On the Role of Input and Output Spillovers when R&D Projects are Risky

Nils Hauenschild*

University of Hamburg

Abstract

This paper analyzes the impact of input and output spillovers on the expected effective cost reductions in a two-stage model of R&D where all R&D projects are risky. It is shown that, relative to the deterministic case, output spillovers tend to reduce expected cost reductions whereas input spillovers tend to increase investment in R&D and hence expected cost reductions. In particular, the relations between cost reductions in the presence of input and output spillovers known from deterministic models may be reversed under certain parameter constellations.

Keywords: research and development, spillovers, uncertainty

JEL classification: D43, L13, O31

*Nils Hauenschild, University of Hamburg, Institute for Statistics and Econometrics, Von-Melle-Park 5, 20146 Hamburg, Germany. Phone: +49-40-428383539, Fax: +49-40-428386326, Email: hauen@econ.uni-hamburg.de

1 Introduction

During the last decade, research and development (henceforth R&D) has been one of the major fields of interest in both theoretical and applied industrial economics. Although there had been some previous studies on this subject, e.g. by Ruff (1969) and Spence (1984), most of the recent (theoretical) research was inspired by the seminal work of d'Aspremont, Jacquemin (1988) who analyzed the R&D activities of oligopolistic firms within the framework of a two-stage game.[1] One of the main features of their model (henceforth referred to as the AJ model) is the presence of output spillovers which means that part of the information about a firm's completed R&D project leaks out costlessly to its competitors who are thus able to benefit from cost reductions resulting from the successful R&D as well. A related line of research is essentially based on work by Kamien, Muller, Zang (1992) whose model (henceforth the KMZ model) is formally rather similar to the AJ model but which assumes input instead of output spillovers. Here, leakages of information occur at the research stage and prior to the discovery of a new technology which does not become available to the rival firms. Despite this formal similarity, the models yield rather different results with respect to R&D performance as the AJ model implies systematically higher cost reductions through R&D than the KMZ model unless there are no spillovers.[2] This surprising and important distinction between the two popular models has only recently been pointed out by Amir (2000).

An important common element of almost all studies building upon one variant or another of the AJ and the KMZ model is that the success of the respective R&D projects is not at stake, i.e., any firm that engages in R&D will innovate with certainty and is also perfectly informed about the extent of the associated cost reduction. In order to analyze the effects of uncertainty with respect to the success of R&D projects one usually turns to stochastic models of racing games, e.g. in the spirit of Loury (1979), Lee, Wilde (1980), Reinganum (1982), and Beath, Katsoulacos, Ulph (1988). There, however, uncertainty refers to the time until one of the firms innovates and all other R&D projects are cancelled once the first innovation occurs. It is not possible that all firms innovate or that all projects fail. Moreover, with the notable exception of Martin (2002), the role of input and output spillovers which is central to the AJ and the KMZ models is not discussed in further detail. Unlike the racing game models, the approaches taken by Choi (1993) and Combs (1993) are conceptually closer to the AJ and the KMZ model, although they still differ in various important aspects. In particular, the product market competition in the second stage of the game is not modelled explicitly.

[1] A comprehensive survey of this line of research is provided by De Bondt (1997).

[2] The original models also differ in their degree of generality because d'Aspremont, Jacquemin (1988) assume linear cost and demand functions whereas Kamien, Muller, Zang (1992) allow for general functional forms. However, the AJ model has been extended to this case by Suzumura (1992).

In this paper we consider simple stochastic versions of the original AJ and KMZ models and analyze how uncertainty with respect to the success of R&D projects influences technological performance in the sense of expected effective cost reductions. The main insight to be brought out in this analysis is that input and output spillovers cause important additional effects that are not present in a deterministic environment, and which influence technological performance in very different ways. While introducing uncertainty leads to a direct decline in R&D activities, input and output spillovers mitigate or enhance, respectively, this general tendency. The respective effects are essentially due to the possibility that the product market competition is asymmetric when only one of the R&D projects is successful whereas the other one fails.

When there are *output* spillovers, a firm with a failing R&D project may still benefit from the successful project of a competitor. From any firm's point of view, output spillovers thus act as a kind of risk sharing device, and it can be expected that some intended cost reduction is also realizable if the own R&D investment is kept at a low level. Consequently, output spillovers tend to *reduce* technological performance and hence add on the general decline in expected cost reductions which is caused by the uncertainty itself. Accordingly, any form of cooperation in R&D in which the spillover rate is set equal to one will result in a comparably low level of R&D activity under uncertainty.

The situation is completely different if there are input spillovers, because they make investment in R&D rather attractive. If a firm is the only one to succeed in R&D it benefits disproportionately more than it suffers if it is the only firm with a failing R&D project. In the former case it has lower costs and an associated advantage in the product market which is even enhanced by the part of the rival's R&D expenditures that has "spilled over", whereas in the latter case it is only faced with a disadvantage in the product market competition. Hence, input spillovers tend to *increase* technological performance and mitigate the decline in expected cost reductions caused by the introduction of uncertainty. Unlike the case of output spillovers, those forms of cooperation in which the spillover rate is set equal to one will thus result in a comparably high level of R&D activity under uncertainty.

The above effects can be so strong that the striking differences between the deterministic AJ and KMZ model derived by Amir (2000) are not significant any more or are even reversed for certain parameter constellations. Under uncertainty, the AJ model does no longer predict dramatically higher R&D levels than the KMZ model. This is particularly true for all forms of cooperation in R&D where the spillover rate is set equal to one, i.e. in research joint ventures, because the opposite effects caused by input and output spillovers are strongest in this case. Thus, introducing uncertainty with respect to the success of R&D projects alters the role of input and output spillovers that are known from deterministic models and has a non-negligible impact on the firms' R&D investments, on expected technological performance, and on the subsequent product market competition.

The paper is organized as follows. In section 2 we introduce and solve the stochastic versions of the AJ and the KMZ model for different types of cooperation and non-cooperation in R&D. Then we contrast the results obtained for the stochastic models with those of their deterministic counterparts by means of a numerical and graphical analysis in sections 3.1 and 3.2. Both models are compared with each other in section 3.3. Finally, some concluding remarks are presented in section 4.

2 The models

In this section we introduce simple stochastic versions of the well-known AJ and KMZ models with output and input spillovers, respectively. Since the corresponding deterministic models are standard in the literature we keep the exposition of their basics rather short, laying emphasis on the modifications caused by a risky R&D technology instead. In both models, two identical firms that produce a homogeneous good compete with each other in a two-stage game. In the first stage, the firms choose investment in R&D so as to reduce their unit production costs in the second stage. Unlike the standard models, however, it is assumed that the R&D technology is stochastic, i.e., the intended cost reduction only becomes effective with some probability less than one while the R&D project may also fail with positive probability.[3] In the second stage, both firms observe the outcome of the R&D projects and compete in the product market by choosing their quantities subject to a linear market demand function and their relevant linear cost functions which depend on whether R&D was successful or not. By assuming that both firms take the expected equilibrium outcome of the second stage subgame into account when deciding on their R&D investment we finally arrive at a subgame perfect equilibrium of the total game.

2.1 The stochastic AJ model

Let us first consider a stochastic version of the "classical" model of d'Aspremont, Jacquemin (1988), the main feature of which is the presence of output spillovers. In this model, the firms' original production technology is summarized by a linear cost function $c_i^0(q_i) = cq_i$, where q_i, $i = 1, 2$, is the output of firm i and where $c > 0$ denotes constant unit (and marginal) costs which are the same for both firms. In the first stage, every firm decides upon a cost reduction that becomes effective in the second stage. If a firm intends a cost reduction of $x_i > 0$, it has to pay the R&D costs $\frac{1}{2}\gamma x_i^2$ in the first stage, where $\gamma > 0$ denotes a parameter. The R&D project

[3] At first sight, this assumption appears somewhat restrictive because it only allows for complete success or complete failure of the R&D projects. However, it is also possible to analyze a variant of this model where the R&D projects may be partially successful, see footnote 9 below. The results obtained in the more general model are qualitatively similar but somewhat less pronounced, so we prefer to work with the simpler version presented here.

is risky and the intended cost reduction only becomes effective with probability θ, $0 < \theta < 1$, whereas the firm maintains its original cost function with probability $1 - \theta$.[4] We assume that both firms have access to the same R&D technology but that they carry out independent R&D projects. Hence, the probability of success θ is the same for both firms and success or failure of one firm does not affect the outcome of the other firm's R&D project. However, there are output spillovers and a cost reduction x_i for firm i also leads to a cost reduction σx_i for firm j, where $0 \leq \sigma \leq 1$ denotes the spillover rate. Hence, the final cost function of firm i is given by $c_i(q_i) = [c - x_i - \sigma x_j] \cdot q_i$, where $x_k = 0$, $k = i, j$, if the R&D project of firm k has failed (or if it did not engage in R&D, of course).

