Should alternative mergers or aquisitions be considered by antitrust authorities?

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Abstract

This paper illustrates how taking alternative mergers into consideration when analyzing the effects of a proposed merger may provide some information to the antitrust authorities. In particular, the use of revealed preference may allow the authorities to establish an expected upper limit on the efficiency gains obtained in a given merger that also increases the participants' market power. Such limit can then be compared to the lower threshold necessary for merger approval. The policy implications of this result are discussed.

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1 Introduction

There is an ongoing debate regarding the way antitrust authorities intervene in the markets. The debate concerns to their objectives (should the authorities aim at increasing consumer surplus or net social welfare?), the scope of their analysis (should an efficiency defense be admitted or not?) as well as the "long term" consequences of a decision (what are the effects of a rejection: the *status quo* or an alternative merger? What is expected to happen after the approval of a merger?). The issue of the authorities' objectives is the subject of recent work by Lyons (2002), Fridolfsson (2001) and Neven and Roller (2000). Nilssen and Sorgard (1998) discussed the possibility of sequential mergers, where the occurrence of a second merger depended on the authorities' decision regarding an earlier one. Some of these aspects are also briefly discussed in Horn and Stennek (2002).

Although theoretically satisfactory, the view that the authorities should foresee the consequences of approving or rejecting a given merger or acquisition in terms of its effect on other concentration operations (that might be prevented or induced) is hard to defend. Economic theory hardly presents a consensus methodology for anticipating which mergers will occur and, even if it did, the informational requirements would be prohibitive.[1] However, this does not necessarily mean that alternative mergers or acquisitions should not be taken into account. In fact, looking at other mergers that might have taken place instead of the merger(s) under analysis may reveal information that would otherwise be unknown to the authorities. This paper explores a way of obtaining such additional information using revealed preference. The revealed preference for a merger that leads to an increase in market power may enable the establishment of conditions that will narrow the admissible range for

[1] For instance, if one were to adopt the core concept as in Horn and Persson (2001) or (2001a), one would need to have information on the cost reductions obtained in any conceivable alternative merger. The full extent of the externalities involved in the formation of any possible coalition(s) would also have to be addressed. Alternatively, mergers may be assumed to result from some non-cooperative game. Such endogenous mergers have been studied, for instance by Bloch (1995) and (1996), but, in most cases, the results depend crucially on the protocol (in particular, on the order firms move).

some parameters that are unknown to the authorities.[2] We are especially concerned with the case in which these unknown variables relate to the extent of the cost reductions generated by the merger, typically considered as unobservable by the authorities (see, for instance, Farrell and Shapiro (1990) and also the 1997 revisions to section 4 of the US Merger Guidelines, where it is stated that "Efficiencies are difficult to verify and quantify in part because much of the information relating to efficiencies is uniquely in the possession of the merging firms. Moreover, efficiencies projected reasonably and in good faith by the merging firms may not be realized").

We will assume that the proposed concentration operation, which is under analysis, is the most profitable one among those considered admissible. We use the fact that a merger for market power is likely to be more profitable than an alternative merger (in which market power plays a lesser role) when efficiencies are not very significant to establish an upper bound on the magnitude of cost reductions. This happens because large unit cost reductions tend to be more profitable when firms produce larger quantities, and firms are more likely to produce a large output after a merger that does not substantially increase market power. One crucial assumption is that the cost reductions under scrutiny are not merger specific, in the sense that these may be obtained in any merger and do not depend on the identity of insiders.

For simplicity we will assume that the authorities are concerned with consumer surplus alone and also that they admit an efficiency defense (otherwise, horizontal mergers should not be approved). We show that the revealed limit on the magnitude of these cost reductions may be sufficient for such authorities to reject the merger. Section 2 presents a well known model that will be used in section 3 to illustrate how some information can be obtained after the authorities are notified of a merger for market power when alternative mergers

[2] Revealed preference has been previously applied to analyze the consistency of merger policy by Nilssen (1997).

were equally plausible. Finally, section 4 concludes.

2 The model

We consider that the authorities were notified of a two-firm merger or acquisition and are presently discussing its approval.[3] This merger is assumed to be the outcome of a sequential game with the following timing of events: (i) *Nature* randomly chooses a firm – the acquiring firm; (ii) the acquiring firm makes a bid to one of its rivals; (iii) the targeted firm accepts or rejects the bid and (iv) firms compete in prices.[4] The equilibrium of this simple game is that the most profitable acquisition will take place, with the acquiring firm bidding slightly above the target's pre-merger profits. Naturally, different industries may correspond more closely to different types of merger formation games: who moves first, how myopic are firms, is there any bargaining or do we have *take it or leave it* offers, are moves sequential or simultaneous, can outsider firms make counter offers? are just a few of the aspects that have been considered in the literature. Nonetheless, a key factor affecting which merger is likely to occur is relative profitability, and it should play an important role in most games with an endogenous merger decision. In this paper we have focused on relative profitability as the main determinant of mergers, although one could extend the argument to other cases.

To show how revealed preference can be used to obtain information regarding non merger-specific efficiencies, we will use the circular city model of Vickrey (1964) and (1999), also referred to as the Salop (1979) model to describe the price competition stage.[5] Horizontal mergers under spatial competition have been previously addressed in the literature.

[3] The results do not apply exclusively to the two-firm case. It is possible to obtain the same type of results for three-firm mergers. It is also possible to show with an example that when one considers more than one merger the same type of outcome can be obtained. These results are available upon request.

[4] Dividing exogenously the set of firms into acquiring firms and targets or into sets of firms deciding whether or not to merge in different moments of time is not uncommon in the merger literature. See, for instance, Nilssen and Sorgard (1998) or Faulí-Oller (2000).

[5] Other models could be used to obtain qualitatively similar results, such as a model of Cournot competition between sellers of differentiated products. For an example, see Brito (2002) and appendix D.

Rothschild, Heywood and Monaco (2000), Rothschild (2000) and Heywood, Monaco and Rothschild (2001) used the well-known Hotelling model to study the relationship between merger activity and location choice, while Levy and Reitzes (1992) and Brito (2003) have analyzed the impact of mergers and the incentives to merge in the circular model. Along similar lines, Braid (1999) analyzes the impact of mergers, considering differentiation along not one, but two dimensions. However, the possibility of the insiders benefitting from cost reductions is not present in any of these contributions. An exception is Braid (2001), which establishes conditions under which horizontal mergers may leave prices unchanged.

With n firms symmetrically distributed on a loop, this model has the nice property that mergers between firms that are, *ex-ante*, symmetric may have very different effects. Naturally, if the study of alternative mergers is to reveal some information, then these alternative mergers cannot have precisely the same effects as the merger under analysis. With this type of model, and assuming n is odd, we will have $(n-1)/2$ different two-firm mergers each with its specific impact on prices, profits and consumer surplus.[6]

As mentioned above, any merger is assumed to reduce the insider firms' marginal costs by the same amount. More particularly, we assume that the marginal cost of each firm is given by $D+ d/k_i$ where k_i is the amount of some cost reducing asset that firm $i = 1, ..., n$ owns.[7] After a merger between firms i and j, the marginal cost of the firm resulting from the merger will be $D+d/(k_i+k_j)$. Assuming firms are initially endowed with the same amount k of the cost reducing asset, insiders' marginal costs will change from $D+d/k$ to $D+d/(2k)$ due to the merger. Let us define $c \equiv D+d/k$ as the pre-merger marginal cost and let ε represent the post-merger reduction in marginal costs, that is, $\varepsilon \equiv D+d/k-D-d/(2k) = d/(2k)$, which is unobservable to the authorities.

As usual, consumers are assumed to be uniformly distributed along the loop (without loss

[6] For n even we would have $n/2$ different types of mergers.

