(Q) q_i + P(Q) - c_i = 0 .$$

By summing FOCs over n, I get:

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[2] Results would not change if the monopolist produces according to a (weakly) convex cost function $C(\cdot)$ when aggregate input is constant in the two pricing regimes. See Yoshida (2000) and Lemma 3 below.

[3] I am ruling out the use of fixed charges. If the monopolist could charge discriminatory two-part input prices the solution would be straightforward. The monopolist would select the most efficient downstream firm for a given cost configuration, charge to that firm a variable component equal to the true marginal cost, and extract all the (monopoly) profit with the fixed component. All the other downstream firms would be excluded from production by offering them excessively high input prices.

(2) $$P'(Q)Q + nP(Q) = n(\bar{k} + \bar{\beta}).$$

Eq. (2) shows that the equilibrium industry output (and consumer surplus) depends only on the sum of downstream marginal costs, not on the distribution. I assume that $(n+1)P'(Q) + P''(Q)Q < 0$ holds globally, which is a sufficient condition for the uniqueness of the Cournot equilibrium.[4] Denote by $Q(\bar{k})$ the solution to eq. (2); $Q(\bar{k})$ is well-defined and it decreases in $\bar{k}$:

$$Q' = \frac{\partial Q}{\partial \bar{k}} = \frac{n}{(n+1)P'(Q) + P''(Q)Q} < 0.$$

Using eq. (1) it is possible to obtain that the expression for the total downstream profit is an increasing function of the variance of the distribution of marginal costs:

(3) $$\pi^d = \sum_{i=1}^{n} \pi_i^d = \frac{\sum_i^n (P(Q(\bar{k})) - c_i)^2}{-P'(Q(\bar{k}))} = \frac{n[(P(Q(\bar{k})) - \bar{c})^2 + \sigma_c^2]}{-P'(Q(\bar{k}))}.$$

From eq. (1) it is also easy to derive the expression taken by welfare, given by the sum of producers' profits and net consumer surplus, when downstream firms produce at equilibrium:

(4) $$W = \int_0^Q P(t)\mathrm{d}t - \sum_i^n \beta_i q_i = \int_0^{Q(\bar{k})} [P(t) - \bar{\beta}]\mathrm{d}t + \frac{n(\sigma_\beta^2 + \sigma_{\beta k})}{-P'(Q(\bar{k}))}.$$

Notice how welfare and downstream profits depend on both the mean and the variance of downstream marginal costs. Hence one has to be careful when basing welfare analysis on changes of final price and output alone.

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[4] I also assume that the downstream costs c_i fall in a range such that the equilibrium output is positive for all firms, which implies that the total number of downstream firms is considered as exogenous in this paper. See Vives (1999) for sufficient conditions that ensure existence and uniqueness of equilibria in Cournot games.

Since the upstream monopolist can appropriate part of downstream revenues using wholesale prices, the previous results suggest that the monopolist will have an interest in manipulating downstream costs in order to alter their variance. To understand the monopolist's incentives, the following result turns out to be very useful (all the proofs are in the Appendix):

Lemma 1. *The upstream monopolist profit can be decomposed into two parts; one that depends only on the average "weighted" input price ($\bar{k}$) and one that depends on the distribution of "weighted" input prices ($k_i = \alpha_i w_i$):*

$$\text{(5)} \qquad \pi^u = \frac{n[(P(Q(\bar{k})) - \bar{\beta})^2 + \sigma_\beta^2]}{-4P'(Q(\bar{k}))} - \frac{\sum_{i=1}^n [k_i - (P(Q(\bar{k})) - \beta_i)/2]^2}{-P'(Q(\bar{k}))}.$$

The separable structure of the monopolist's profit given by eq. (5) points to a two-step determination of the input prices when price discrimination is allowed. In the first step the monopolist determines the optimal distribution of prices for a given average weighted input price; in the second step the monopolist sets the optimal $\bar{k}$.

The first step simply corresponds to the minimization of the loss associated with the second term in eq. (5), that is:

$$\min_{k_i} \sum_{i=1}^n [k_i - \frac{P(Q(\frac{k_1+...+k_n}{n})) - \beta_i}{2}]^2$$
$$\text{s.t.}\,(k_1 + ... + k_n)/n - \bar{k} = 0$$

obtaining:

$$k_i - (P(Q(\bar{k})) - \beta_i)/2 = \text{constant.}$$

By summing the previous equation over *n*, one gets:

$$\text{(6)} \qquad k_i = \alpha_i w_i = \bar{k} - (\beta_i - \bar{\beta})/2,$$

which shows that lower cost firms are charged higher 'weighted' input prices. This is a generalization of the result of DeGraba (1990), obtained in the setting of a duopoly with linear demand and identical α-efficiency.