In the product market the firms are faced with the linear demand function $p(q) = a - bq$, where $q = q_i + q_j$ is total output and $a, b > 0$ are parameters. Having observed the outcome of both R&D projects, the firms decide upon outputs in the second stage so as to maximize profits π_i, where

$$\pi_i = [a - c + x_i + \sigma x_j - b(q_i + q_j)] \cdot q_i \tag{1}$$

and where failed R&D projects are formally included for $x_i = 0$ and/or $x_j = 0$.[5] Standard calculations show that the optimal quantities in a Nash-equilibrium are given by

$$q_i = \frac{1}{3b} [a - c + (2 - \sigma)x_i + (2\sigma - 1)x_j], \tag{2}$$

$$q_j = \frac{1}{3b} [a - c + (2 - \sigma)x_j + (2\sigma - 1)x_i]. \tag{3}$$

Note that the product market equilibrium is in general *not* symmetric even if both firms have chosen the same intended cost reduction $x = x_i = x_j$ because one of the projects may have failed which implies $q_i \neq q_j$. Only *expected* quantities will be equal if the first stage equilibrium is symmetric. Inserting (2) and (3) into (1), second stage profits are given by $\pi_i = bq_i^2$.

In the first stage firms maximize expected profits by choosing the intended cost reduction x_i, thereby taking the associated costs $\frac{1}{2}\gamma x_i^2$ and the resulting expected equilibrium outcome in the product market into account. Thus, they choose x_i to maximize

$$E(\Pi_i) = E\left(\pi_i - \frac{1}{2}\gamma x_i^2\right) = b \cdot E(q_i^2) - \frac{1}{2}\gamma x_i^2, \tag{4}$$

[4] One might be tempted to consider probabilities that also depend on the firms' R&D activities as in Choi (1993) and Combs (1993), but the model becomes analytically rather intractable then. Note that the other models are simplified by the fact the the firms' payoffs after innovation do not depend on their R&D expenditures explicitly. Any kind of combination of the different models is well beyond the scope of this paper, see also section 4 on this point.

[5] In order to maintain mathematical precision, one would have to discuss all four possible combinations of failure and success separately, but this is omitted for the sake of simplicity.

where

$$\begin{aligned} E(q_i^2) &= \theta^2\left(\frac{a-c+(2-\sigma)x_i+(2\sigma-1)x_j}{3b}\right)^2+\theta(1-\theta)\left(\frac{a-c+(2-\sigma)x_i}{3b}\right)^2 \\ &\quad +\theta(1-\theta)\left(\frac{a-c+(2\sigma-1)x_j}{3b}\right)+(1-\theta)^2\left(\frac{a-c}{3b}\right)^2, \end{aligned} \tag{5}$$

because both R&D projects are independent and have the same probability of success. Setting the first derivative of (4) with respect to x_i equal to zero and solving for x_i yields firm i's best response function

$$x_i = \frac{\frac{2}{9}(2-\sigma)\theta\left[a-c+\theta(2\sigma-1)x_j\right]}{b\gamma-\frac{2}{9}(2-\sigma)^2\theta}. \tag{6}$$

Analogously we obtain

$$x_j = \frac{\frac{2}{9}(2-\sigma)\theta\left[a-c+\theta(2\sigma-1)x_i\right]}{b\gamma-\frac{2}{9}(2-\sigma)^2\theta} \tag{7}$$

for firm j's best response function. Equilibrium in the first stage is symmetric, hence both firms choose the intended cost reduction $x^N = x_i = x_j$ given by

$$\begin{aligned} x^N &= \frac{\frac{2}{9}(2-\sigma)(a-c)\theta}{b\gamma-\frac{2}{9}(2-\sigma)(2\sigma-1)\theta^2-\frac{2}{9}(2-\sigma)^2\theta} \\ &= \frac{\frac{2}{9}(2-\sigma)(a-c)\theta}{b\gamma-\frac{2}{9}(2-\sigma)(1+\sigma)\theta^2-\frac{2}{9}(2-\sigma)^2\theta(1-\theta)}, \end{aligned} \tag{8}$$

which coincides with the well-known solution of the deterministic AJ model for $\theta = 1$. The total *expected* effective cost reduction (including spillovers) for each firm, X^N, is given by

$$X^N = \theta^2(1+\sigma)x^N+\theta(1-\theta)x^N+\theta(1-\theta)\sigma x^N+(1-\theta)^2\cdot 0 = \theta(1+\sigma)x^N, \tag{9}$$

where the superscript N refers to the expected R&D performance in the non-cooperative equilibrium analyzed so far and where x^N has to be substituted by (8).[6]

Of course, one is also interested in the (expected) R&D performance when firms cooperate in R&D. As is standard in the literature, we maintain the assumption that the firms compete in the product market, such that the Nash-equilibrium outputs (2) and (3) remain valid under any form of cooperation.[7] In the first case, both

[6] As noted by Amir (2000, p. 1021) one is mainly interested in statements about effective R&D (i.e. X^N) instead of each firm's autonomous R&D levels (i.e. x^N). This is even more true when R&D projects may fail because *intended* cost reductions are of rather limited relevance then. Thus, we focus on expected effective cost reductions in all our comparisons between the different scenarios and models.

[7] The following scenarios were already analyzed in Kamien, Muller, Zang (1992), and they were applied to the AJ model by Amir (2000). Here, we also use their notations to identify the different forms of cooperation.

firms carry out their own R&D project but cooperate by letting the other firm fully participate in the R&D outcome, i.e., by setting the spillover rate equal to one. Thus, the firms build a (competitive) research joint venture (RJV) which will subsequently be referred to as case NJ (non-cooperative, joint research). Under this kind of cooperation, the total expected cost reduction for each firm is given by

$$X^{NJ} = \frac{\frac{4}{9}(a-c)\theta^2}{b\gamma - \frac{4}{9}\theta^2 - \frac{2}{9}\theta(1-\theta)}, \tag{10}$$

which may be obtained by setting $\sigma = 1$ in (8) and (9).

In the second case, both firms build an R&D cartel, i.e., they choose the same intended cost reduction so as to maximize their joint profits but leave the spillover rate unaffected. In view of (2)-(5) and $x = x_i = x_j$ we have

$$\begin{aligned} E(\Pi_i + \Pi_j) &= 2\theta^2 \frac{1}{9b}\left(a - c + (1+\sigma)x\right)^2 + 2\theta(1-\theta)\frac{1}{9b}\left(a - c + (2-\sigma)x\right)^2 \\ &+ 2\theta(1-\theta)\frac{1}{9b}\left(a - c + (2\sigma - 1)x\right)^2 + 2(1-\theta)^2\frac{1}{9b}\left(a-c\right)^2 - \gamma x^2. \end{aligned} \tag{11}$$

Setting the partial derivative of (11) with respect to x equal to zero and solving for x we obtain

$$x^C = \frac{\frac{2}{9}(1+\sigma)(a-c)\theta}{b\gamma - \frac{2}{9}(1+\sigma)^2\theta^2 - \frac{2}{9}\left[(2-\sigma)^2 + (2\sigma-1)^2\right]\theta(1-\theta)}, \tag{12}$$

and the total expected effective cost reduction in an R&D cartel is given by

$$X^C = \theta(1+\sigma)x^C, \tag{13}$$

where the superscript C refers to the cartel solution.

Finally, the firms may set up an RJV cartel in which they maximize joint profits *and* set the spillover rate equal to one. This kind of cooperation is labelled as case CJ, and its total expected cost reduction is given by

$$X^{CJ} = \frac{\frac{8}{9}(a-c)\theta^2}{b\gamma - \frac{8}{9}\theta^2 - \frac{4}{9}\theta(1-\theta)}, \tag{14}$$

which is obtained from (12) and (13) by setting $\sigma = 1$.[8] In section 3 we will analyze in detail how the expected cost reductions in the four scenarios relate to their deterministic counterparts and to each other.[9]

[8] Observe that all expected cost reductions are equal to those derived by Amir (2000) for the deterministic AJ model for $\theta = 1$.