[7] This type of cost function is very similar to the one by Perry and Porter (1985). This particular function has been previously used in merger analysis by Persson (1999).

of generality, of unit length) and to face quadratic transportation costs, with ty^2 measuring the cost of travelling distance y. Consumers are assumed to have unit demands with a valuation V that is high enough to guarantee that they always purchase one unit of the good. The no-merger equilibrium has each firm setting a price $P_i = c + t/n^2$ and having individual profits of $\Pi_i = t/n^3$. Note that this is precisely the profit any firm will have when there is a merger involving non-neighboring firms and when mergers do not lead to any cost savings, that is, when $d = 0$.

The following subsections present the equilibrium prices and profits for mergers between any pair of neighboring firms and between any two non-neighboring firms. These mergers differ because the former type of merger will also have a market power motive while the latter is motivated by cost reductions alone. After characterizing the effects of the alternative mergers, the third section will discuss the policy implications of the revealed information on the extent of cost reductions.

2.1 Merger between adjoining firms

In this subsection we analyze the effects of a merger between adjoining firms. Note that, in this setting, only such merger is likely to concern the authorities, especially if cost reductions are sufficiently small. In what follows, and without loss of generality, it will be assumed that the insiders are firm 1 and firm 2. After such merger, the insiders' aggregate profit function is:

$$\Pi_{1+2} = (P_1 - c_1)\left(\frac{1}{n} + n\frac{P_n - P_1}{2t} + n\frac{P_2 - P_1}{2t}\right) + (P_2 - c_2)\left(\frac{1}{n} + n\frac{P_1 - P_2}{2t} + n\frac{P_3 - P_2}{2t}\right). \tag{1}$$

In appendix A we show that the equilibrium prices are the following:

$$P_1 = P_2 = c + \frac{t}{n^2} + \frac{1}{3}\frac{t}{n^2}\frac{-3+\sqrt{3}a^n - \sqrt{3} + 3a}{a^n - a} - \frac{a + a^n}{a^n - a}x, \tag{2}$$

$$P_i = \frac{a^{i-1} + a^{2+n-i}}{a - a^n}\left(x - \frac{t}{n^2}\frac{\sqrt{3}}{3}\right) + \frac{t}{n^2} + c, i = 3, ..., n. \tag{3}$$

where $x = \varepsilon/\sqrt{3}$ and $a = (2+\sqrt{3})$. These results extend those obtained by Levy and Reitzes (1992) to include the possibility of insiders' unit cost reductions after a merger.

The effect this merger has on prices is not straightforward. On the one hand each insider has a clear incentive to raise its price because some of the consumers will shift to the other insider firm. On the other hand, lower costs induce insiders to lower their prices.

Note that this merger is always worse for consumers than the alternative merger between non-adjoining firms characterized in the next subsection, meaning that the optimal policy would be to reject it. However, the authorities are not presently empowered to reject a merger with the justifications that there is a better alternative. In most countries, the authorities can only reject a merger on the grounds that its *per se* impact on competition is negative.

As compared to the pre-merger prices, post-merger prices decrease if and only if (note that this condition for the insiders' prices to decrease also implies that average prices will decrease):[8]

$$x > \underline{x} \equiv \frac{1}{\sqrt{3}}\frac{t}{n^2} \tag{4}$$

As no merger decreases transportation costs (and these are minimized at the pre-merger equilibrium where demand between any two firms is shared equally between them), a necessary condition for the merger to be allowed is that prices decrease, that is, $x > \underline{x}$.

[8] See Nevo (2000) for an estimation in a particular industry (ready-to-eat cereals) of the percent reduction in marginal costs required for no change in predicted post-merger prices. The fact that the cost reduction necessary to leave prices unchanged after the merger is equal to the pre-merger markup was pointed out by Braid (2001).

Corresponding equilibrium profits for the insiders can be shown to be equal to

$$\Pi_{1+2} = \left(\left(1 + \frac{\sqrt{3}}{3}\right) \frac{a^n - 1}{a^n - a} \frac{t}{n^2} - (\frac{a + a^n}{a^n - a} - \sqrt{3})x \right)^2 \frac{n}{t}. \tag{5}$$

2.2 Merger between nonadjoining firms

We will now characterize the alternative mergers that might have taken place instead of the one under evaluation. In the appendix we show that a merger between firm 1 and firm l (with $2 < l < n$) will lead to the following equilibrium prices

$$P_i = \frac{\left(a^{n-l+i} + a^{i-1} + a^{n+1-i} + a^{l-i}\right)}{(1 - a^n)} x + \frac{t}{n^2} + c, i = 1, ..., l, \tag{6}$$

The way to obtain the prices set by firms $l + 1$ to n is simply to redefine l as firm $n + 2 - l$, thus obtaining

$$P_{n+2-i} = \frac{\left(a^{l-2+i} + a^{i-1} + a^{n+1-i} + a^{n-l+2-i}\right)}{(1 - a^n)} x + \frac{t}{n^2} + c, i = 2, ..., n + 2 - l. \tag{7}$$

It is easy to check that $P_1 = P_l$, for all l, and that $P_{1+j} = P_{l-j}$, for $1 < j < l$.

The merged firm, firm $1 + l$, will have joint profits given by (see appendix B):

$$\Pi_{1+l} = 2 \left(\left(\frac{\left(a^{n-l+1} + 1 + a^n + a^{l-1}\right)}{(1 - a^n)} + \sqrt{3} \right) x + \frac{t}{n^2} \right)^2 \frac{n}{t} \tag{8}$$

Insiders' profits increase with P_1 which is a function of l and is maximized for $l^* = \frac{1}{2}n + 1$ when n is even or $l^* = (n + 1)/2$ when n is odd (in this case a merger between firm 1 and firm $(n + 3)/2$ would also yield the same profits). Hence, the most profitable alternative merger involves firms at the highest possible distance. The intuition is the following: a firm with lower costs will set a lower price. This will directly affect only the two neighboring firms, which will also set a lower price. The lower the neighbor's price is, the less will the firm with the lower costs benefit. But this happens if the neighboring firm has an insider on each side. In that case it will lower its prices substantially, thus hurting both insiders. In contrast to the mergers between adjoining firms, all outsiders will see their profits decline after the merger between firm 1 and firm l, especially those located near the insiders.

The insiders' aggregate profits after the optimal alternative merger (that is the one that increases profits the most) are

$$\Pi_{1+l^*} = 2\left(\left(\frac{1+a^n+2a^{\frac{n}{2}}}{1-a^n}+\sqrt{3}\right)x+\frac{t}{n^2}\right)^2\frac{n}{t} \tag{9}$$

if the number of firms is even or

$$\Pi_{1+l^*} = 2\left(\left(\frac{1+a^n+2a^{\frac{n-1}{2}}}{1-a^n}+\sqrt{3}\right)x+\frac{t}{n^2}\right)^2\frac{n}{t} \tag{10}$$

if the number of firms is odd.

3 Revealed Preference

In this section we use the assumption that the merger taking place is the one in which insiders' profits increase the most. The merger between any two adjoining firms is more profitable than the most profitable alternative (with n even) if and only if

$$\left(\left(1+\frac{\sqrt{3}}{3}\right)\frac{a^n-1}{a^n-a}\frac{t}{n^2}-\left(\frac{a+a^n}{a^n-a}-\sqrt{3}\right)x\right)^2\frac{n}{t} > 2\left(\left(\frac{1+a^n+2a^{\frac{n}{2}}}{1-a^n}+\sqrt{3}\right)x+\frac{t}{n^2}\right)^2\frac{n}{t} \tag{11}$$

which can be simplified to:

$$x < \overline{x_{even}} \equiv \frac{t}{n^2}\frac{\left(1+\frac{\sqrt{3}}{3}\right)\frac{a^n-1}{a^n-a}-\sqrt{2}}{\left(\frac{a+a^n}{a^n-a}-\sqrt{3}\right)+\sqrt{2}\left(\frac{1+a^n+2a^{\frac{n}{2}}}{1-a^n}+\sqrt{3}\right)}. \tag{12}$$

We have therefore established an upper bound on the magnitude of the marginal cost reductions obtainable through merger.[9]

The following lemma illustrates that considering alternative mergers may provide the authorities with sufficient information to reject a given merger between neighboring firms.