Given eq. (6), we can manipulate eq. (5) to obtain the following expression for the profit of the monopolist in the second step of the maximization procedure:

$$(7) \qquad \pi^{u,d}(\bar{k}) = \frac{n[\bar{k}(P(Q(\bar{k})) - \bar{k} - \bar{\beta}) + \sigma_\beta^2 / 4]}{-P'(Q(\bar{k}))},$$

where the superscript d stands for “discrimination”. We can also use eq. (6) to obtain the total downstream profit under discrimination:

$$(8) \qquad \pi^{d,d}(\bar{k}) = \frac{n[(P(Q(\bar{k})) - \bar{k} - \bar{\beta})^2 + \sigma_\beta^2 / 4]}{-P'(Q(\bar{k}))}.$$

Finally, the total industry profit and total welfare under discrimination are:

$$\pi^{tot,d}(\bar{k}) = \pi^{u,d}(\bar{k}) + \pi^{d,d}(\bar{k}) = \frac{n[(P(Q(\bar{k})) - \bar{k} - \bar{\beta})(P(Q(\bar{k})) - \bar{\beta}) + \sigma_\beta^2 / 2]}{-P'(Q(\bar{k}))}$$

$$(9) \qquad W^d(\bar{k}) = \int_0^{Q(\bar{k})} [P(t) - \bar{\beta}]\mathrm{d}t + \frac{n\sigma_\beta^2 / 2}{-P'(Q(\bar{k}))}.$$

In case discrimination is not allowed, the following result holds true:

Lemma 2. *When input prices are uniform, the monopolist profit, the total downstream profit, the total industry profit and welfare can be written as:*

$$(10) \qquad \pi^{u,u}(\bar{k}) = \pi^{u,d}(\bar{k}) - L(\bar{k}) / (-P'(Q(\bar{k})))$$

$$(11) \qquad \pi^{d,u}(\bar{k}) = \pi^{d,d}(\bar{k}) + [L(\bar{k}) + G(\bar{k})] / (-P'(Q(\bar{k})))$$

$$\pi^{tot,u}(\bar{k}) = \pi^{tot,d}(\bar{k}) + G(\bar{k}) / (-P'(Q(\bar{k})))$$

$$(12) \qquad W^u(\bar{k}) = W^d(\bar{k}) + G(\bar{k}) / (-P'(Q(\bar{k})))$$

where

$$L(\bar{k}) = n(\bar{k}^2 \sigma_\alpha^2 / \bar{\alpha}^2 + \sigma_\beta^2 / 4 + \rho \bar{k} \sigma_\alpha \sigma_\beta / \bar{\alpha}) > 0$$

$$G(\overline{k}) = n(\sigma_\beta^2 / 2 + \rho\overline{k}\sigma_\alpha\sigma_\beta / \overline{\alpha}) .$$

The decomposition that I have conducted allows one to obtain quite neat results in terms of the differences between price discrimination and uniform pricing. The monopolist's incentive to induce different levels of final output in the two regimes impinges entirely upon the extra term in eq. (10). It is clear that the two regimes differ so long as there is some diversity in the cost parameters of downstream firms. If downstream firms were identical, then $L(\cdot) = 0$ and there would be no reason for the upstream monopolist to discriminate. From now onwards I will assume that there is some diversity among downstream firms. In particular, it can be shown that $L(\cdot) > 0$, independently from the correlation between cost parameters. $L(\cdot)$ can be interpreted as a "loss function" due to the reduced instruments at the monopolist's disposal under a uniform pricing regime. This interpretation is not very precise, however, since it is not clear at all if the monopolist would choose the same average weighted charge with and without discrimination.

Call k^d the optimal average weighted input price when discrimination is allowed (from the maximization of eq. (7)) and k^u the optimal solution when discrimination is not permitted (from the maximization of eq. (10)). Recall that total output depends only on the weighted average price. Whether the monopolist would choose different weighted prices in the two regimes depends on the second term in eq. (10). We are now in a position to state the following:

Proposition 1. *When demand is linear and there is no α-difference ($\sigma_\alpha = 0$), then $k^d = k^u$ and total output is unchanged in the two regimes. However total industry profits and total welfare are both decreased by price discrimination.*

Other authors (Katz, 1987; DeGraba, 1990; Yoshida, 2000) have already obtained this result, according to various degrees of generalization. The intuition for the basic case is simple. Imagine there are only two downstream competitors that pay the same input price to the upstream monopolist. The firm with the lower cost of the other inputs sells more units than the one with the higher cost. Now suppose the monopolist raises the input price by a small amount to the low cost firm and lowers the input price by the same amount to the high cost firm. In the new equilibrium, average downstream costs are unchanged and so is equilibrium output. However, the monopolist's profits increase because he makes additional profits on the units sold by the low cost firm that more than compensate for the lower profits from the high cost firm. This is detrimental to total welfare since total output is unchanged but the "wrong" firm is now producing more than before. This argument cannot be replicated in such a clean way when firms differ also in α-efficiency or when demand is not linear. This is why it is helpful to decompose the problem. By looking at the extra term in eq. (10), it allows one to state the following results:

Proposition 2. *When demand is strictly concave and there is no α-difference ($\sigma_\alpha = 0$), then $k^d > k^u$ and consumer surplus is decreased by price discrimination. Limited concavity is a sufficient condition for welfare to decrease as well under discrimination.*

Proposition 3. *When demand is (weakly) concave or not "too" convex and there is no β-difference ($\sigma_\beta = 0$), then $k^d > k^u$ and both consumer surplus and welfare are decreased by price discrimination.*

Proposition 4. *When demand is (weakly) concave or not "too" convex, a sufficient condition for price discrimination to decrease consumer surplus is positive correlation between cost parameters. This is also sufficient to produce a negative impact on welfare for limited convexity/concavity of the demand function.*

It is worth stressing that the decomposition method that we have employed allows us to obtain welfare comparisons without having to rely on direct maximization of both eq. (7) and eq. (10). Of course, the optimal weighted input prices in the two regimes could be derived, but welfare comparisons would be more cumbersome. Propositions 2-4 are quite general and encompass all the findings of Yoshida (2000) that obtains them with linear demand and when the downstream firms can be ordered in α-β-efficiency, a special case of positive correlation between cost parameters. If demand is linear, we can obtain the following:

Proposition 5. *Suppose demand is linear and there is no change in output, then the change in welfare due to discrimination is strictly negative, unless there is perfectly negative correlation ($\rho = -1$), in which case there is also no change in welfare.*

Proposition 5 highlights a difference between the 2-firm case and the case with 3 or more firms. With 2 firms, if cost parameters are negatively correlated, they are obviously perfectly negatively correlated, in which case if there is no change in output then there is also no change in total welfare. On the other hand, with 3 or more firms, the condition that output is unaffected by discrimination implies that welfare decreases, unless we are in the very peculiar situation of perfectly negative correlation.[5] Sticking to the linear demand case, it is possible to obtain a more complete characterization of the equilibria and their properties:

Lemma 3. *Let demand be linear, P(Q) = A – BQ. The optimal "weighted" input prices under a discriminatory regime and under a uniform regime, and the change in total welfare between the two regimes are:*

$$(13) \qquad k^d = (A - \overline{\beta})/2$$

$$(14) \qquad k^u = \frac{\overline{\alpha}[\overline{\alpha}(A-\overline{\beta}) - (n+1)\rho\sigma_\alpha\sigma_\beta]}{2[\overline{\alpha}^2 + (n+1)\sigma_\alpha^2]}$$

$$(15) \qquad \Delta W = W^d - W^u = (k^d + k^u + B\frac{Q^d + Q^u}{n})\frac{Q^d - Q^u}{2} - n(k^u \rho \frac{\sigma_\alpha}{\overline{\alpha}} + \frac{\sigma_\beta}{2})\frac{\sigma_\beta}{B}.$$

The aggregate input X supplied by the monopolist does not change in the two regimes.

Notice that to obtain the input price charged to firm *i* under a discriminatory regime it is sufficient to substitute eq. (13) into eq. (6). Lemma 3 finds again the result of Yoshida (2000) that price discrimination has no effect on the aggregate quantity *X* supplied by the upstream monopolist. In addition, a complete welfare analysis is now possible. We are able to determine whether quantity and welfare rise or fall by price discrimination in general cases of any order in α-β-efficiency, as it is shown in the following final result:

Proposition 6. *When demand is linear, a sufficient condition for discrimination to have a negative impact on welfare is that correlation is not too negative (*$-0.943 \approx -\sqrt{8/9} < \rho \leq 1$*). When cost parameters are sufficiently negatively correlated (*$-1 \leq \rho < -\sqrt{8/9}$ *) welfare may only increase in cases where final output falls.*

[5] I do not want to argue that negative correlation is unlikely, since one could counterargue that if an active firm in a market is significantly α-inefficient, it must be rather β-efficient to remain in the market, and then the case of negative correlation is reasonable. However, the case of perfectly negative correlation does seem a bit of a knife-edge case. Also, notice how welfare results here depend on the value taken by the correlation coefficient of cost parameters. This type of information is easier to obtain from industry studies, international benchmarks, etc., than from observing directly downstream costs.

At first sight, the last part of Proposition 6 seems in stark contrast with Varian's finding that an output increase is a necessary condition for price discrimination to improve welfare (Varian, 1985). The difference comes from the fact that the economic environments considered by his paper and by this one are not directly comparable. Varian presents a model of third-degree price discrimination in a final good market, while I consider an input market, i.e., a situation where buyers are strategic since they compete against each other once they have purchased the essential input from the upstream monopolist. Still, it is of some interest to see why his approach could produce different results here. In my model, there is only one final market, so it is obvious that the change of consumer surplus is bounded as follows:

$$Q^d (P^u - P^d) \geq \Delta CS = CS^d - CS^u \geq Q^u (P^u - P^d).$$

In Varian's model there are many segmented final markets, but he shows that the previous inequalities still hold.[6] In my model, the upstream good is produced at zero cost, hence total industry profits in any regime are given by $PQ - \sum_{i=1}^{n} \beta_i q_i$ and bounds on welfare become:

$$\sum_{i=1}^{n} (P^u - \beta_i) \Delta q_i \geq \Delta W = W^d - W^u \geq \sum_{i=1}^{n} (P^d - \beta_i) \Delta q_i$$

where $\Delta q_i = q_i^d - q_i^u$ corresponds to the change in downstream firm *i*'s output in my model and to the change in market *i*'s output in Varian's model. In Varian's work, production costs are identical in all markets (say equal to β): it is immediate to derive from the first inequality that $(P^u - \beta) \sum_{i=1}^{n} \Delta q_i \geq \Delta W$, which gives his necessary condition: price discrimination happens in the final markets and it misallocates the marginal units across markets, hence an increase in overall output is necessary to compensate for this inefficiency.

[6] To be precise, Varian works with indirect utility rather than with consumer surplus and applies duality theory. Moreover, since he has many markets, the expressions should be written in vector notation, e.g., $\boldsymbol{Q}^u (\boldsymbol{P}^u - \boldsymbol{P}^d)$ is the scalar product between the vector of quantities in the various market under price uniformity and the vector of changes in prices in the corresponding market, and so on.