[9] In footnote 3 we have claimed that the above model could easily be extended to the case where the R&D projects may also be partially successful without changing its central implications. Formally, one could extend the model in this direction by multiplying the respective R&D outcomes in (2) and (3) with some stochastically independent (possibly continuous) random variables η_i, $i = 1, 2$ (e.g. with support $[0, 1]$) instead of assuming a probability of success θ. Here, a realized value of $\eta_i = 0,5$ (say) means that half of the intended cost reduction becomes effective. In this case, perfect analogues of equations (9), (10), (13) and (14) are obtained, where θ^2 and $\theta(1-\theta)$ have to be substituted by $E(\eta_i)^2$ and $Var(\eta_i)$, respectively. In fact, the model considered here is the special case where η_i is a discrete random variable with possible realizations zero and one and with $\Pr(\eta_i = 1) = \theta$.

2.2 The stochastic KMZ model

Let us now turn to a version of the model introduced by Kamien, Muller, Zang (1992). Although its basic structure is rather similar to that of the AJ model at first sight, it differs in two important aspects. Firstly, the firms do not choose their (intended) cost reductions directly but decide on R&D expenditures instead, which are transformed into (unit) cost reductions via an R&D production function. Secondly, there are no output spillovers ($\sigma = 0$) but firms directly (and costlessly) benefit from their rival's R&D expenditures that add to the own investment as an input in the R&D production function, i.e., there are *input* spillovers. Since we are aiming at a comparison of the AJ and the KMZ model we subsequently assume a particular functional form for the R&D production function due to Amir (2000), for which the AJ and the KMZ model are equivalent in the absence of spillovers. Under this specification a successful R&D project leads to a cost reduction of $\sqrt{(2/\gamma)y}$ if total R&D investment is equal to $y \geq 0$. Let y_i, $i = 1, 2$, denote the R&D expenditures of firm i. In the presence of input spillovers, total R&D investment for firm i (say) is then given by $y_i + \beta y_j$, where $0 \leq \beta \leq 1$ denotes the spillover rate. As in the AJ model of section 2.1 we assume a stochastic R&D technology, such that the cost reduction $\sqrt{(2/\gamma)(y_i + \beta y_j)}$ only becomes effective with probability θ, $0 < \theta < 1$. On the other hand, a firm's cost function remains completely unaffected if its own R&D project fails, which happens with probability $1 - \theta$.

Regarding the market demand functions and the firms' original cost functions c_i^0 we maintain all assumptions and notations that were introduced in section 2.1. Having observed the outcome of the R&D projects, both firms thus determine their outputs in the second stage so as to maximize profits given by

$$\pi_i = \left[a - c + \sqrt{\frac{2}{\gamma}(y_i + \beta y_j)} - b(q_i + q_j)\right] \cdot q_i, \tag{15}$$

where the case of a failed R&D project is *formally* included for $y_i = y_j = 0$.[10] As before, standard calculations yield

$$q_i = \frac{1}{3b}\left[a - c + 2\sqrt{\frac{2}{\gamma}(y_i + \beta y_j)} - \sqrt{\frac{2}{\gamma}(y_j + \beta y_i)}\right], \tag{16}$$

$$q_j = \frac{1}{3b}\left[a - c + 2\sqrt{\frac{2}{\gamma}(y_j + \beta y_i)} - \sqrt{\frac{2}{\gamma}(y_i + \beta y_j)}\right], \tag{17}$$

where only expected quantities are equal if $y_i = y_j$, but the *realized* product market equilibrium is generally asymmetric. When deciding upon their R&D expenditures in the first stage, both firms anticipate the expected equilibrium outcome of the second stage and thus choose the R&D investment y_i that maximizes expected profits given by

$$E(\Pi_i) = b \cdot E(q_i)^2 - y_i, \tag{18}$$

[10] Again, one would have to consider the cases of failure and success separately to be mathematically precise but this is not done for simplicity.

where

$$
\begin{aligned}
E(q_i^2) &= \theta^2 \left(\frac{1}{3b} \left[a - c + 2\sqrt{\frac{2}{\gamma}(y_i + \beta y_j)} - \sqrt{\frac{2}{\gamma}(y_j + \beta y_i)} \right] \right)^2 \\
&+ \theta(1-\theta) \left(\frac{1}{3b} \left[a - c + 2\sqrt{\frac{2}{\gamma}(y_i + \beta y_j)} \right] \right)^2 \\
&+ \theta(1-\theta) \left(\frac{1}{3b} \left[a - c - \sqrt{\frac{2}{\gamma}(y_j + \beta y_i)} \right] \right)^2 + (1-\theta)^2 \left(\frac{a-c}{3b} \right)^2 . \quad (19)
\end{aligned}
$$

Setting the first order partial derivative of (18) with respect to y_i equal to zero and using the fact that $y_i = y_j = y$ in a symmetric equilibrium, straightforward calculations yield

$$\sqrt{\frac{2}{\gamma}(1+\beta)y} = \frac{\theta\frac{2}{9}(a-c)(2-\beta)}{b\gamma - \frac{2}{9}(2-\beta)\theta^2 - \frac{2}{9}(4+\beta)\theta(1-\theta)}. \quad (20)$$

Observe that equation (20) only gives a firm's total cost reduction if its R&D project is successful. Otherwise, the cost reduction is equal to zero because firms do not benefit from a rival's successful R&D project in the absence of output spillovers. Hence, the *expected* effective cost reduction for each firm in the non-cooperative equilibrium, Y^N, is given by

$$Y^N = \frac{\frac{2}{9}(a-c)(2-\beta)\theta^2}{b\gamma - \frac{2}{9}(2-\beta)\theta^2 - \frac{2}{9}(4+\beta)\theta(1-\theta)}. \quad (21)$$

Naturally, the firms may also cooperate within the KMZ model, where the three cases NJ, C, and CJ are exactly as described in the previous section. By replication of the derivations in the AJ model we now obtain the following expected total cost reductions for the KMZ model:[11]

$$Y^{NJ} = \frac{\frac{2}{9}(a-c)\theta^2}{b\gamma - \frac{2}{9}\theta^2 - \frac{10}{9}\theta(1-\theta)}, \quad (22)$$

$$Y^{C} = \frac{\frac{2}{9}(a-c)(1+\beta)\theta^2}{b\gamma - \frac{2}{9}(1+\beta)\theta^2 - \frac{10}{9}(1+\beta)\theta(1-\theta)}, \quad (23)$$

$$Y^{CJ} = \frac{\frac{4}{9}(a-c)\theta^2}{b\gamma - \frac{4}{9}\theta^2 - \frac{20}{9}\theta(1-\theta)}. \quad (24)$$

3 R&D performance under uncertainty

In this section we will start by analyzing the AJ and the KMZ model separately and discuss two important issues. Firstly, we investigate whether the possibility of

[11] As in the AJ model the expected cost reductions given in (21)-(24) are identical to those derived by Amir (2000, p. 1020) for the deterministic case if $\theta = 1$.

a failure of R&D projects enhances or reduces the firms' R&D activities or rather their expected cost reductions, compared to the case of a non-risky R&D technology. Secondly, we analyze whether the relation between R&D performances under the four different scenarios of cooperative and non-cooperative behaviour that hold in a deterministic environment extend to the stochastic case. It turns out that the AJ and the KMZ model yield quite different results with regard to both aspects. This will become even more evident when we proceed to comparing expected effective cost reductions across the respective models in section 3.3.

3.1 The AJ model

An analytical comparison of the different expected effective cost reductions given in equations (9), (10), (13), and (14) among each other and in relation to their deterministic counterparts is mostly inconclusive because of the large variety of possible parameter constellations. This is even more true regarding the influence of the probability of R&D success and the spillover rate on those relations, so we restrict ourselves to a numerical and graphical analysis.[12] In figures 1-3 we summarize the results concerning the relation of the expected cost reductions in the presence of a risky R&D technology to the certain cost reductions in the corresponding deterministic model (indicated by the index "det"), which are obtained by setting $\theta = 1$ in equations (9), (10), (13) and (14). The different curves in those figures (as well as in all subsequent figures) are based on the normalized parameter values $a - c = 10$ and $b\gamma = 1$. All relevant results are qualitatively robust against variations in both parameters, where the term $a - c$ is completely innocent as it merely represents a scaling factor. The parameter $b\gamma$ is somewhat more critical because too small values lead to infeasible cost reductions ($X < 0$). If $b\gamma$ is too large, it dominates the other terms in the denominator of the various X, and the effects described below are less evident, although they are still present. For $b\gamma = 1$, the (expected) cost reductions as well as the associated equilibrium quantities are feasible and all important results may easily be identified.[13] Observe that the cost reductions for the cases NJ and CJ are included in the figures for $\sigma = 1$.