Lemma 1 *Provided that $n > 6$, the fact that the merger for market power (i.e. the merger between neighboring firms) is more profitable than the best alternative merger reveals that cost reductions are insufficient to lower prices and, therefore, to raise consumer surplus.*

Proof. Recall from above that the merger reduces prices if and only if cost reductions are significant, that is, if and only if (4) is verified. Assuming that n is even, the merger between adjoining firms is more profitable than any alternative if and only if (12) holds. These two conditions cannot both be true if

$$\frac{t}{n^2}\frac{\left(1+\frac{\sqrt{3}}{3}\right)\frac{a^n-1}{a^n-a}-\sqrt{2}}{(\frac{a+a^n}{a^n-a}-\sqrt{3})+\sqrt{2}\left(\frac{1+a^n+2a^{\frac{n}{2}}}{1-a^n}+\sqrt{3}\right)}<\frac{t}{\sqrt{3}n^2}, \tag{13}$$

which is equivalent to

$$f(n)\equiv\left(1+\sqrt{3}\right)\frac{a^n-1}{a^n-a}-\sqrt{6}-(\frac{a+a^n}{a^n-a}-\sqrt{3})-\sqrt{2}\left(\frac{1+a^n+2a^{n/2}}{1-a^n}+\sqrt{3}\right)<0. \tag{14}$$

Note that

$$\frac{\partial f(n)}{\partial n}=-\sqrt{2}\frac{a^{2n}\left(a^n-\sqrt{3}a\right)+a^2+a^{n+1}\sqrt{3}+2\sqrt{a^n}\left(a^n-a\right)^2}{\left(a^n-a\right)^2\sqrt{a^n}\left(a^n-1\right)^2}\frac{\partial(a^n)}{\partial n}<0 \tag{15}$$

As $f(6)\approx 3.4816\times10^{-2}$ and $f(8)\approx -6.0089\times10^{-3}$ this proves the result: $f(n)<0$, $\forall n>6$ and n even. When n is odd a similar proof establishes that $f(n)<0$, $\forall n>6$. ■

[9] If n is odd the corresponding condition is $x<\overline{x_{odd}}\equiv\frac{\left(\left(1+\frac{\sqrt{3}}{3}\right)\frac{a^n-1}{a^n-a}-\sqrt{2}\right)}{(\frac{a+a^n}{a^n-a}+\sqrt{2}\frac{1+a^n+2a^{\frac{n-1}{2}}}{1-a^n}-\sqrt{3}+\sqrt{6})}\frac{t}{n^2}$

In order to explain these results let us consider the following: let B_i be the post-merger set of products sold by firm i. Naturally, different mergers will result in different B_i's. When cost reductions are non-existent, we assume that the merger between closer competitors is more profitable. As cost reductions increase above zero, both mergers will lead to higher profits but the rate at which profits will grow is different. If the insiders' profit after the merger between closer competitors grows with ε at a slower rate then, after some point, the best alternative merger will become more profitable.

Letting $D_j(\overrightarrow{p})$ denote the demand for brand j as a function of all competing brands' prices $\overrightarrow{p}$, firm i's profits can be written as

$$\pi_i = \sum_{j \in B_i} (p_j - c_j) D_j(\overrightarrow{p}) \tag{16}$$

To check how these profits change with ε let us analyze

$$\frac{d\pi_i}{d\varepsilon} = \sum_{j \in B_i} \frac{\partial \pi_i}{\partial p_j} \frac{\partial p_j}{\partial \varepsilon} + \sum_{l \notin B_i} \frac{\partial \pi_i}{\partial p_l} \frac{\partial p_l}{\partial \varepsilon} + \frac{\partial \pi_i}{\partial \varepsilon} = \sum_{l \notin B_i} \left(\sum_{j \in B_i} \pounds_j \epsilon_{jl} \frac{p_j D_j(\overrightarrow{p})}{p_l} \right) \frac{\partial p_l}{\partial \varepsilon} + \sum_{j \in B_i} D_j(\overrightarrow{p}), \tag{17}$$

where $\pounds_j = \frac{p_j - c_j}{p_j}$ and ϵ_{jl} is the cross-elasticity of demand for brand j with respect to brand l.

Note that the second term in (17), measuring the *direct effect* of a reduction in costs, is always larger when the merger does not involve very close competitors, that is, under the "alternative" merger. This is true for different types of models, provided that the dimension of the markets in which both types of targets operate is similar. If firms compete in prices and sell close substitutes there is a clear incentive to increase prices, given that part of the demand that will be lost will shift to other products of the merged firm. The higher the equilibrium price, the lower the sales for insiders will be and therefore the lower the *direct effect*. Under Cournot competition, for instance, firms have an incentive to lower output as well, because the resulting increase in price will benefit their "partner" in the merger whereas before the concentration it would benefit a competitor. Naturally, an alternative merger with a firm not selling in the same market will not have this effect.

The first term in (17) measures the *strategic effect* and is negative. By lowering costs, insiders induce rivals to lower their prices ($\partial p_l / \partial \varepsilon < 0$). This will have a negative impact on insiders' profits because some demand will shift away towards these rivals, provided that the cross-elasticities are strictly positive. Comparing this *strategic effect* under each merger is not as straightforward. After a merger for market power, the insiders' markups ($\mathcal{L}_j$'s) are typically larger than after the alternative merger. But if a merger for market power takes place it is natural that the cross elasticities of the insiders' demand with respect to the outsiders' prices will be smaller. This happens because after a merger for market power the products with the larger ϵ_{jl} elasticities will be sold by the same firm whereas the alternative merger, in which market power plays a less important role, will involve firms with smaller ϵ_{jl}'s. In particular, in the circular city model addressed in this paper, after the merger for market power has taken place there will be two positive cross-elasticities with respect to he outsiders whereas in the sequence of the alternative merger there would be four. This makes the negative *strategic effect* stronger in the case of the merger between non-adjoining firms. Nevertheless, the *direct effect* prevails in the circular model and insiders' profits increase more with ε when the merger involves the distant firms. This means that for a sufficiently high level of cost savings ε, the merger for market power should not have been the outcome of the merger formation stage given that there is an alternative merger that is more profitable. If the merger involves adjoining firms and all our assumptions hold, it must be that cost reductions are below the level presented in (12).

As the number of firms decreases, the merger for market power becomes relatively more profitable than the alternative. Therefore, the involved cost reductions would have to be very large for the alternative merger to be even more profitable. Although it is still possible to establish a limit to the extent of cost reductions, this limit does not necessarily imply that cost reductions are smaller than the minimum needed for prices to decrease. Even so, it could still be used by the authorities to update their conviction about the distribution of the unknown ε. This happens for the cases of $4 \leqq n \leqq 6$: the revealed upper bound ($\overline{x_{even}}$ or $\overline{x_{odd}}$) does not exclude the possibility of the merger between adjoining firms leading to a decrease in prices. Despite the fact that the revealed threshold is not incompatible with a decrease in prices, this bound could still be used to update the authorities prior beliefs regarding the true extent of ε and eventually correct an initial decision.

Figure 1 presents the revealed upper bounds ($\overline{x_{even}}$ or $\overline{x_{odd}}$) as well as the thresholds ($\underline{x}$) below which the merger under analysis increases prices. The upper limit on x that ensures that there is a positive demand for each firm ($\overline{x_1}$) is also plotted (see next subsection).