In my model there is no reason to discriminate in the final market, since there is only one market. Price discrimination happens in the wholesale market, where the upstream monopolist can exploit different elasticities of the downstream firms' derived demands. Since downstream costs are different, the price-cost margin cannot be factored out from the left inequality above and Varian's necessary condition does not apply here. Using the results derived in this section, one can derive an upper bound on welfare change for the problem of input price discrimination under linear demand:

$$\Delta W \leq \sum_{i=1}^{n}(P^u - \beta_i)\Delta q_i = \sum_{i=1}^{n}(P^u - \beta_i)\frac{\Delta P - \Delta c_i}{-P'} =$$
$$\sum_{i=1}^{n}(P^u - \beta_i)\frac{(\alpha_i - \overline{\alpha})\frac{\overline{k}^u}{\overline{\alpha}} + \frac{\beta_i - \overline{\beta}}{2} - P'\Delta Q / n}{-P'} = (P^u - \overline{\beta})\Delta Q - \frac{n}{-P'}(\frac{\sigma_\beta^2}{2} + \frac{\overline{k}^u}{\overline{\alpha}}\sigma_{\alpha\beta}).$$

It is then clear that *if* total output decreases, and there is positive (or not 'too' negative) correlation between cost parameters, then consumer surplus and welfare both unambiguously decrease.

To see why discrimination indeed produces negative effects on output and welfare under many circumstances, consider eq. (6). Imagine first that all firms have the same α-efficiency. We know from Proposition 1 that total output does not change, but the more efficient firms (low β) are penalized by discrimination since they are charged more than the less efficient rivals. There is a primary source of productive inefficiency here that causes a distortion of output among downstream firms. The firms with high β's tend to produce too much under discrimination.

Imagine now there is a positive correlation between the cost parameters. The previous effect gets even worse, since the actual input price paid by a firm is $w_i = k_i/\alpha_i$, hence a 'good' firm ends up being overcharged. In fact - on top of the misallocation of output among downstream firms - total output decreases making consumers worse off (Proposition 4).

Since the essential input is produced at zero cost by the upstream monopolist, only β-efficiency matters for productive efficiency; however, the downstream strategic interaction among firms implies that α-efficiency has an impact on final output. This is the key to understand why, under some special circumstances, it may be the case that welfare is increased under discrimination. This can happen only when there is a strong negative correlation between cost parameters, so that a firm with an efficient β parameter, despite being penalized by discrimination on the 'weighted' input price (eq. (6)), may end up paying a lower input price $w_i = k_i/\alpha_i$. This can have a positive effect on welfare since output is reallocated - in a discriminatory regime - from high to low β firms. This positive reallocation effect is not present in Varian's analysis.

However, it is not always true that the input price is decreased to a low β firm under discrimination and strong negative correlation. In fact the reverse can also happen, which explains the apparent departure form Varian's result. In the case of negative correlation, price discrimination can cause an increase in total output (so that consumers benefit), but the market shares of downstream firms are distorted: this situation arises when discrimination favors a firm that is α-efficient but β-inefficient. Since only the latter type of efficiency matters for total welfare on the production side, the welfare gain from a higher total output is offset by a worse reallocation of production.[7]

3. Extension: product differentiation and demand shocks

In this section I introduce a model with product differentiation and idiosyncratic demand and cost shocks. In particular, I assume a quadratic utility function, quasilinear with respect to the numeraire good (money) y:

$$U = \sum_{i=1}^{n}(A-\theta_i)q_i - \frac{1}{2}[(B+D)\sum_{i=1}^{n}q_i^2 + B\sum_{i=1}^{n}q_i\sum_{\substack{j=1\\ j\neq i}}^{n}q_j] + y,$$

generating the following demand for firm i:

$$P_i = A - \theta_i - (B+D)q_i - B\sum_{\substack{j=1\\ j\neq i}}^{n}q_j$$

where θ_i is a firm-specific parameter and $D > 0$ is a parameter of product differentiation. I consider this case for two related reasons. Firstly, I want to show that the previous

[7] The Appendix contains some numerical results that describe the case of perfectly negative correlation.

analysis can be extended to take into account firm-specific factors that arise also from the demand side rather than from the cost side alone. Secondly, the example is solved to show that the decomposition methodology can be applied also to Bertrand games with symmetric product differentiation. Averaging over the various inverse demands:

$$\overline{P} = A - \overline{\theta} - (nB + D)\overline{q},$$

$$P_i - \overline{P} = -(\theta_i - \overline{\theta}) - D(q_i - \overline{q}).$$

The cost structure is as in Section 2. The profit of a generic downstream firm is $\pi_i^d = (P_i - c_i)q_i$, and the relative first-order condition w.r.t. quantity is:[8]

$$\text{(16)} \qquad \frac{\partial \pi_i^d}{\partial q_i} = -(B + D)q_i + P_i - c_i = 0.$$

By summing FOCs over *n*, I get:

$$\text{(17)} \qquad \overline{q}(\overline{k}) = (\overline{P} - \overline{c})/(B + D) = (A - \overline{\theta} - \overline{\beta} - \overline{k})/[(n+1)B + 2D].$$

Once again, equilibrium total output is determined only by average demand and cost values and not by their distribution. Comparison of eq. (16) and eq. (17) leads to:

$$q_i - \overline{q}(\overline{k}) = -\frac{(\theta_i - \overline{\theta}) + (c_i - \overline{c})}{B + 2D},$$

$$\sigma_q^2 = \frac{\sigma_\theta^2 + \sigma_c^2 + 2\sigma_{c\theta}}{(B + 2D)^2}.$$

where σ_q^2 denotes the variance of equilibrium downstream quantities. Total downstream profits and net consumer surplus are (notice that I am now working with

[8] I still work with quantity competition simply in order to obtain results that are directly comparable with Section 2. However, it is immediate to notice that, switching the role of prices and quantities, one gets a model where $q_i = A' - \theta_i' - D'P_i - nB'\overline{P}$. Letting $B' < 0$ one has a model where products are gross substitutes (and competition of the strategic complementarity variety) and the analysis would proceed along the same lines illustrated below.