In figures 1 and 2 we have depicted the expected effective cost reductions X^N and X^C as a function of the spillover rate σ for various probabilities θ, where the deterministic values (X^N_{det} and X^C_{det}) correspond to the case $\theta = 1$. Figures 3a and 3b contain isoratio curves in the (σ, θ)-space for different levels of the ratios X^N/X^N_{det}

[12] It is in fact possible to establish some of the relations rigorously (e.g. $X^C < X^{CJ}$). In those cases, the relevant inequalities follow from the requirement of feasible R&D levels and quantities (i.e., $x \geq 0$, $q \geq 0$) which impose several restrictions on the important relation of $b\gamma$ to θ and σ. However, this analysis is not very instructive and is, therefore, omitted.

[13] Regarding the feasibility of the possible equilbrium quantities it is obvious from equations (2) and (3) that all R&D levels smaller than or equal to 10 lead to feasible quantities (i.e. $q_k \geq 0$, $k = i, j$) because we have set $a - c = 10$. Moreover, R&D levels larger than 10 are feasible for

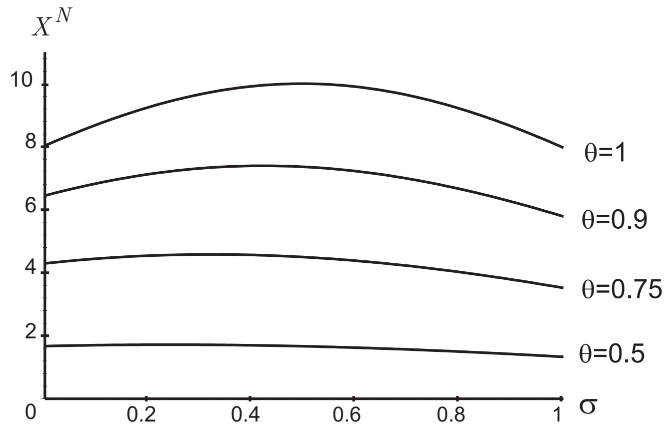

Fig. 1: X^N for different probabilities of success
$(a - c = 10,\ b\gamma = 1)$

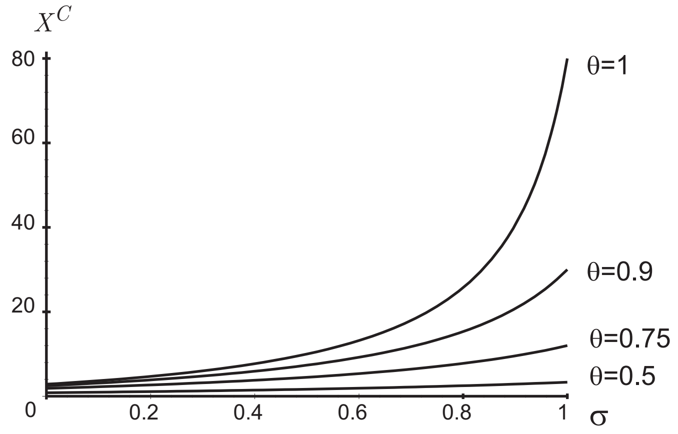

Fig. 2: X^C for different probabilities of success
$(a - c = 10,\ b\gamma = 1)$

and X^C/X^C_{det}, respectively, giving all combinations of σ and θ that yield the same ratio of the expected effective cost reductions compared to the deterministic values. Not surprisingly, we find that the expected effective cost reductions are increasing in the probability of success in all four scenarios and irrespective of the spillover rate. Furthermore, the expected cost reductions are always below the certain cost reductions in the deterministic model. While this might have been expected as well, we also obtain the stronger result that even the highest possible cost reductions in the stochastic model, i.e., the effective cost reductions when *both* firms carry out successful R&D projects, are lower than the deterministic cost reductions in all types of R&D cooperation (and non-cooperation).[14] Obviously, uncertainty with respect to the success of R&D projects is detrimental to technological performance.

$\sigma \geq 1/2$.

[14]This may be shown numerically by dividing all R&D levels X^k, $k = N, NJ, C, CJ$, by the respective probability θ, cf. equation (9).

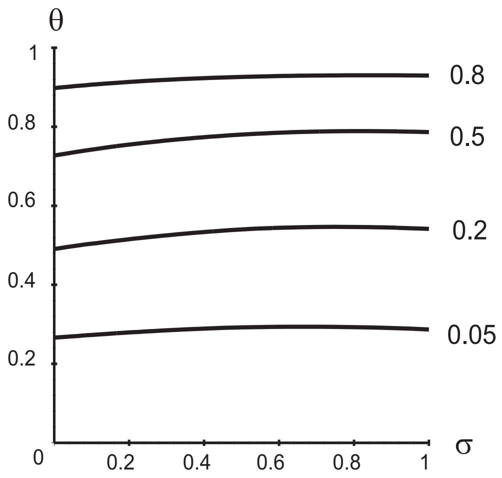

Fig. 3a: Isoratio curves for X^N/X^N_{det} $(a - c = 10,\ b\gamma = 1)$

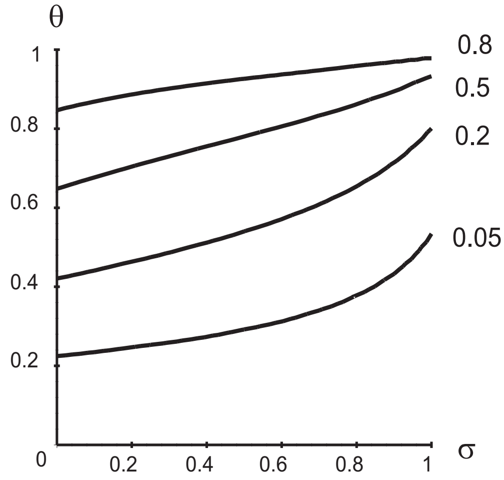

Fig. 3b: Isoratio curves for X^C/X^C_{det} $(a - c = 10,\ b\gamma = 1)$

Having a look at the impact of variations in the spillover rate for fixed θ, we learn from figure 1 that the non-cooperative R&D level X^N increases as σ rises from zero and then declines as σ becomes larger and tends to one, just as its deterministic counterpart X^N_{det}. Unlike the deterministic case, however, the maximum is not reached for $\sigma = 1/2$ but for a *lower* value of the spillover rate, and expected cost reductions for a high spillover rate ($\sigma > 1/2$) are *smaller* than for the "associated" low spillover rate $(1 - \sigma)$. In addition we observe that X^N increases more slowly than X^N_{det} as σ rises but does not decline stronger as σ becomes larger.[15] Similar results are obtained for the cooperative R&D levels X^C. Just as in the deterministic case X^C increases as the spillover rate rises, but this increase is much slower than in case of safe R&D projects, see figure 2.[16] Hence, the expected cost reductions in the cooperative RJV are much smaller compared to their deterministic counterparts. In particular, setting the spillover rate equal to one in a cooperative RJV cartel (case CJ) induces a much smaller increase in cost reductions than if all R&D projects were not risky.

Those findings are supported by the isoratio curves given in figure 3b, as they reveal a positive relationship between the probability of success and the spillover rate with regard to constant ratios: in order to obtain the same ratio of the expected effective cost reductions to the deterministic values, both higher probabilities and higher spillovers are required. Since higher probabilities increase expected cost reductions, higher spillovers obviously *reduce* them relative to the deterministic model, i.e., θ and σ actually work into different directions regarding cost reductions. More-

[15] This may also be seen from figure 3a, because for fixed θ the ratio X^N/X^N_{det} falls as σ rises from zero and then recovers or stays rather constant for $\sigma > 1/2$.

[16] This is also obvious from figure 3b, because the ratio X^C/X^C_{det} declines sharply as σ rises for fixed θ.

over, this effect is more pronounced for high values of the spillover rate, which is also apparent from the isoratio curves in figure 3b that become steeper as σ rises. For the non-cooperative scenario depicted in figure 3a we obtain roughly the same results, although the isoratio curves are much flatter and even slightly decreasing for large spillovers. This effect is due to the asymmetric and non-monotonic behaviour of X^N with respect to changes in σ, and we will give an appropriate interpretation below.