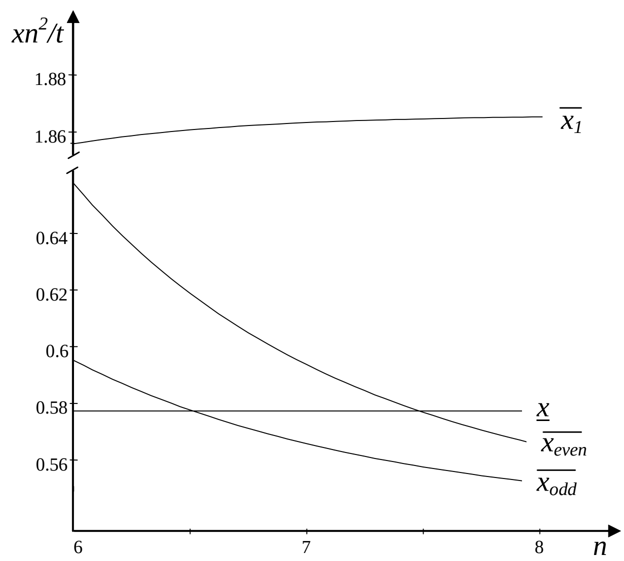

Figure 1: Revealed upper bounds ($\overline{x_{even}}$ and $\overline{x_{odd}}$) and minimum threshold for decrease in prices ($\underline{x}$)

3.1 "Large" cost reductions

Note that throughout the analysis it was implicitly assumed that cost reductions were sufficiently small so that, for all two-firm mergers considered, any firm would still sell a positive amount in equilibrium. For any outsider verifying its first-order condition we can see that having a positive demand is equivalent to pricing above its marginal cost c. Any outsider to any merger between non-adjoining firms sells above marginal cost if and only if:[10]

$$x < \overline{x}_1 \equiv \frac{1}{2}\frac{a^n - 1}{a^{n-1} + a}\frac{t}{n^2} \tag{18}$$

For the nearest outsiders to the merger between adjoining firms, the analogous condition is

$$x < \overline{x_2} \equiv \left(\frac{a^n - a}{a^2 + a^{n-1}} + \frac{\sqrt{3}}{3}\right)\frac{t}{n^2} \tag{19}$$

It is easy to check that $\overline{x}_1 < \overline{x_2}$ meaning that we must assume that $x < \overline{x}_1$ for the results above to be valid.

Assume now that the cost reduction x is very large, such that in *any* of the two mergers considered (between adjoining firms or between the most distant firms), both outsiders located near the merger would exit the market.[11] This means that

$$x \;>\; \overline{x_2} \equiv \left(\frac{a^n - a}{a^2 + a^{n-1}} + \frac{\sqrt{3}}{3}\right)\frac{t}{n^2} \tag{20}$$

$$x \;>\; \overline{x_3} \equiv \frac{a^n - 1}{\left(a^{\frac{1}{2}n+1} + a + a^{n-1} + a^{\frac{1}{2}n-1}\right)}\frac{t}{n^2} \tag{21}$$

where (20) refers to the merger between adjoining firms and (21) refers to the merger between the most distant firms.[12] It is easy to show that $\overline{x_2} > \overline{x_3}$, that is, if cost reductions are such

[10] This condition is obtained for firm 2 in the case of a merger between firms 1 and firm $l = 3$.

[11] Note that we are considering that there is some cost of re-entering the market so that, after exiting, a firm will not be able to re-enter immediately if prices increase.

[12] Note that we are not claiming that this is the most profitable alternative but merely that it is an alternative concentration.

that after a merger by two adjoining firms, nearby rivals will exit the market ($x < \overline{x_3}$) then nearby rivals would also exit the market when the merger involved nonadjoining firms ($x < \overline{x_2}$). This is an expected result given that prices will be higher when the merger involves firms that compete for the same customers and this will benefit outsiders, making them less prone to exit the market.

In appendix C we show that for $x > \overline{x_2}$ the merger between non-adjoining firms is always more profitable, independently of the extent of cost reductions. For $\overline{x_3} < x < \overline{x_2}$ rivals would not exit the market following the merger between adjoining firms, meaning that the insider's aggregate profit would be even smaller than in the previous case. Hence, in either case, the revealed preference argument cannot be applied in the presence of "large" cost reductions because the alternative distant merger is actually more profitable than the one under scrutiny.

4 Conclusions

We showed, with a simple example, that taking alternative mergers or acquisitions into account may be relevant for the antitrust authorities. It has been argued in the literature that the authorities should not be myopic and should consider the future consequences of a merger in terms of leading the way for (or, instead, preempting) other mergers. However, a better understanding of the incentives firms have to merge is necessary in order to foresee the long term consequences of approving or rejecting a proposed merger. We claim that such understanding may also enable the authorities to obtain information regarding the short term effects of the merger under scrutiny. Knowing why one merger was chosen over another may enable the authorities to establish several conditions regarding the extent of cost reductions. Using a simple circular city model where Perry and Porter (1985) type of cost reductions are assumed to exist, we showed that the announcement of a merger in which both market power and cost reductions are present is sufficient to establish that such a merger will increase prices, provided that there are more than six firms. The main assumptions needed for this result to hold are the following: (i) the criteria that leads to one merger being chosen over another is profitability (that is, the more profitable merger is the one taking place), (ii) cost reductions are identical, regardless of the identity of insiders.

The qualitative results do not change substantially when other types of models are used. The results are driven by the fact that a merger for market power will lead to a lower aggregate output than an alternative merger in which insiders sell more differentiated products and this does not depend on the type of model considered. The main argument does not hold solely for the case in which the merging firms are the closest competitors or for the cases such that the alternative mergers do not create any increase in market power at all. To make possible the use of this type of argument, it is only needed that the insiders would produce a sufficiently larger output in the case of the alternative merger, for the same level of unit costs.

For any proposed real world acquisition involving sellers of sufficiently close competitors it is, in general, possible to identify alternative targets where the potential increase in market power is more limited. These targets may operate in different geographic markets or sell products that are not perceived as close substitutes to those of the acquiring firm or at least not as close as those sold by the actual target firm. In some countries, authorities already analyze the probability of entry into the market *after* a merger or acquisition takes place. Although slightly different, establishing the existence and likelihood of an alternative concentration involving a firm selling a poorer substitute should not require any additional know-how: alternative concentration operations can be interpreted as entry by the acquiring firm into another (although related) market. Having identified a likely alternative target, the authorities should then check whether the alleged unit cost reductions present in the merger under analysis would also be likely to hold for the alternative concentration.

Naturally, this methodology does not apply to all types of mergers. Cases in which insiders claim some cost savings and show that these are specific to the insiders involved are not covered by the analysis presented here. However, when insiders fail to substantiate these claims or when the authorities can expect similar marginal cost reductions in alternative mergers, the revealed preference argument presented in this paper may be used. This may happen, for instance, when firms own complementary assets, (and the acquiring firm owns an asset that is different from the one owned by any other firms') or when differentiation arises solely from advertising or location decisions by firms that use the same production technology.

If nothing else, this analysis highlights the relevance for future research of considering alternative mergers, of understanding the mechanisms that lead to one merger taking place instead of another and of establishing whether alleged cost reductions due to merger are indeed firm specific or not.

A Merger between adjoining firms

A.1 Equilibrium prices

After a merger between firm 1 and firm 2, insiders' aggregate profits are:

$$\Pi_{1+2} = (P_1 - c_1)\left(\frac{1}{n} + n\frac{P_n - P_1}{2t} + n\frac{P_2 - P_1}{2t}\right) + (P_2 - c_2)\left(\frac{1}{n} + n\frac{P_1 - P_2}{2t} + n\frac{P_3 - P_2}{2t}\right) \tag{22}$$

and the corresponding first-order conditions can be simplified to

$$\begin{bmatrix} -4 & 2 & 0 & ... & 0 & 1 \\ 2 & -4 & 1 & 0 & ... & 0 \\ 0 & 1 & -4 & 1 & 0... & 0 \\ : & & & & & : \\ 0 & ... & 0 & 1 & -4 & 1 \\ 1 & 0 & ... & 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ : \\ P_{n-1} \\ P_n \end{bmatrix} = \begin{bmatrix} -t2/n^2 - 2c_1 + c_2 \\ -t2/n^2 - 2c_2 + c_1 \\ -t2/n^2 - 2c_3 \\ : \\ -t2/n^2 - 2c_{n-1} \\ -t2/n^2 - 2c_n \end{bmatrix}$$

Denoting the i^{th} row by l_i, the following operations over the rows

$$l_1 - \sum_{j=1}^{n-2} u_j l_{n-j} \text{ and } l_2 - \sum_{j=1}^{n-3} u_j l_{j+3} - u_{n-2} l_1 \tag{23}$$

where each coefficient u_j can be obtained recursively from $u_0 = 0, u_1 = 1$ and $u_j = 4u_{j-1} - u_{j-2}$, will result in two equations that can be solved to yield (following the same steps as in appendix B and setting $c_1 = c_2 = c - \varepsilon$ and $c_3 = ... = c_n = c$):

$$P_1 = P_2 = c + \left(1 + \frac{\sqrt{3}}{3}\right) \frac{a^n - 1}{a^n - a} \frac{t}{n^2} - \frac{a + a^n}{a^n - a} x \tag{24}$$

$$P_i = \frac{a^{i-1} + a^{2+n-i}}{a - a^n} \left(x - \frac{t}{n^2} \frac{\sqrt{3}}{3}\right) + \frac{t}{n^2} + c, i = 3, ..., n \tag{25}$$

where $x = \varepsilon/\sqrt{3}$ and $a = (2 + \sqrt{3})$.