quantities rather than with price/cost differences - but this is entirely equivalent to the previous analysis):

(18) $$\pi^d = \sum_{i=1}^n \pi_i^d = (B+D)\sum_i^n (q_i)^2 = n(B+D)(\overline{q}(\overline{k})^2 + \sigma_q^2),$$

(19) $$CS = U - \sum_{i=1}^n P_i q_i = [B(\sum_i^n q_i)^2 + D\sum_i^n (q_i)^2]/2 = n[(nB+D)\overline{q}(\overline{k})^2 + D\sigma_q^2]/2.$$

Eq. (18) corresponds to the similar eq. (3) in Section 2 and it has a similar interpretation. On the other hand, eq. (19) shows a new feature, due to product differentiation. Consumer surplus now depends not only on the average of downstream parameters, but also on their variance. The latter effect is bigger the more differentiated the products are. Turning to the upstream monopolist, its profit is:

(20) $$\pi^u = \sum_{i=1}^n w_i x_i = \sum_{i=1}^n k_i(\overline{q} + q_i - \overline{q}) = n\overline{k}\overline{q}(\overline{k}) - \sum_{i=1}^n k_i \frac{(\theta_i - \overline{\theta}) + (k_i + \beta_i - \overline{k} - \overline{\beta})}{B+2D} =$$
$$= n\overline{k}\overline{q}(\overline{k}) + \frac{n\overline{k}(\overline{k} + \overline{\theta} + \overline{\beta})}{B+2D} - \frac{\sum_{i=1}^n k_i(k_i + \theta_i + \beta_i)}{B+2D}$$

We find a result here analogous to Lemma 1 and we can apply the same procedure, i.e. the monopolist sets the distribution of prices for a given average. In the present context, this reduces to $\min_{k_i} \sum_{i=1}^n k_i(k_i + \theta_i + \beta_i)$ s.t. $\sum_{i=1}^n k_i = n\overline{k}$, getting:

(21) $$k_i = \overline{k} - (\beta_i - \overline{\beta})/2 - (\theta_i - \overline{\theta})/2.$$

This result parallels eq. (6): a "better" firm is charged higher input prices, where better now stands both for having a lower cost and a higher demand intercept. Eq. (21) allows manipulation of the last term in eq. (20) to obtain the monopolist's profits under discrimination:

$$\pi^{u,d} = n\overline{k}\overline{q}(\overline{k}) + \frac{n}{4}\frac{\sigma_\theta^2 + \sigma_\beta^2 + 2\sigma_{\beta\theta}}{B+2D},$$

where $\bar{q}(\bar{k})$ is given by eq. (17). On the other hand, if there is no discrimination, then one can directly manipulate the last term in eq. (20), using $k_i = \alpha_i \bar{k} / \bar{\alpha}$, to get:

$$\pi^{u,u} = \pi^{u,d} - L(\bar{k})/(B+2D)$$

(22) $$L(\bar{k}) = n(\bar{k}^2 \sigma_\alpha^2 / \bar{\alpha}^2 + \sigma_\beta^2 / 4 + \sigma_\theta^2 / 4 + \bar{k}\sigma_{\alpha\beta} / \bar{\alpha} + \bar{k}\sigma_{\alpha\theta} / \bar{\alpha} + \sigma_{\beta\theta} / 2) .$$

As in Section 2, we can obtain the variance of downstream shares with and without discrimination, resulting in the following expressions for the downstream total profits:

$$\pi^{d,d} = n(B+D)[\bar{q}(\bar{k})^2 + (\sigma_\theta^2 / 4 + \sigma_\beta^2 / 4 + \sigma_{\beta\theta} / 2)/(B+2D)^2]$$

$$\pi^{d,u} = \pi^{d,d} + [L(\bar{k}) + G(\bar{k})](B+D)/(B+2D)^2$$

$$CS^u = CS^d + D[L(\bar{k}) + G(\bar{k})]/[2(B+2D)^2]$$

$$G(\bar{k}) = n(\sigma_\beta^2 / 2 + \sigma_\theta^2 / 2 + \bar{k}\sigma_{\alpha\beta} / \bar{\alpha} + \bar{k}\sigma_{\alpha\theta} / \bar{\alpha} + \sigma_{\beta\theta}) .$$

In general, whether the monopolist would choose different values for average *k* depends on the term $L(\cdot)$ given by eq. (22). We are not interested here to solve the full case (although it is feasible in this example with linear demand). However we can immediately obtain an interesting result for a simpler case. Imagine there is no α-difference. As far as the other idiosyncratic parameters are concerned, one cannot claim a priori that β and θ should be positively or negatively correlated. A firm may be better than the rival in both components (positive correlation). Conversely, a firm with a higher cost may be of a better quality, having a higher demand intercept (low θ, hence a case of negative correlation). Still, welfare results are not ambiguous. To see why, notice that with no α-difference, $L(\bar{k}) = n(\sigma_\beta^2 + \sigma_\theta^2 + 2\sigma_{\beta\theta})/4 = G(\bar{k})/2$. In addition, $L(\bar{k}) > 0$ for any value of the correlation between the two parameters. Since $L(\cdot)$ does not depend on $\bar{k}$, the ability to discriminate will not affect the average *k*. However, the monopolist reduces the variance of downstream market shares and its gain $L/(B + 2D)$ cannot compensate for the loss of downstream firms that amounts to $(L + G)(B + D)/(B$

$+ 2D)^2 = 3L(B + D)/(B + 2D)^2$, resulting in lower total industry profits that decrease by $L(2B + D)/(B + 2D)^2$. In the limit, when products are homogenous ($D = 0$), the downstream loss can be three times as much as the monopolist's gain. In addition, consumers are negatively affected when $D > 0$.[9] The following proposition summarizes the results of this section.