The economic rationale behind the above results lies in an additional effect of output spillovers on the firms' (expected) profits which is only present in the case of a risky R&D technology. If we have a look at equation (5) which contains the crucial component $E(q_i^2)$ of the expected profits we find that the firms arrive in the standard product market equilibrium with or without R&D if they both succeed or fail in R&D, respectively. Thus, any deviating results must be due to the asymmetric cases where only one firm has a successful R&D project. If only firm i (say) succeeds, the second term in equation (5) makes evident that output spillovers are detrimental to the firm's profits because part of the cost reduction leaks out to the competitor who also benefits from i's success. On the other hand, if only j is successful, then the third term in equation (5) shows that firm i is better off as well if there are output spillovers.[17] Consequently, both possible asymmetric outcomes in the product market provide an incentive to invest *less* in R&D because a firm will find it more attractive to free ride on its competitor's success in case of a failure instead of supporting the rival in case of success. From another point of view that lays emphasis on the possibility to participate in the rival's success in case of the own failure, we may interpret output spillovers as a kind of risk sharing device because they ensure cost reductions for both firms unless all projects fail. Thus, both firms expect to realize some given intended cost reductions with less investment in R&D, such that overall R&D expenditures and hence expected effective cost reductions will decline. Of course, this effect is stronger the higher the spillover rate (cf. the middle terms in (5)), and hence it is economically plausible that expected cost reductions are lowest compared to the deterministic model in the case CJ, see figure 2. As already indicated above, matters are a little different for the non-cooperative scenarios N and NJ because of the non-monotonic and asymmetric shape of the X^N-curves. However, the risk sharing and hence R&D reducing effect of large spillovers is also visible here. For small spillovers the ratio X^N/X^N_{det} is decreasing in σ. If there was *no* risk sharing, one would expect that uncertainty merely "squeezes" the X^N-curves downward such that the ratio X^N/X^N_{det} would be increasing for $\sigma > 1/2$ and the isoratio curves in figure 3a would be bending downward stronger for large σ. It is the risk sharing effect that reduces X^N further as σ rises, and both X^N/X^N_{det} and the isoratio curves stay roughly constant for large spillovers.

[17] If those spillovers are large ($\sigma > 1/2$), then firm i is even better off than in the case that both R&D projects fail.

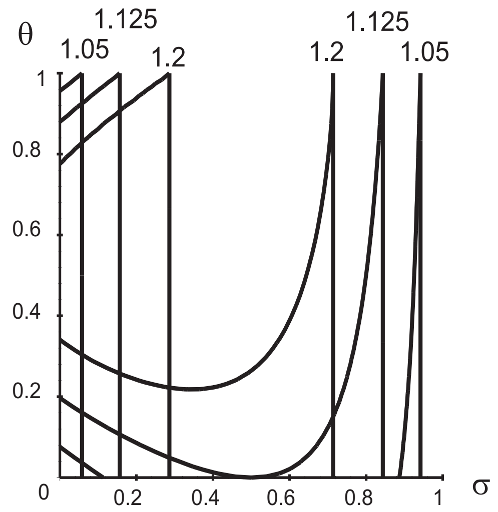

Fig. 4a: Isoratio curves for X^N/X^{NJ} $(a-c=10,\, b\gamma=1)$

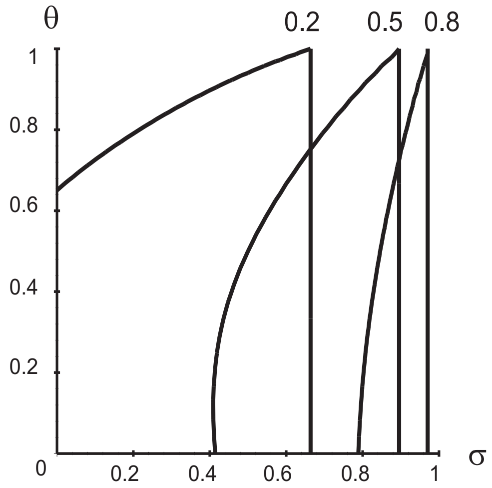

Fig. 4b: Isoratio curves for X^C/X^{CJ} $(a-c=10,\, b\gamma=1)$

Figures 4a and 4b shed some additional light on the above results and interpretations. In those figures we have depicted isoratio curves for X^N/X^{NJ} and X^C/X^{CJ}, respectively, giving all combinations of the probability of success and the spillover rate that yield the same ratio of the "standard" cases (N and C) to the RJVs that set the spillover rate equal to one (NJ and CJ). Let us first consider the cooperative scenarios C and CJ. In the deterministic model, every σ determines a unique value of the ratio $X^C_{\text{det}}/X^{CJ}_{\text{det}}$ which is represented by the vertical lines in figure 4b. If uncertainty prevails, both X^C and X^{CJ} are reduced, but the impact on X^{CJ} is stronger if $\sigma < 1$. Again, this can be attributed to the risk sharing (or diversificational) effect of output spillovers. Setting σ equal to one provides a higher degree of risk sharing than in the "original" situation where only the "true" spillover rate $\sigma < 1$ is effective. Hence, the ratio X^C/X^{CJ} increases for fixed σ and, since X^C is increasing in σ, a *smaller* spillover rate is required to achieve the same ratio X^C/X^{CJ} as in the deterministic model. Moreover, we see that the isoratio curves are much flatter for smaller values of σ which reflects the fact that the wedge between X^C and X^{CJ} is higher the less risk sharing is provided by the true spillover rate in case C. Hence, the above effect is most significant for small σ.[18]

The corresponding isoratio curves for the non-cooperative cases N and NJ are a little harder to interpret, because the non-monotonic and asymmetric behaviour of X^N induces counterintuitive effects for large spillovers, see above. Nevertheless, figure 4a makes evident that the same interpretation as for the cases C and CJ applies for $\sigma < 1/2$: setting the spillover rate equal to one in an RJV leads to a much lower level of X^{NJ} than of X^{NJ}_{det} because the risk sharing effect of output

[18] Numerical calculations demonstrate that X^C/X^{CJ} is up to eight times as large as its deterministic counterpart for small σ while it is less than twice as high as $X^C_{\text{det}}/X^{CJ}_{\text{det}}$ for σ close to one.

spillovers is rather strong in that case. The corresponding impact of the uncertainty on X^N is comparably moderate, such that X^N is larger relative to X^{NJ} than in the deterministic model. If σ is larger, this effect is also present but cannot outweigh the opposite effects resulting from the asymmetry in X^N, such that the ratio X^N/X^{NJ} is even smaller than the respective ratio in the deterministic model.

Summing up all results and interpretations we may conclude that output spillovers provide a kind of risk sharing in the presence of uncertain R&D projects, thereby triggering a general decline in R&D activities and expected cost reductions that adds on the effect already caused by the uncertainty itself.[19]

3.2 The KMZ model

Matters are quite different for the KMZ model with input spillovers. As for the AJ model, comparing the different expected cost reductions given by (21)-(24) and deriving the impact of variations in the relevant parameters β and θ is analytically inconclusive in most cases, so we only provide numerical and graphical results. At first, we consider the benchmark case $a - c = 10$ and $b\gamma = 1$. As before, all conclusions are qualitatively robust with respect to variations in $a - c$ and for all $b\gamma > 1$. Unlike the AJ model, however, some surprising results emerge as $b\gamma$ becomes smaller, which will be emphasized at the appropriate places.

It is obvious from figures 5 and 6 that the expected cost reductions are increasing in the probability of success, θ, no matter which kind of R&D cooperation is established and irrespective of the spillover rate β, unless both β and θ are close to one in case C.[20] For every fixed probability of success we see that the non-cooperative R&D level Y^N declines and the cooperative R&D level Y^C increases as the spillover rate rises from zero to one, just as in the deterministic case. However, the decline of the expected R&D levels in the non-cooperative scenario is less significant in the stochastic model and the increase in the cooperative level is stronger when R&D is risky, at least if θ is not too small in case C.[21]

It is also quite instructive to consider the isoratio curves for Y^N/Y^N_{det} and Y^C/Y^C_{det} depicted in figures 7a and 7b, respectively. They reveal a negative relationship between the probability of success and the spillover rate with regard to constant

[19] In the analysis of the various scenarios of cooperative and non-cooperative behaviour we have omitted the comparison of the cases N and C. By drawing isoratio curves as in figure 4 it is possible to show that all results are qualitatively similar to the deterministic case and that no additional insights can be gained.