A.2 Equilibrium profits

After such merger, insiders' profits are

$$\Pi_{1+2} = (P_1 - c_1) \left(\frac{1}{n} + n\frac{P_n - P_1}{2t}\right) + (P_2 - c_2) \left(\frac{1}{n} + n\frac{P_3 - P_2}{2t}\right) \tag{26}$$

Using the first-order conditions, this boils down to

$$(P_1 - c_1)\left(\frac{1}{n} + n\frac{(-2t/n^2 + P_1 - c_1)}{2t}\right) + (P_2 - c_2)\left(\frac{1}{n} + n\frac{-2t/n^2 + P_2 - c_2}{2t}\right) = (P_1 - c_1)^2 \frac{n}{t}. \tag{27}$$

Replacing P_1 and c_1 with the equilibrium values results in (5).

B Merger between nonadjoining firms

B.1 Equilibrium prices

Firm i's problem is to maximize its profit and the set of n first-order conditions is

$$P_{i+1} - 4P_i + P_{i-1} = \frac{-2t}{n^2} - 2c_i, i = 1, ..., n. \tag{28}$$

These can be simplified and written in matrix terms (after multiplying by n) as

$$\begin{bmatrix} -4 & 1 & 0 & ... & 0 & 1 \\ 1 & -4 & 1 & 0 & ... & 0 \\ 0 & 1 & -4 & 1 & 0... & 0 \\ : & & & & & : \\ 0 & ... & 0 & 1 & -4 & 1 \\ 1 & 0 & ... & 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ : \\ P_{n-1} \\ P_n \end{bmatrix} = \begin{bmatrix} \frac{-2t}{n^2} - 2c_1 \\ \frac{-2t}{n^2} - 2c_2 \\ \frac{-2t}{n^2} - 2c_3 \\ : \\ \frac{-2t}{n^2} - 2c_{n-1} \\ \frac{-2t}{n^2} - 2c_n \end{bmatrix}$$

Denote the row concerning P_i by l_i. Performing the following operations over the rows

$$l_1 - \sum_{j=1}^{n-2} u_j l_{n-j} \text{ and } l_n + \sum_{j=1}^{n-2} u_{j+1} l_{n-j} \tag{29}$$

results respectively in the following two equations in P_1 and P_2 :

$$-(4+u_{n-2})P_1+(1+u_{n-1})P_2 = \frac{-2t}{n^2}-2c_1+\sum_{j=1}^{n-2}u_j(\frac{2t}{n^2}+2c_{n-j}) \tag{30}$$

$$(1+u_{n-1})P_1-u_nP_2 = \frac{-2t}{n^2}-2c_n-\sum_{j=1}^{n-2}u_{j+1}\left(\frac{2t}{n^2}+2c_{n-j}\right) \tag{31}$$

Noting that $u_j=\frac{a^j-a^{-j}}{2\sqrt{3}}$ with $a=(2+\sqrt{3})$ we can write these equations as

$$\begin{aligned}-(4+\frac{a^{n-2}-a^{-n+2}}{2\sqrt{3}})P_1+(1+\frac{a^{n-1}-a^{-n+1}}{2\sqrt{3}})P_2 &= \frac{-2t}{n^2}-2c_1\\ &+\frac{t}{n^2}\sum_{j=1}^{n-2}\frac{a^j-a^{-j}}{\sqrt{3}}+\sum_{j=1}^{n-2}\frac{a^j-a^{-j}}{\sqrt{3}}c_{n-j}\end{aligned} \tag{32}$$

$$\begin{aligned}(1+\frac{a^{n-1}-a^{-n+1}}{2\sqrt{3}})P_1-\frac{a^n-a^{-n}}{2\sqrt{3}}P_2 &= \frac{-2t}{n^2}-2c_n\\ &-\frac{t}{n^2}\sum_{j=1}^{n-2}\frac{a^{j+1}-a^{-j-1}}{\sqrt{3}}-\sum_{j=1}^{n-2}\frac{a^{j+1}-a^{-j-1}}{\sqrt{3}}c_{n-j}\end{aligned} \tag{33}$$

It is straightforward to show that

$$\sum_{j=1}^{n-2}\frac{a^j-a^{-j}}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}\frac{1-a^{n-1}-a^{-n+2}+a}{1-a} \tag{34}$$

$$\sum_{j=1}^{n-2}\frac{a^{j+1}-a^{-j-1}}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}\frac{1-a^{n+1}-a^{-n+2}+a^3}{a\,(1-a)} \tag{35}$$

Let the insiders, firms 1 and l (by assumption nonadjoining firms) see their unit costs decline by ε. All other firms will have unit costs c. Therefore both equations can be simplified and solved to obtain

$$P_1 = \left(\frac{a^{l-1}+a^{n-l+1}+a^n+1}{1-a^n}\right)x+\frac{t}{n^2}+c \tag{36}$$

$$P_2 = \left(\frac{a^{l-2}+a^{n-l+2}+a^{n-1}+a}{1-a^n}\right)x+\frac{t}{n^2}+c \tag{37}$$

where $x=\varepsilon/\sqrt{3}$.

For all other firms up to firm $l-1$ we can write

$$P_{i+1} = \frac{-2t}{n^2} - 2c + 4P_i - P_{i-1},\ i \geq 2 \tag{38}$$

Solving this difference equation we obtain:

$$P_i = \frac{\left(a^{n-l+i} + a^{i-1} + a^{n+1-i} + a^{l-i}\right)}{(1-a^n)} x + \frac{t}{n^2} + c, i = 1, ..., l \tag{39}$$

Redefining l as $n+2-l$ allows us to obtain the prices for the remaining firms.

B.2 Equilibrium profits

From the first-order conditions it is clear that individual profits are

$$\Pi_i = (P_i - c_i)^2 \frac{n}{t} \tag{40}$$

Thus, after the merger, insiders' aggregate equilibrium profits are given by

$$\Pi_{1+l} = \left(\left(P_1 - c + x\sqrt{3}\right)^2 + \left(P_l - c + x\sqrt{3}\right)^2\right)\frac{n}{t} = 2\left(\left(\frac{a^{n-l+1} + 1 + a^n + a^{l-1}}{1-a^n} + \sqrt{3}\right)x + \frac{t}{n^2}\right)^2 \frac{n}{t} \tag{41}$$

Aggregate insiders' profits increase with P_1 which is a function of l. We will now establish which l maximizes P_1 and consequently profits. With this purpose the function $\left(a^{n-l+1} + a^{1-1} + a^{n+1-1} + a^{l-1}\right)$ will be minimized:

$$\frac{\partial\left(a^{n-l+1} + a^{1-1} + a^{n+1-1} + a^{l-1}\right)}{\partial l} = 0 \Leftrightarrow l = \frac{1}{2}n + 1 \tag{42}$$

The second order conditions for minimum are verified:

$$(\ln(a))^2 \left(a^{n-l+1} + a^{l-1}\right) > 0 \tag{43}$$

Hence, equilibrium P_1 (and insiders' profits) is maximized for $l^* = \frac{1}{2}n + 1$ for n even. For an odd number of firms the most profitable merger would involve firm 1 and either firm $(n+1)/2$ or firm $(n+3)/2$.