Proposition 7. *When downstream firms are equally efficient in treating the essential input, but have different costs of other inputs and different demand intercepts, then the upstream firm's ability to discriminate negatively affects total industry profits and consumer surplus, no matter how firm-specific demand and cost factors are correlated.*

4. Conclusion and discussion

In this paper, I have reconsidered the question of third-degree price discrimination, originally addressed in simpler frameworks by Katz (1987) and DeGraba (1990). I have adopted the same model of Yoshida (2000), but I have relied on a different solution method that decomposes the upstream monopolist's profit into two parts, one that depends on average input prices, and one that depends on their distribution. This method allows one to obtain rather general results that extend also to other settings. I have found that, under reasonable conditions, there is a rationale for banning input price discrimination since it would otherwise lead to a decrease in both consumer surplus and total welfare. Notice that this rationale is not usually mentioned in cases related to network industries, where the main concern is rather that an integrated incumbent could leverage its upstream market power to the downstream market. In this paper, I have discussed the complementary notion that input price discrimination can be harmful even if the upstream monopolist has no interest to favor a particular downstream firm.

Rather strikingly, in the linear demand case, a necessary but not sufficient condition for discrimination to have a positive impact on welfare, is to have a decrease in final total output. There could be a positive impact on welfare only in the particular situation of very strong negative cost correlation among downstream firms, i.e. the firm that is most efficient at using the monopolist's input should also be the least efficient at

[9] Distortions for consumers are non-monotonic in D. When $D = 0$ consumer surplus is not affected by discrimination. It then decreases relative to consumer surplus under no discrimination, for higher values of the differentiation parameter. However, it then converges again to the value taken by consumer surplus under no discrimination for very high values of D (in the limit, when $D \rightarrow \infty$, markets are completely separate and discrimination plays no role).

using other complementary inputs. It can be argued that this situation is unlikely. On the contrary, I have shown how, under reasonable assumptions (e.g., positive or not 'too' negative correlation among cost parameters, product differentiation with demand shocks), the ability to discriminate would have a negative impact on consumer surplus and total welfare. All in all, a simple test based on changes in final output, which is common to analyze third-degree price discrimination among final goods, can also be applied in the context of input price discrimination as a first step. If total final output decreases, there would be *prima facie* a strong case for not allowing price discrimination in input markets.

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Appendix: proofs

Proof of Lemma 1. The profit of the upstream firm is $\pi^u = \sum_{i=1}^n w_i x_i = \sum_{i=1}^n k_i q_i =$

$$\sum_{i=1}^n [P-\beta_i-(P-c_i)]q_i = P'[-\frac{\sum_{i=1}^n (P-\beta_i)q_i}{-P'} + \sum_{i=1}^n (q_i - \frac{P-\beta_i}{-2P'} + \frac{P-\beta_i}{-2P'})^2] =$$

$$P'[-\sum_{i=1}^n (\frac{P-\beta_i}{-2P'})^2 + \sum_{i=1}^n (q_i - \frac{P-\beta_i}{-2P'})^2] = \frac{\sum_{i=1}^n (P-\beta_i)^2}{-4P'} - \frac{\sum_{i=1}^n (P-c_i-\frac{P-\beta_i}{2})^2}{-P'}$$

that can be transformed into eq. (5).

Proof of Lemma 2. When discrimination is not permitted, one can write $w = \bar{k}/\bar{\alpha}$. Consider now the numerator of the last term in eq. (5). It can be rewritten as follows:

$$\sum_{i=1}^n [\bar{k}-(P-\bar{\beta})/2 + \bar{k}(\alpha_i-\bar{\alpha})/\bar{\alpha} + (\beta_i-\bar{\beta})/2]^2 =$$

$$\sum_{i=1}^n [\bar{k}-(P-\bar{\beta})/2]^2 + \sum_{i=1}^n [\bar{k}(\alpha_i-\bar{\alpha})/\bar{\alpha}]^2 + \sum_{i=1}^n [(\beta_i-\bar{\beta})/2]^2 + \sum_{i=1}^n [\bar{k}(\alpha_i-\bar{\alpha})(\beta_i-\bar{\beta})/\bar{\alpha}].$$

The last three terms represent the function $L(\cdot)$ in Lemma 2. Notice that L is increasing in ρ, hence $L \geq n(\bar{k}^2\sigma_\alpha^2/\bar{\alpha}^2 + \sigma_\beta^2/4 - \bar{k}\sigma_\alpha\sigma_\beta/\bar{\alpha}) = n(\bar{k}\sigma_\alpha/\bar{\alpha} - \sigma_\beta/2)^2 > 0$. To get eq.

(11) and (12), start with eq. (3) and (4) and note that the variance of downstream cost $c_i = w\alpha_i + \beta_i$ is $\sigma_c^2 = \bar{k}^2 \sigma_\alpha^2 / \bar{\alpha}^2 + \sigma_\beta^2 + 2\rho \bar{k} \sigma_\alpha \sigma_\beta / \bar{\alpha}$. Comparisons with eq. (9) and (11) give the result.