[20] For an economic interpretation of this phenomenon see below. Moreover, unlike the AJ model, it is *not* always true that the highest possible cost reduction, i.e., if both R&D projects are successful, is below the certain cost reduction in the deterministic model. Here, the former may exceed the latter if both β and θ are large.

[21] The fact that Y^C rises more slowly than Y^C_{det} for small θ is counterintuitive and hard to interpret. Presumably, the threat of failure dominates if success of the R&D projects appears to be too improbable.

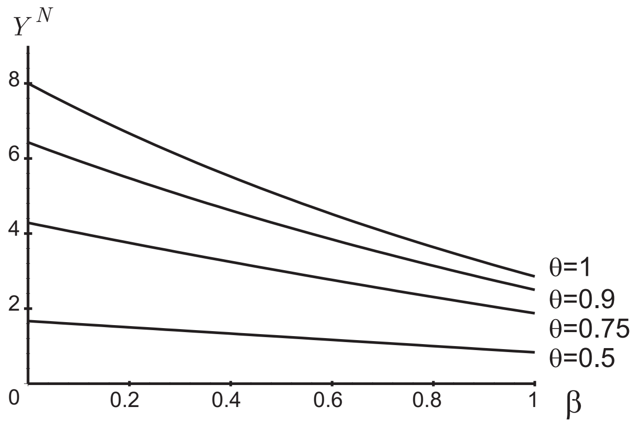

Fig. 5: Y^N for different probabilities of success
($a - c = 10$, $b\gamma = 1$)

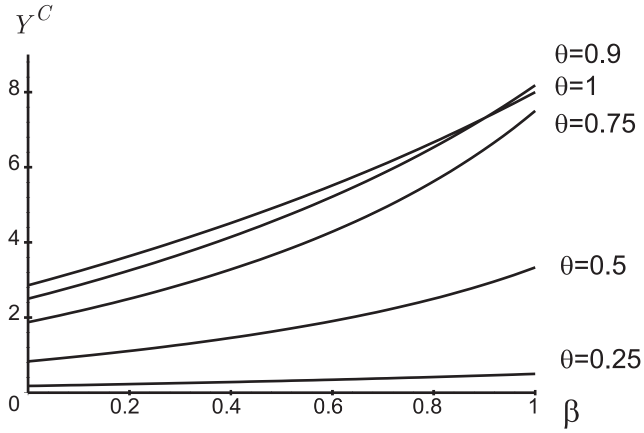

Fig. 6: Y^C for different probabilities of success
($a - c = 10$, $b\gamma = 1$)

ratios:[22] a higher probability of success and a smaller spillover rate are required to obtain the same ratio of the expected effective cost reductions in the stochastic model compared to their deterministic counterparts. As higher probabilities increase expected cost reductions, smaller spillovers obviously reduce them relative to the deterministic model, i.e., θ and β actually work into the *same* direction in the KMZ model.[23] It is thus not surprising that the most significant results are obtained if β and θ are large. For example, Y^C even *exceeds* Y^C_{det} for $\beta = 1$ and $\theta = 0.9$. In fact, if $b\gamma$ is smaller than one, there are much more parameter constellations with large β and large θ where the ratio Y^C/Y^C_{det} exceeds one, and the same also holds true for Y^N/Y^N_{det}, see table 1 for some examples.[24] Consequently, the expected cost

[22] Remember that the corresponding result for the AJ model is exactly opposite, see figure 3.

[23] There is an exception for case C if θ is small, see also footnote 21.

[24] Admittedly, those cases are somewhat "extreme" as they produce striking results for some (β, θ)-constellations but also yield infeasible solutions for y and q in others. Hence, those cases are not presented for the full range of parameter values.

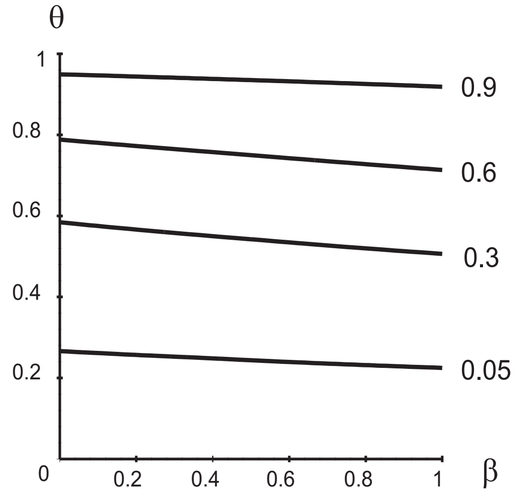

Fig. 7a: Isoratio curves for Y^N/Y^N_{det}
($a - c = 10$, $b\gamma = 1$)

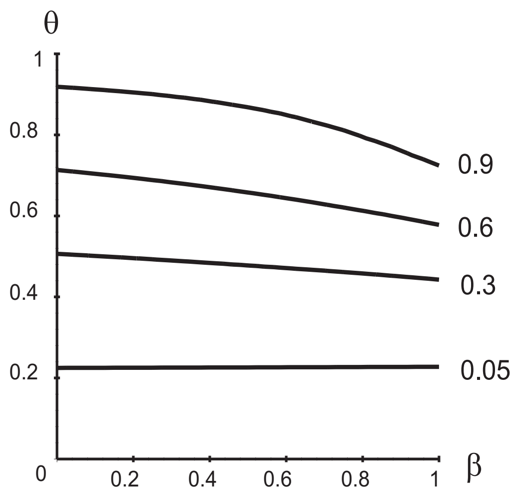

Fig. 7b: Isoratio curves for Y^C/Y^C_{det}
($a - c = 10$, $b\gamma = 1$)

reductions in the RJVs that set the spillover rate equal to one, i.e. Y^{NJ} and Y^{CJ}, are relatively high compared to the deterministic model, and setting $\beta = 1$ when the spillover rate is small implies a weaker decline or a stronger increase in cost reductions within the cases NJ and CJ, respectively.

Obviously, high rates of input spillovers increase technological performance in the sense of higher expected effective cost reductions in relation to the certain cost reductions in the deterministic model. While this result is in sharp contrast to the essentially opposite findings for the AJ model, it is economically highly plausible.

$b\gamma$	β	θ	Y^N	Y^N_{det}	Y^N/Y^N_{det}	Y^C	Y^C_{det}	Y^C/Y^C_{det}
0,45	0,9	0,75	12,692	11,892	1,0673			
0,45	0,9	0,9	12,857	11,892	1,0812			
0,45	1	0,75	10,714	9,7561	1,0982			
0,45	1	0,9	10,5882	9,7561	1,0853			
0,5	0,9	0,9	9,7059	9,5652	1,0147			
0,5	1	0,9	8,1818	8	1,0227			
0,7	0,5	0,75				9,375	9,0909	1,0313
0,7	0,5	0,9				9,6429	9,0909	1,0607
0,9	0,9	0,75				8,9063	8,8372	1,0078
0,9	0,9	0,9				9,2935	8,8372	1,0516
0,9	1	0,75				10,7143	9,7561	1,0982
0,9	1	0,9				10,5882	9,7561	1,0853

Table 1: Expected effective cost reductions in the KMZ model
($a - c = 10$, different values of $b\gamma$)

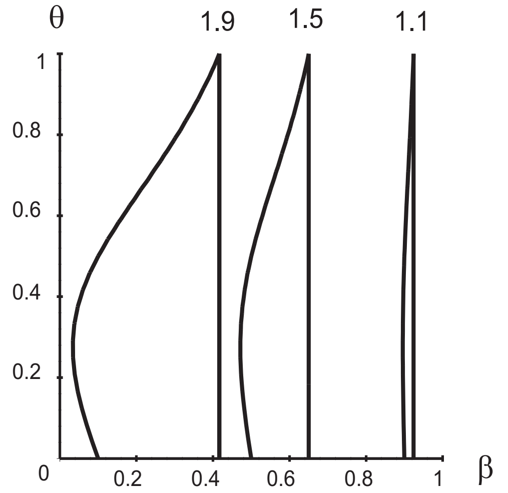

Fig. 8a: Isoratio curves for Y^N/Y^{NJ}
($a - c = 10$, $b\gamma = 1$)