C "Large" cost reductions

Assume that $x > \overline{x_2}$, meaning that the outsiders located near the merger will exit the market after the merger between the adjoining firms and also after the merger between the most distant firms.

Lemma 2 *After a merger by two adjoining firms that leads the two firms closer to the merger location to exit the market, insiders' profit maximizing prices are, for* $n \geq 5$.

$$P_1^c - c + \varepsilon = \frac{22\sqrt{3}+120}{249} \frac{83a^n - 6158 - 3555\sqrt{3}}{13a^n - 1862 - 1075\sqrt{3}} \frac{t}{n^2} - \frac{1}{3} \frac{a^n\left(2\sqrt{3}-21\right) + 2023\sqrt{3} + 3504}{13a^n - 1862 - 1075\sqrt{3}} \varepsilon \tag{44}$$

Proof. After such merger and after two firms have abandoned the market, the remaining firms will no longer be symmetrically located on the loop. The insiders will be located at a larger distance from their closest competitors. This distance, $2/n$, is twice as large as the distance separating any other firms in the market. Let d_i represent the distance between firm i and $i+1$. Let us assume, without loss of generality, that the merging firms are firms 1 and 2 and that the firms that exit the market are firms 3 and n. Then, the problem facing any firm $i = 5, ..., n-1$ is to maximize its profit, given by

$$\pi_i = (P_i - c)\left(\frac{P_{i+1} - P_i}{2t/n} + \frac{P_{i-1} - P_i}{2t/n} + \frac{1}{n}\right) \tag{45}$$

and the f.o.c. are

$$P_{i-1} - 4P_i + P_{i+1} = -(2\frac{t}{n^2} + 2c) \tag{46}$$

while firms 4 and $n-1$ have the following profit function

$$\pi_i = (P_i - c)\left(\frac{P_{i+1} - P_i}{2t/n} + \frac{P_{i-1} - P_i}{4t/n} + \frac{3}{2n}\right) \tag{47}$$

and the corresponding f.o.c. are

$$P_{i-1} - 6P_i + 2P_{i+1} = -6\frac{t}{n^2} - 3c \tag{48}$$

Finally, the insiders have the following profit function

$$\begin{aligned} \pi_i \quad &= \quad (P_1 - c + \varepsilon)\left(\frac{P_2 - P_1}{2t(1/n)} + \frac{P_{n-2} - P_1}{2t(2/n)} + \frac{1}{2}\left((2/n) + (1/n)\right)\right) + \\ &\quad +(P_2 - c + \varepsilon)\left(\frac{P_2 - P_3}{-2t(2/n)} + \frac{P_1 - P_2}{2t(1/n)} + \frac{1}{2}\left((1/n) + (2/n)\right)\right) \end{aligned} \tag{49}$$

with the f.o.c. for P_1 given by

$$-6P_1 + 4P_2 + P_{n-1} = -6\frac{t}{n^2} - (c - \varepsilon) \tag{50}$$

In matrix terms, the system of $n-2$ f.o.c. is expressed as:

$$\begin{bmatrix} -6 & 4 & 0 & 0 & ... & 0 & 1 \\ 4 & -6 & 1 & 0 & ... & 0 & 0 \\ 0 & 1 & -6 & 2 & ... & 0 & 0 \\ 0 & 0 & 1 & -4 & ... & 0 & 0 \\ : & : & : & : & ... & : & : \\ 0 & 0 & 0 & 0 & ... & -4 & 1 \\ 1 & 0 & 0 & 0 & ... & 2 & -6 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_4 \\ P_5 \\ : \\ P_{n-2} \\ P_{n-1} \end{bmatrix} = \begin{bmatrix} -6\frac{t}{n^2} - (c-\varepsilon) \\ -6\frac{t}{n^2} - (c-\varepsilon) \\ -6\frac{t}{n^2} - 3c \\ -2\frac{t}{n^2} - 2c \\ : \\ -2\frac{t}{n^2} - 2c \\ -6\frac{t}{n^2} - 3c \end{bmatrix}$$

Let $m = n-2$ represent the new number of active firms. Let l_i denote the matrix i^{th} row and let us perform the following operations over the rows:

$l_1 - \sum_{i=1}^{m-4} u_i l_{m-i} - \frac{u_3}{2} l_3 + (-u_{m-4} + 2u_{m-3})$

This results in an equation in P_1 and P_2 (which, by symmetry, are equal) that can be solved to obtain

$$P_1 = P_2 = \frac{(1 - 3u_{m-3} + u_{m-4})\left(-\left(6\frac{t}{100} + c - \varepsilon\right)\right) + \frac{u_{m-3}}{2}(t\frac{6}{100} + 3c) + (\frac{t}{100}2 + 2c)\sum_{j=1}^{m-4} u_j}{-2 + 5.5u_{m-3} - 2u_{m-4}} \tag{51}$$

with $u_j = \frac{a^j - a^{-j}}{2\sqrt{3}}$ and $\sum_{j=1}^{n} u_j = \frac{1}{2\sqrt{3}}\left(\frac{a^{n+1} - a + a^{-n} - 1}{a-1}\right)$. This can be simplified to

$$P_1 = P_2 = c + \frac{t}{n^2}\frac{2}{249}\left(11\sqrt{3} + 60\right)\frac{83a^{m-3} - 211 + 112\sqrt{3}}{13a^{m-3} - 19 + 8\sqrt{3}} - \frac{2}{3}\left(\sqrt{3} + 9\right)\frac{a^{m-3} - 2 + \sqrt{3}}{13a^{m-3} - 19 + 8\sqrt{3}}\varepsilon \tag{52}$$

Further simplification yields the result. ■

Lemma 3 *After a merger by two firms at the maximum possible distance from each other that leads the two closest neighbors to each insider to exit the market, insiders' profit maximizing prices are (provided that n is even and $n \geq 8$)*

$$P_1^{nc} - c + \varepsilon = \frac{(\frac{84}{13} + \frac{18}{13}\sqrt{3})\left(-13a^{n/2} + 229 + 132\sqrt{3}\right)}{3\left(-13a^{n/2} + 499 + 288\sqrt{3}\right)}\frac{t}{n^2} + \left(\frac{2\left(\sqrt{3} + 9\right)\left(a^{n/2} - 26 - 15\sqrt{3}\right)}{3\left(-13a^{n/2} + 499 + 288\sqrt{3}\right)} + 1\right)\varepsilon \tag{53}$$

Proof. After such merger firms will no longer be symmetrically located on the loop. Each insider will be located at a larger distance from both closest competitors. This distance, $2/n$, is twice as large as the distance separating any other firms. Again, let d_i represent the distance between firm i and $i+1$. Let us assume, without loss of generality, that the merging firms are firm 1 and $n/2+1$ and the firms that exit the market are firms 3 and n (near firm 1) and $n/2$ and $n/2+2$ (near firm $n/2+1$). Then, the problem facing any firm $i = 4, ..., n/2-1$ and $i = n/2+3, ..., n-3$ is to maximize its profit, given by

$$\pi_i = (P_i - c)\left(\frac{P_{i+1} - P_i}{2t/n} + \frac{P_{i-1} - P_i}{2t/n} + \frac{1}{n}\right) \tag{54}$$

and the f.o.c. are

$$P_{i-1} - 4P_i + P_{i+1} = -(2\frac{t}{n^2} + 2c) \tag{55}$$

Firms 3, $n/2-2$, $n/2+3$ and $n-2$ will have the following profit function (exemplified here for firm 3)

$$\pi_3 = (P_3 - c)\left(\frac{P_1 - P_3}{2t(2/n)} + \frac{P_4 - P_3}{2t(1/n)} + \frac{1}{2}\left((2/n) + (1/n)\right)\right) \tag{56}$$

with the f.o.c.