Proof of Proposition 1. Consider eq. (10). The FOC without discrimination is:

$$\frac{\partial \pi^{u,u}}{\partial \bar{k}} = \frac{\partial \pi^{u,d}}{\partial \bar{k}} + \frac{L'P' - P''Q'L}{(P')^2} = 0$$

$$L' = \partial L / \partial \bar{k} = n(2\bar{k}\sigma_\alpha / \bar{\alpha} + \rho \sigma_\beta)\sigma_\alpha / \bar{\alpha}.$$

By simple inspection, if demand is linear and $\sigma_\alpha = 0$, then $L' = 0$ (hence output is unchanged) but $G > 0$, hence total profits and welfare decrease under discrimination.

Proof of Proposition 2. Working along the lines of the previous proof, if $\sigma_\alpha = 0$, then $L' = 0$ and $G > 0$. If demand is concave ($P'' < 0$) then the FOC under uniformity differs from the FOC under discrimination by an additional negative term (recall that $Q' < 0$), hence $k^d > k^u$. From eq. (9) and eq. (12) the impact on welfare under discrimination of an increase in $\bar{k}$ depends on: $\partial W^d / \partial \bar{k} = Q'[(P - \bar{\beta}) + P'' n \sigma_\beta^2 / (2P'^2)]$. In general, the sign is indeterminate when demand is concave. Using eq. (2), a *sufficient* condition for having a negative impact on welfare reduces to $-P'' < 2Q(-P')^3 / (n\sigma_\beta)^2$, i.e., concavity can be negative but small in absolute value.

Proof of Proposition 3. If $\sigma_\beta = 0$, then $L' > 0$ and $G = 0$. The impact on welfare under discrimination of an increase in $\bar{k}$ is unambiguous: $\partial W^d / \partial \bar{k} = Q'(P - \bar{\beta}) < 0$. Hence $k^d > k^u$ is a necessary and sufficient condition for discrimination to adversely affect both welfare and consumer surplus. This happens so long as $L' - Q'LP''/P' > 0$, which is always true when demand is linear, concave or not "too" convex.

Proof of Proposition 4. Sufficient conditions for price discrimination to decrease output (and consumer surplus) as well as total welfare are: (i) $k^d > k^u$, (ii) $G > 0$, and (iii)

$\partial W^d / \partial \bar{k} < 0$. When demand is concave, (i) and (ii) are satisfied when $L' \geq 0$ and $G >$ 0. The previous inequalities are all trivially verified when $\rho \geq 0$. The result is still valid when $P'' > 0$ so long as $L' - Q'LP''/P' > 0$, i.e., demand is not "too" convex, in which case (iii) is also satisfied. If demand is concave we need the same condition on limited concavity as in Proposition 2 to satisfy (iii).

Proof of Proposition 5. From Lemma 2 we know that in general the monopolist sets the same average weighted input price if the extra term in eq. (10) does not affect the FOC. Hence when demand is linear, $\Delta Q = 0$ only if $L'(k^u) = 0$, and $L'(k^u) = 0$ only if (i) $\sigma_\alpha = 0$ (Proposition 1) or (ii) $k^u = -\rho\bar{\alpha}\sigma_\beta/(2\sigma_\alpha)$. Condition (ii) makes sense only in case of negative correlation. Assume it is indeed the case and $\bar{k}$ takes the previous value. As a result, final output is the same and the change of total profits and welfare is:

$$G(k^u) = \frac{n\sigma_\beta^2}{2}(1-\rho^2) > 0 \Rightarrow \Delta W = W^d - W^u = \Delta\pi^{tot} = \pi^{tot,d} - \pi^{tot,u} = -G(k^u) < 0.$$

Proof of Lemma 3. From eq. (2) it is possible to obtain the following expression for the price when demand is linear: $P = (A + n\bar{k} + n\bar{\beta})/(n+1)$. After substituting the previous expression into eq. (7) and eq. (10), direct maximisation w.r.t. $\bar{k}$ allows to obtain eq. (13) and (14) for the two regimes. Summing profits and consumer surplus gives total welfare $W = (A+P)Q/2 - \sum_{i=1}^n \beta_i q_i$. From the FOC given by eq. (1) and from eq. (6), one gets $Bq_i^d = P^d - k^d - (\beta_i + \bar{\beta})/2$ and $Bq_i^u = P^u - \alpha_i k^u/\bar{\alpha} - \beta_i$, hence:

$$\begin{aligned}
&\Delta W = (P^u + P^d)(Q^d - Q^u)/2 - \sum\nolimits_{i=1}^n \beta_i [P^d - P^u - k^d + \alpha_i k^u/\bar{\alpha} + (\beta_i - \bar{\beta})/2]/B = \\
&(P^u + P^d)(Q^d - Q^u)/2 - [n\bar{\beta}(P^d - P^u - k^d) + nk^u(\rho\sigma_\alpha\sigma_\beta + \bar{\alpha}\bar{\beta})/\bar{\alpha} + n\sigma_\beta^2/2]/B = \\
&(k^u + \bar{\beta} + BQ^u/n + k^d + \bar{\beta} + BQ^d/n)(Q^d - Q^u)/2 - \bar{\beta}(Q^d - Q^u) - n(\rho\sigma_\alpha\sigma_\beta k^u/\bar{\alpha} + \sigma_\beta^2/2)/B
\end{aligned}$$

which gives eq. (15). Finally, start from Bq_i^d and Bq_i^u previously expressed, multiply by α_i and sum over n to get:

$$-P'X^{d} = n(\overline{\alpha}P^{d} - \overline{\alpha}k^{d} - \overline{\alpha}\overline{\beta} - \rho\sigma_{\alpha}\sigma_{\beta}/2)$$
$$-P'X^{u} = n(\overline{\alpha}P^{u} - \overline{\alpha}k^{u} - \overline{\alpha}\overline{\beta} - \rho\sigma_{\alpha}\sigma_{\beta} - k^{u}\sigma_{\alpha}^{2}/\overline{\alpha}).$$

After substituting the equilibrium value for prices and weighted input prices, simple manipulations show that the RHS of the last two equations are identical, and that total input under both regimes is: $X = n[(A-\overline{\beta})\overline{\alpha} - (n+1)\rho\sigma_{\alpha}\sigma_{\beta}]/[2B(n+1)]$.