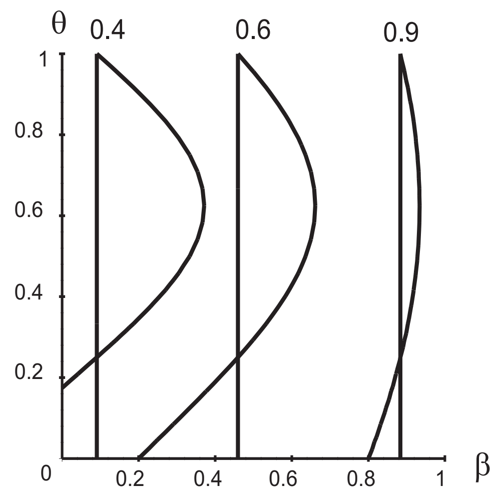

Fig. 8b: Isoratio curves for Y^C/Y^{CJ}
($a - c = 10$, $b\gamma = 1$)

First of all, the risk sharing effect of output spillovers that reduces technological performance in the AJ model is not present here. Instead, a firm only benefits from its rival's R&D investment if its *own* R&D project is successful, whereas it merely supports the other firm if the competitor's project is the only one to succeed. If both projects are successful or if both fail, the product market equilibrium will be symmetric, so each firm essentially has to trade off the two asymmetric cases against each other when deciding upon its own R&D expenditures. The economic rationale behind the results contained in figures 5-7 now becomes evident if we keep in mind that both firms choose the same R&D investment in equilibrium and if we have a look at the two middle terms in equation (19). If firm i, say, is the only one with a successful R&D project it obviously gains *more* (relative to firm j) than it loses if the rival firm is the only one to be successful. This is because successful R&D has two different effects on the firms' profits. The *direct* effect is the cost reduction itself while the *indirect* effect results from the associated advantage in the product market competition. A successful firm benefits from both effects whereas a failing firm is only harmed by the indirect effect. Thus, even though parts of the own R&D effort leaks out to the competitor, each firm has a strong incentive to increase its own investment in risky R&D projects. In fact, this incentive is even enhanced for large spillovers because they imply that the benefit to a successful firm is disproportionately higher than the disadvantage in case of a failure.

As in the AJ model, the above findings are supported by the isoratio curves for Y^N/Y^{NJ} and Y^C/Y^{CJ} depicted in figures 8a and 8b. Regarding the non-cooperative scenarios N and NJ, every spillover rate determines a specific value $Y^N_{\text{det}}/Y^{NJ}_{\text{det}}$ for the deterministic model represented by the vertical lines. Under uncertainty, Y^{NJ} is not as much reduced relative to Y^N as in the deterministic case, which can be attributed to the R&D intensifying impact of input spillovers indicated above. Hence, the ratio Y^N/Y^{NJ} is smaller than the corresponding ratio under certainty, and a smaller spillover rate is required to reach the same level, because Y^N is a decreasing function of β. Moreover, this effect diminishes as β rises since the R&D intensifying impact on Y^N becomes stronger as well. Similar results are also obtained in the two cooperative scenarios C and CJ, see figure 8b. If the probability of success, θ, is not too small, the ratios are smaller than those for the deterministic model, indicating the fact that Y^{CJ} (the case with β set equal to one) declines less than Y^C, relative to the deterministic case. Since Y^C is an increasing function of β, a higher spillover rate is required to reach the same ratio as under certainty.[25] As β rises, this effect becomes less significant, which is due to the fact that with large spillover rates Y^C also tends to decline less.[26]

Summing up the preceding discussion, we may conclude that the presence of input spillovers provides an incentive for firms to engage more in R&D if the R&D technology is risky, thus mitigating the decline in R&D activity caused by the uncertainty. If a firm happens to be the only successful it participates in the rival's investment and gains more than compared to what it loses by supporting the other firm if only its project fails.[27]

3.3 Comparing the models

The analysis in this section is very much inspired by a recent paper by Amir (2000) who provides an extensive comparison between the AJ and the KMZ model at a formal level. Among other things, he shows[28] that effective (total) cost reductions tend to be significantly higher in the AJ model, at least for the same level of spillovers. In particular, we have (using our notation) $X^N_{\text{det}} > Y^N_{\text{det}}$ unless $\sigma = \beta = 0$, $X^C_{\text{det}} > Y^C_{\text{det}}$ if $\beta < \sigma(\sigma+2)$, and $X^{CJ}_{\text{det}} > 2Y^{CJ}_{\text{det}}$ irrespective of the spillover rate. In view of our above results that output spillovers tend to decrease (expected) effective cost reductions whereas input spillovers tend to increase them when the R&D technology is risky, the question arises whether the results obtained by Amir (2000) for the deterministic model are still valid in a stochastic environment. Table 2 summarizes our previous results by giving the ratios of the expected cost reductions in both models for the same level of spillovers. We see that the wedge between the cost reductions in the AJ and the KMZ model is much smaller under uncertainty. In accordance with the economic interpretations given in sections 3.1 and 3.2 this effect is stronger the larger the spillover rates. In particular, X^{NJ} and Y^{NJ} as well as X^{CJ} and Y^{CJ} are pretty close together, and one can observe that the inequality $X^{CJ}_{\text{det}} > 2Y^{CJ}_{\text{det}}$ does

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[25] As above, the case of a small θ is opposite and difficult to interpret.

[26] Note that the results presented in figure 8 are essentially opposite to those expressed in figure 4 for the AJ model.

[27] As for the AJ model we have omitted a comparison between Y^N and Y^C because it does not provide additional insights.

[28] See proposition 3.1 in Amir (2000).

σ	β	θ	X^N/Y^N	$X^N_{\text{det}}/Y^N_{\text{det}}$	X^C/Y^C	$X^C_{\text{det}}/Y^C_{\text{det}}$
0,1	0,1	0,1	1,0888	1,1867	1,0702	1,1368
0,1	0,1	0,25	1,0762	1,1867	1,0310	1,1368
0,1	0,1	0,5	1,0732	1,1867	1	1,1368
0,1	0,1	0,75	1,1062	1,1867	1,0372	1,1368
0,1	0,1	0,9	1,1487	1,1867	1,0918	1,1368
0,25	0,25	0,1	1,2195	1,4865	1,1720	1,3830
0,25	0,25	0,25	1,1856	1,4865	1,0744	1,3830
0,25	0,25	0,5	1,1765	1,4865	1	1,3830
0,25	0,25	0,75	1,2609	1,4865	1,0938	1,3830
0,25	0,25	0,9	1,3761	1,4865	1,2452	1,3830
0,5	0,5	0,1	1,4316	2	1,3368	2
0,5	0,5	0,25	1,3571	2	1,1429	2
0,5	0,5	0,5	1,3333	2	1	2
0,5	0,5	0,75	1,5	2	1,2	2
0,5	0,5	0,9	1,7455	2	1,5818	2
0,75	0,75	0,1	1,6380	2,4595	1,5016	3,3478
0,75	0,75	0,25	1,5182	2,4595	1,2126	3,3478
0,75	0,75	0,5	1,4737	2,4595	1	3,3478
0,75	0,75	0,75	1,7087	2,4595	1,3462	3,3478
0,75	0,75	0,9	2,0696	2,4595	2,1636	3,3478
0,9	0,9	0,1	1,7599	2,6805	1,6032	5,5506
0,9	0,9	0,25	1,6110	2,6805	1,2578	5,5506
0,9	0,9	0,5	1,5510	2,6805	1	5,5506
0,9	0,9	0,75	1,8172	2,6805	1,4770	5,5506
0,9	0,9	0,9	2,2308	2,6805	2,8391	5,5506
1	1	0,1	1,8405	2,8	1,6729	10
1	1	0,25	1,6716	2,8	1,2903	10
1	1	0,5	1,6	2,8	1	10
1	1	0,75	1,8824	2,8	1,6	10
1	1	0,9	2,3226	2,8	3,6667	10

Table 2: Comparing the AJ and the KMZ model ($a - c = 10$, $b\gamma = 1$)

no longer hold for X^{CJ} and Y^{CJ}. These results are also illustrated by the isoratio curves given in figure 9. Starting from the deterministic case for some spillover rate (represented by a vertical line), introducing uncertainty leads to a significant reduction of X^N because of the risk sharing effect, whereas Y^N declines only slightly due to the R&D intensifying impact of input spillovers. Hence, the ratio X^N/Y^N is smaller under uncertainty, and the fact that X^N/Y^N is an increasing function of