$$P_1 - 6P_3 + 2P_4 = -6\frac{t}{n^2} - 3c \tag{57}$$

Finally, the insiders have the following profit functions

$$\begin{aligned}&(P_1 - c + \varepsilon)\left(\frac{P_3 - P_1}{2t(2/n)} + \frac{P_{n-1} - P_1}{2t(2/n)} + \frac{1}{2}\left((2/n) + (2/n)\right)\right) + \\ &+(P_1 - c + \varepsilon)\left(\frac{P_{n/2+3} - P_{n/2+1}}{2t(2/n)} + \frac{P_{n/2-1} - P_{n/2+1}}{2t(2/n)} + \frac{1}{2}\left((2/n) + (2/n)\right)\right)\end{aligned} \tag{58}$$

Insider's f.o.c for P_1 can be simplified to

$$P_{n-1} - 4P_1 + P_3 = -2(4\frac{t}{n^2} + c - \varepsilon) \tag{59}$$

Note that this can be simplified due to the symmetry of this game: As $P_{1+j} = P_{n/2+1+j} = P_{n/2+1-j} = P_{n+1-j}$ we can write this system as

$$\begin{bmatrix} -4 & 1 & 0 & ... & 0 & 0 & 1 \\ 1 & -6 & 2 & ... & 0 & 0 & 0 \\ 0 & 1 & -4 & ... & 0 & 0 & 0 \\ : & : & : & ... & : & : & : \\ 0 & 0 & 0 & ... & -4 & 1 & 0 \\ 0 & 0 & 0 & ... & 1 & -4 & 1 \\ 1 & 0 & 0 & ... & 0 & 2 & -6 \end{bmatrix} \begin{bmatrix} P_1 \\ P_3 \\ P_4 \\ : \\ P_{n/2-3} \\ P_{n/2-2} \\ P_{n/2-1} \end{bmatrix} = \begin{bmatrix} -2(\frac{t}{n^2}4 + c - \varepsilon) \\ -3(\frac{t}{n^2}2 + c) \\ -2(\frac{t}{n^2} + c) \\ : \\ -2(\frac{t}{n^2} + c) \\ -2(\frac{t}{n^2} + c) \\ -3(\frac{t}{n^2}2 + c) \end{bmatrix}$$

Making the following operations over the matrix rows,

$l_1 - \sum_{j=1}^{n/2-5} u_j l_{j+2} - \frac{u_{n/2-4}}{2} l_{n/2}$

we have,

$$\begin{aligned} &(-4 - \frac{1}{2} * u_{\frac{n}{2}-4})P_1 + (-u_{\frac{n}{2}-5} + 1 + 3 * u_{\frac{n}{2}-4})P_3 = \\ &\qquad = -2(\frac{t}{n^2}4 + c - \varepsilon) - \sum_{j=1}^{\frac{n}{2}-5} u_j \left(-2(\frac{t}{n^2} + c)\right) - \frac{u_{\frac{n}{2}-4}}{2} \left(-3(\frac{t}{n^2}2 + c)\right) \end{aligned}$$

and

$$-4P_1 + 2P_3 = -2(\frac{t}{n^2}4 + c - \varepsilon) \tag{60}$$

where (60) is merely the f.o.c. for P_1. This can be solved in order to P_1, yielding

$$P_1 = \frac{2\left(\sqrt{3}+9\right)\left(-a^{n/2} + 26 + 15\sqrt{3}\right)\varepsilon - \left(\frac{84}{13} + \frac{18}{13}\sqrt{3}\right)\left(-13a^{n/2} + 229 + 132\sqrt{3}\right)t/n^2}{-3\left(-13a^{n/2} + 499 + 288\sqrt{3}\right)} + c \tag{61}$$

■

Lemma 4 *Assuming that cost reductions are such that after a merger the firms closer to the merger location will exit the market, post-merger insiders' individual profits are given by*

$$\pi_i = \frac{1}{2}\frac{n}{t}(P_i - c_i)^2,\ i = 1, l \tag{62}$$

when insiders are nonadjoining firms and by

$$\pi_1 = \pi_1 = (P_1 - c + \varepsilon)^2 \left(\frac{1}{2t(2/n)}\right) \tag{63}$$

when firms are adjoining.

Proof. A given store's profit is given by:

$$\pi_i = (P_i - c_i)\left(\frac{P_i - P_{i+1}}{-2td_i} + \frac{P_{i-1} - P_i}{2td_{i-1}} + \frac{1}{2}(d_{i-1} + d_i)\right) \tag{64}$$

When a merger involves nonadjoining firms, its profit maximizing conditions are

$$\left(\frac{P_i - P_{i+1}}{-2td_i} + \frac{P_{i-1} - P_i}{2td_{i-1}} + \frac{1}{2}(d_{i-1} + d_i)\right) = (P_i - c_i)(\frac{1}{2td_i} + \frac{1}{2td_{i-1}}) \tag{65}$$

from where we get each store's equilibrium profits

$$(P_i - c_i)^2 \frac{d_{i-1} + d_i}{2td_i d_{i-1}} = \frac{1}{2}\frac{n}{t}(P_i - c_i)^2 \tag{66}$$

When a merger involves neighboring firms we have the following f.o.c., for P_1

$$\left(\frac{P_2 - P_1}{2t(1/n)} + \frac{P_{n-2} - P_1}{2t(2/n)} + \frac{1}{2}\left((2/n) + (1/n)\right)\right) +$$
$$+ (P_1 - c + \varepsilon)\left(\frac{-1}{2t(1/n)} + \frac{-1}{2t(2/n)}\right) + (P_2 - c + \varepsilon)\frac{1}{2t(1/n)} = 0$$

As $P_1 = P_2$ we get

$$\frac{P_2 - P_1}{2t(1/n)} + \frac{P_{n-2} - P_1}{2t(2/n)} + \frac{1}{2}\left((2/n) + (1/n)\right) = (P_1 - c + \varepsilon)\left(\frac{1}{2t(2/n)}\right) \tag{67}$$

Therefore, firm 1's individual profit is given by

$$\pi_1 = (P_1 - c + \varepsilon)^2 \left(\frac{1}{2t(2/n)}\right) \tag{68}$$

■

Proposition 5 *Independently of the marginal cost reductions resulting from merging, the merger between non-adjoining firms is always more profitable, provided that* $n \geqslant 8$*.*

Proof. Using the expressions above for post-merger profits, we conclude that the merger between non-adjoining firms is more profitable if

$$(P_1^c - c + \varepsilon)^2 \frac{1}{4}\frac{n}{t} < \frac{1}{2}\frac{n}{t}(P_i^{nc} - c_i)^2 \Leftrightarrow (P_1^c - c + \varepsilon) < \sqrt{2}(P_i^{nc} - c + \varepsilon) \tag{69}$$

where $P_1^c(P_i^{nc})$ denotes equilibrium prices when the merger involves adjoining (non-adjoining) firms. Using the lemmas above we have

$$\sqrt{2}\,(P_1^{nc} - c + \varepsilon) = k_1 \frac{t}{n^2} + k_2 \varepsilon \tag{70}$$
$$P_1^c - c + \varepsilon = k_3 \frac{t}{n^2} + k_4 \varepsilon \tag{71}$$

with

$$k_1 = \frac{\sqrt{2}(\frac{84}{13}+\frac{18}{13}\sqrt{3})\left(-13a^{n/2}+229+132\sqrt{3}\right)}{3\left(-13a^{n/2}+499+288\sqrt{3}\right)} \tag{72}$$

$$k_2 = \sqrt{2}(\frac{2\left(\sqrt{3}+9\right)\left(a^{n/2}-26-15\sqrt{3}\right)}{3\left(-13a^{n/2}+499+288\sqrt{3}\right)}+1) \tag{73}$$

$$k_3 = \frac{2}{249}\left(11\sqrt{3}+60\right)\frac{83a^n-6158-3555\sqrt{3}}{(13a^n-1862-1075\sqrt{3})} \tag{74}$$

$$k_4 = (-\frac{2}{3}\left(\sqrt{3}+9\right)\frac{a^n-97-56\sqrt{3}}{13a^n-1862-1075\sqrt{3}}+1) \tag{75}$$

We will show that $k_1 > k_3$ and $k_2 > k_4$ for $n \geqq 8$, meaning that (69) holds.