Proof of Proposition 6. Using eq. (13) and eq. (14) gives:

$$sign[\Delta Q] = sign[k^{u} - k^{d}] = sign[-(A-\overline{\beta})\sigma_{\alpha} - \rho\overline{\alpha}\sigma_{\beta}].$$

The previous expression is decreasing in A and, if correlation is negative, there is a value of the intersect parameter that makes ΔQ = 0. Denote such value by A^*. Manipulations of eq. (15) (not reported here for the sake of brevity) show that ΔW is quadratic in A, and in particular it is concave in A and may be at first increasing in A and then decreasing if the correlation parameter is negative enough (see figure 1 for a two-firm case, i.e., a case where ρ = -1). Moreover, $sign[\frac{\partial \Delta W}{\partial A}\big|_{A=A^*}] = -sign[\rho]$. In line with Proposition 5, one also gets $sign[\Delta W\big|_{A=A^*}] = sign[-(1-\rho^{2})]$. In other words, when ΔQ > 0, then necessarily ΔW < 0. On the other hand, if ΔQ < 0, then welfare may increase. To get the second part of the Proposition, ΔW takes this value at its maximum in A:

$$\Delta W^{\max} = -n\sigma_{\beta}^{2}\frac{\overline{\alpha}^{2}[(16+24n+8n^{2})(1-\rho^{2}) - n^{2}\rho^{2}] + 4\sigma_{\alpha}^{2}(n+1)^{2}(4+n)(1-\rho^{2})}{8B(n+1)[2\overline{\alpha}^{2}(2+n) + (4+5n+n^{2})\sigma_{\alpha}^{2}]}.$$

The previous expression is always negative unless the correlation parameter is negative and very high in absolute value, exceeding the following value:

$$\rho_{\lim}^{2} = \frac{4(n+1)[2\overline{\alpha}^{2}(2+n) + (4+5n+n^{2})\sigma_{\alpha}^{2}]}{\overline{\alpha}^{2}(4+3n)^{2} + 4(1+n)^{2}(4+n)\sigma_{\alpha}^{2}}.$$

The previous expression is increasing in the variance of α, hence a lower bound is found when σ_α = 0, obtaining $\rho^2_{\lim} = \dfrac{8(n+1)(2+n)}{(4+3n)^2}$ which, in turn, is decreasing in n. As a result, for discrimination to have a negative impact on aggregate welfare it is sufficient that the correlation parameter lies in the interval $\rho_{\lim} < \rho \leq 1$ where the left bound is obtained as n goes to infinity, getting $\rho^2_{\lim} = 8/9$.

Table 1 reports the equilibria for a two-firm case. Firm 1 is the low cost (i.e., low β) firm in this example. Two cases are presented according to the value taken by the intercept parameter A. In both cases, the upstream monopolist obviously benefits from the ability to input price discriminate. The first two rows (A = 1) describe a case where discrimination increases welfare. Firm 1 benefits from discrimination, via a lower input price. The low cost firm is then allocated more production of the total output and this has a positive effect on welfare, despite the reduction in total output. The last two rows (A = 0.6), on the other hand, present a case where total output increases, but such an increase is the result of price discrimination that subsidizes the α-efficient firm that is exceedingly β-inefficient, explaining the decrease in total welfare.

[Fig. 1 - An example with 2 firms. Parameters: α_1 = .15, α_2 = .12, β_1 = .05, β_2 = .2, B = 1][10]

Regime	A	w	w_1	w_2	q_1	q_2	Q	π_1^d	π_2^d	π^u	CS	W
Uniformity	1	3.215			.1738	.1202	.2940	.0302	.0145	.1301	.0432	.2180
Discrimination			3.167	3.333	.1833	.1083	.2917	.0336	.0117	.1304	.0425	.2183
Uniformity	0.6	1.786			.1262	.0298	.1559	.0159	.0009	.0402	.0121	.0692
Discrimination			1.833	1.667	.1167	.0417	.1583	.0136	.0017	.0404	.0125	.0683

[Table 1 - An example with 2 firms. Parameters as in Fig.1]

[10] The intercept parameter A has to be sufficiently high to guarantee an interior equilibrium. In this example A > .47 is sufficient to have an interior solution both under discrimination and under uniformity.

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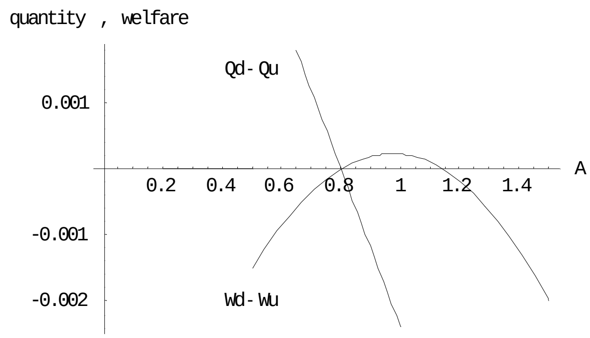

[Fig. 1 - An example with 2 firms. Parameters: α_1 = .15, α_2 = .12, β_1 = .05, β_2 = .2, B = 1]