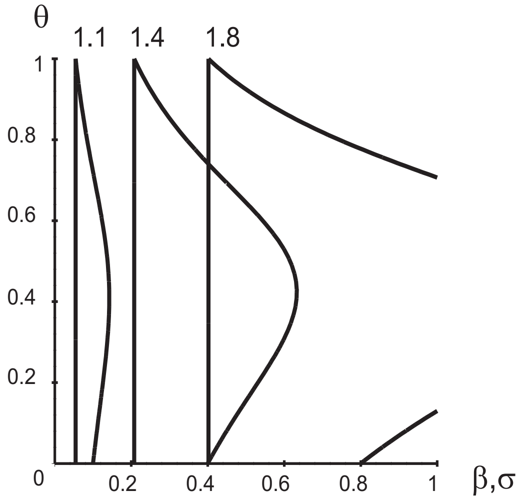

Fig. 9a: Isoratio curves for X^N/Y^N ($a-c=10$, $b\gamma=1$)

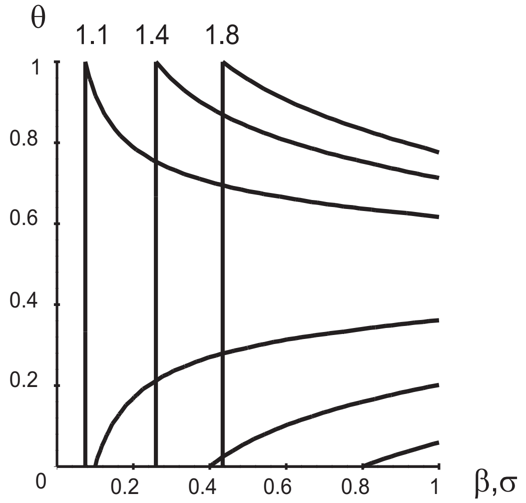

Fig. 9b: Isoratio curves for X^C/Y^C ($a-c=10$, $b\gamma=1$)

the spillover rate requires a higher value of $\beta=\sigma$ to reach the original ratio level. As the relevant isoratio curves are flatter for higher spillover rates, we may conclude that the effect is strongest for this case. Exactly the same reasoning holds for the ratio X^C/Y^C. While all this could have been expected given the results of the previous sections, we may also show that the various inequalities are even reversed for certain parameter constellations, i.e., the expected cost reductions are *larger* in the presence of input spillovers. Several examples for those cases are given in table 3.[29] If $b\gamma$ is sufficiently small, X^N may be smaller than Y^N, we have $X^C<Y^C$ for parameter constellations with $\beta<\sigma(\sigma+2)$ as well, and even $X^{CJ}<Y^{CJ}$ is possible.[30] Economically, the value of the parameter γ is crucial for this result. If γ is (very) small, then R&D expenditures are close to being negligible compared to the expected cost reductions. In the KMZ model, this implies that a firm that invested a lot in its R&D project is extremely better off if it is the only one to be successful, cf. the second term in (19). Hence, R&D levels will be disproportionately high in the presence of input spillovers if γ is small. In the AJ model, a small γ only implies lower costs for R&D projects but the risk sharing effect caused by the output spillovers remains entirely unaffected by γ, see equation (5). Hence, smaller values of γ imply that expected effective cost reductions in the KMZ model decline significantly less relative to those in the AJ model, which may lead to a reversion of the inequalities known from the deterministic models. It should be kept in mind, however, that those cases are primarily of theoretical interest because they require rather special parameter constellations for which most of the other cases lead to infeasible (negative) cost reductions.

[29] For all cases considered in table 3 we obtain feasible solutions for both the R&D levels and the equilibrium quantities.

[30] Remember that we have $X^{CJ}_{\text{det}} > \mathbf{2}Y^{CJ}_{\text{det}}$.

$b\gamma$	σ	β	θ	X^N/Y^N	X^C/Y^C	$X^C_{\mathrm{det}}/Y^C_{\mathrm{det}}$	X^{CJ}/Y^{CJ}
0,35	0,1	0,1	0,25	0,9853			
0,35	0,25	0,25	0,25	0,9675			
0,35	0,5	0,5	0,25	0,9444			
0,35	0,75	0,75	0,25	0,9263			
0,35	0,9	0,9	0,25	0,9168			
1,2	0,4	1	0,1		0,8531	0,9686	
1,2	0,4	1	0,25		0,6956	0,9686	
1,2	0,4	1	0,5		0,5521	0,9686	
1,2	0,4	1	0,75		0,6173	0,9686	
1,2	0,4	1	0,9		0,7887	0,9686	
1,2	0,5	1	0,1		0,9739	1,2143	
1,2	0,5	1	0,25		0,7907	1,2143	
1,2	0,5	1	0,5		0,6316	1,2143	
1,2	0,5	1	0,75		0,7273	1,2143	
1,2	0,5	1	0,9		0,96	1,2143	
0,9	arb.	arb.	0,35				0,9855
0,9	arb.	arb.	0,4				0,9078
0,9	arb.	arb.	0,5				0,8235
0,9	arb.	arb.	0,6				0,8732
0,9	arb.	arb.	0,65				0,9764

Table 3: Comparing the AJ and the KMZ model
($a - c = 10$, different values of $b\gamma$, arb.: arbitrary)

4 Concluding remarks

In this paper we have extended the well-known models of d'Aspremont, Jacquemin (1988) and Kamien, Muller, Zang (1992) to the case of a risky R&D technology. The R&D projects of both firms may fail (independent of each other) with some positive probability. When deciding upon its R&D investment or rather the intended cost reductions, each firm has to take into account the possibility that none, both, or exactly one of the firms may innovate. In this environment we have identified two very different effects of input and output spillovers. The latters act as a kind of risk sharing device because a firm whose project has failed may still benefit from cost reductions caused by its successful competitor even though the own R&D investment is lost. If a firm is the only one to be successful, on the other hand, output spillovers reduce its profits and support the rival firm. These effects tend to *reduce* R&D investment and hence expected effective cost reductions of both firms.

The effects caused by input spillovers are completely different. Here, a "winner" benefits disproportionately more than a "loser" suffers. This is because a winner enjoys the cost reduction due to the successful innovation *and* has an associated strategic advantage in the subsequent product market competition. The loser only suffers from the corresponding disadvantage in the product market, so there is an incentive to *extend* R&D investment. Since a successful firm also benefits from the rival's expenditures this effect is stronger the larger the spillover rate.

Notably, the general tendency that output spillovers tend to imply lower expected cost reductions while input spillovers tend to enhance R&D performance relative to the deterministic case holds for all scenarios of non-cooperative and cooperative R&D. As a rule, however, they are stronger in case of cooperative RJVs, where the firms choose R&D levels to maximize joint profits (such that it is most important that someone innovates), and they are, of course, most pronounced in those RJVs in which the spillover rate is set equal to one.

The present analysis could be generalized into various directions, where two extensions appear to be close at hand and promising. First, one might incorporate both input and output spillovers in a single model and analyze how they interact with each other and which of the above effects dominates. Second, one could allow the probability of a successful project to depend on the R&D investment, where higher expenditures should lead to higher probabilities of success. In this case, additional effects will emerge and influence the firms' decisions on their R&D activities. As for the model with fixed probabilities one would expect that the asymmetric outcomes with only one successful project are crucial in this respect. In the AJ model, being the only successful firm is not too advantageous while being the only one with a failing project is not really problematical because of the risk sharing effect, see section 3.1. Thus, pushing the success of the own R&D project by increasing the corresponding probability of success via higher R&D expenditures does not seem very attractive. In the KMZ model with input spillovers matters are quite different, because being the only firm with a successful project is extremely advantageous, see section 3.2. Hence, there is yet another incentive to engage more in R&D so as to increase the probability of success of the own project. In addition, with input spillovers those higher R&D expenditures of the firms increase both probabilities of success significantly, which in turn contributes to a further increase in R&D levels. It can thus be suspected that the results established for the KMZ model in this paper will even be enhanced if the probability of success depends on R&D expenditures. For the AJ model it seems that all above results extend to this model framework, even though it is not clear whether they will be enhanced as well. Unfortunately, the models will become rather intractable for both modifications, so the corresponding extensions are left for future research.[31]

[31] First steps into those directions were already taken by Choi (1993), Combs (1993) and Martin (2002), though in different model setups.

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