We will start by defining $v \equiv a^{n/2} > a^4 = 193.994\,8$ and noting that $v^2 > a^8 = 37634.0$..It is easy to show that, for $n \geqq 8$

$$k_1 - k_3 > 0 \Leftrightarrow \frac{g_1(v)}{\left(13v-499-288\sqrt{3}\right)\left(13v^2-1862-1075\sqrt{3}\right)} > 0 \tag{76}$$

with

$$\begin{aligned} g_1(v) = &-2978\,475\sqrt{3} - 5158\,870 - 4127\,096\sqrt{2}\sqrt{3} - 7148\,340\sqrt{2} - \\ &- v\left(675\,759\sqrt{3} + 436\,678\sqrt{2}\sqrt{3} + 756\,360\sqrt{2} + 1170\,470\right) + \\ &+ v^2\left(171\,000\sqrt{3} + 131\,400\sqrt{2}\sqrt{3} + 227\,646\sqrt{2} + 296\,239\right) + 923v^3 \end{aligned}$$

Note that $g_1(v)$ is increasing in v for $v > 1.808\,394$ and also that $g_1(a^4) > 0$. Also,

$$k_2 - k_4 > 0 \Leftrightarrow \frac{g_2(v)}{\left(13v-499-288\sqrt{3}\right)\left(13v^2-1862-1075\sqrt{3}\right)} > 0 \tag{77}$$

with

$$g_2(v) = 2193\,218 + 1266\,255\sqrt{3} + v\left(-12\,390 - 7153\sqrt{3} + 4672\sqrt{2}\sqrt{3} + 8092\sqrt{2}\right) + \\ - v^2\left(5672\sqrt{3} + 1252\sqrt{2}\sqrt{3} + 9825 + 2168\sqrt{2}\right) + 143v^3$$

This function $g_2(v)$ is increasing in v for $v > 120.\,232\,3$ and $g_2(a^4) > 0$. Thus, for $n \geqq 8$ we have $k_1 > k_3$ and $k_2 > k_4$, which proves the result. ■

D Mergers under Cournot competition

In order to illustrate that the results carry over to other types of competition, let us consider the well known case of the linear model of Cournot competition. In order to have mergers with different degrees of market power let us consider w *identical* and independent markets.

Let us assume that the inverse demand function in each market is given by $P(X) = A - X$, that n independent firms compete in each market and that the pre-merger and post-merger costs are defined as in the text.

It is easy to show that after a k-firm merger in any of these markets the equilibrium outputs are

$$X(k, \varepsilon) = \frac{(A - c)(n - k + 1) + \varepsilon}{n - k + 2} \tag{78}$$

$$X_I(k, \varepsilon) = \frac{A - c + \varepsilon(n - k + 1)}{n - k + 2} \tag{79}$$

where $X(k, \varepsilon)$ and $X_I(k, \varepsilon)$ represent aggregate output and insiders' output, respectively. We assume that the alternative acquisition to this k-firm concentration is to acquire one firm in each of $k - 1$ markets. The merger for market power is more profitable if and only if ε is sufficiently small:[13]

$$\left(\frac{A-c+\varepsilon\left(n-k+1\right)}{n-k+2}\right)^2 > k\left(\frac{A-c+\varepsilon n}{n+1}\right)^2 \Leftrightarrow \frac{\varepsilon}{A-c} < \frac{k+\sqrt{k}-1-n}{\sqrt{k}+1+n\left(n-k+2\right)} \tag{80}$$

We now turn to the lower bound on ε such that welfare will increase. Calculating welfare from the equilibrium above, we have that social welfare increases following the merger if and only if

$$\frac{\varepsilon}{A-c} > \frac{-\left(n-k+3\right)+\frac{n-k+2}{n+1}\sqrt{-2k^2+4kn+6k+n^2}}{2\left(k-n\right)^2+6(n-k)+3} \tag{81}$$

Comparing both thresholds for $\varepsilon/(A-c)$ we have the following result

Proposition 6 *Assume that there are w equal markets in which firms compete a la Cournot with constant marginal costs. If there are no barriers to entry through acquisition in other markets and efficiency gains are the same independently of which firm is acquired, then an acquisition in the same market the acquiring firm operates in reveals that efficiency gains are insufficient to increase welfare, for any number of firms operating in each market, n, or the number of acquired firms, k.*

Proof. It is necessary to prove that

$$\frac{-\left(n-k+3\right)+\frac{n-k+2}{n+1}\sqrt{-2k^2+4kn+6k+n^2}}{\left(2\left(k-n\right)^2+6(n-k)+3\right)} > \frac{k+\sqrt{k}-1-n}{\sqrt{k}+1+n\left(n-k+2\right)} \tag{82}$$

[13] One should also make the assumption that the proposed merger is strictly profitable in addition to being more profitable than the alternative considered here. To ensure merger profitability one should have i) a sufficiently high number of insiders, ii) sufficiently high unit cost reductions ε or iii) sufficiently high fixed cost savings. If one does not allow for any fixed cost savings, merger profitability amounts to the following condition on ε :

$$\frac{\varepsilon}{A-c} > \frac{\sqrt{k}\left(n-k+2\right)-\left(n+1\right)}{\left(n-k+1\right)\left(n+1\right)}$$

A necessary condition for the existence of an $\varepsilon/(A-c)$ such that the merger between competitors is both profitable and the most profitable is that at least 80% of the firms in the market are insiders. Note that this last condition could be discarded if one assumed that profitability originated from fixed cost reductions.

But this is equivalent to

$$\sqrt{-2k^2+4kn+6k+n^2} > \frac{\left(2k^2+2k\sqrt{k}-3\sqrt{k}-3kn-4k+n-2n\sqrt{k}+n^2\right)(n+1)}{-\sqrt{k}-1-n(n-k)-2n} \tag{83}$$

If the right-hand side is negative this proves the result. If it is positive the inequality above is equivalent to (after squaring both sides and simplifying)

$$g(k,n) \equiv \sqrt{k}\left(4kn+3kn^2+2k+1-8n^2-4n-4n^3\right)+2\left(k\left(2n^2+2+3n\right)-n(n+1)^2\right)<0 \tag{84}$$

As

$$\begin{aligned}
g(1,n) &= -(n-1)\left(6n^2+11n+7\right)<0 & (85)\\
g(n,n) &= -4\left(\sqrt{n}\right)^5-\left(\sqrt{n}\right)^7-2\left(\sqrt{n}\right)^3+\sqrt{n}+2n^3+2n+2n^2<0 \text{ for } n \geq 2 & (86)\\
\frac{\partial g^2(k,n)}{\partial k^2} &> 0 & (87)
\end{aligned}$$

we have that (84) is verified for all k and n. ■

An alternative is to check the necessary cost reductions for a decrease in price after the merger, instead of an increase in welfare. Prices decrease after the k-firm merger if and only if

$$\frac{A+c(n-k+1)-\varepsilon}{n-k+2} < \frac{A+cn}{n+1} \Leftrightarrow \frac{\varepsilon}{(A-c)} > \frac{k-1}{n+1} \tag{88}$$

It is easy to check that

$$\frac{k-1}{n+1} > \frac{k+\sqrt{k}-1-n}{\sqrt{k}+1+n(n-k+2)} \Leftrightarrow nk-\sqrt{k}>0 \tag{89}$$

which is always true. Thus, we have established that price will increase and welfare will decrease following a merger for market power, if the cost reductions involved are such that this merger is more profitable than the one between a similar number of firms operating in different markets.

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