The Design, Implementation, Performance, and Tâtonnement Dynamics of a Large, Multi-Market, Policy Constrained Auction

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The paper reports on the theory, design and outcome of a large, multiple market auction built on principles of general equilibrium. The auction distributed 18,788 entitlements to operate electronic gaming machines in 176 interconnected markets to 363 potential buyers representing gaming establishments subject to policy constraints across multiple dimensions of the allocation. The multi-round auction, conducted in one day, produced over $600M in revenue. Revealed dynamics of interim allocations and new statistical tests provide evidence of multiple market convergence hypothesized by classical theories of general equilibrium. Results support considering tâtonnement as an empirical process and not a purely theoretical construct.

¹ The financial support for analyzing the data and developing this report provided to Plott by the Rising Tide Foundation (Grant Number: RTF-19-500) and the John Templeton Foundation (Grant Number: 58067) are gratefully acknowledged as is the technical support of the Caltech Laboratory for Experimental Economic and Political Science. Limited data analysis for scientific purposes was allowed by the Victorian government. The cooperation and help of William Stevenson of Intelligent Market Systems LLC were fundamental.
Section 1: Introduction
The paper reports on the design and field implementation of a large, multiple market and policy constrained auction based on general equilibrium theory as opposed to a traditional theory of auctions. The auction involved the sale of 18,788 ten-year entitlements for the use of electronic gaming machines in Victoria Australia, in May, 2010. Policy issues dictated the operation of 176 interconnected markets to allocate sales of these licenses to 363 potential buyers representing licensed gaming establishments. The auction outcomes satisfied policy constraints, was conducted in one day, and produced over $600M in revenue. The design rested on principles of an exchange economy in which bidders are assumed to have well-formed preferences with choices guided by the classical tâtonnement model of market adjustment. The multiple market interdependencies created by policy constraints, the size of the problem, and policy constrained limitations on timing challenged any obvious application of traditional forms of auctions. The theoretical framework used to interpret the results is informed by competitive market principles that demonstrate convergence to an equilibrium with many features predicted by the classical theory. The paper reviews the policy background, the theoretical architecture, some key features of the laboratory experimental testbedding, and detailed discussions of results and dynamic performance, providing the first field demonstration of tâtonnement as an empirical model of price formation.

We pose two broad, overriding questions to evaluate the mechanism’s performance relative to policy-focused market designs in which simplified and limited models play a fundamental role. The first question is a form of proof of principle evaluating a basic question about the policy’s implementation. (1) “Was the implementation successful in satisfying the policy goals? Or, equivalently, did the implementation do what it was supposed to do?” The second question investigates consistency in relation to the underlying theory. (2) “Were the models used to guide the design successful in the sense that the observed market behaviors are

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2 Methodology and strategies for using alternative auction formats are open questions beyond the scope of our analysis. We present an auction that addressed the given allocation problem. In doing so, we demonstrate the successful application of the theoretical principles underlying the auction’s design and empirical properties of price formation and general equilibration across multiple markets. How or whether it is possible to adapt other types of auctions to address the allocation problem are left as open questions.

3 These questions were first posed by Plott (1994) as a methodology for capturing the relationship between experimental testbeds and policy implementation. The literatures addressing alternative methodologies for addressing design issues requires reviews beyond the scope of this paper and played no role in the design.
consistent with the principles used in the design? Or, equivalently, did it do what it did for the right reasons?"

In the light of those questions three results stand out. First, the auction outcomes satisfied all of the complex policy constraints. Second, the auction provides substantial field support, the first of its kind, for basic principles of classical general equilibrium and competitive economic theory including price discovery and convergence dynamics. Third, the data support the application of these principles together with the auction architecture that guided the gaming machines design.

The auction was the result of the Victorian government’s efforts to change policies regarding gambling operations and regulations in the state. In 2008, the government initiated a reorganization of this industry by changing the method of allocating the entitlements to operate electronic gaming machines (e.g. poker and slot machines) and the method of finance. Historically, the distribution of gaming machines was managed by two large corporations. The machines were allocated to businesses consistent with local policies governing their use. Finance had been based on the revenue produced by the machines with roughly a third going to the local establishment, a third to the managing company, and a third to the government.

Governmental concerns with the historical policy reflected a desire to better control gambling and concomitant social problems, gambling-related government public finance, and a desire for conformity with frameworks used for economic regulation. The transition policy was intended to accommodate several broad economic objectives that evolved through legislative and legal discussions. Auction-style mechanisms emerged as possible tools. The policy objectives did not include revenue maximization, which would have resulted in a supply reductions to monopolistic determined levels. Instead, the aggregate supply quantity was dictated to be near historical levels. The mechanism design goal was to discover demand for entitlements and allocate them by an auction process. Authorities wished to create minimal economic dislocations and a climate where future regulatory efforts could be based on principles of decentralized competition and operator profits. The auction was also implemented to allow for possible entry and shifting of entitlements from past use to reflect underlying economic value rather than historical administrative practices.

The resulting auction mechanism and its implementation present a remarkable success for the decades of abstract theorizing about general equilibrium in classical economics. As
demonstrated in Section 2’s presentation of the auction structure, general equilibrium theory proved to be quite useful in practice when defining the mechanism. Further, Section 3 and the associated Appendix A demonstrate the auction procedures and illustrate how lessons learned from testbed experiments in a laboratory environment provided insights for its field implementation. Partial equilibrium analysis in Section 4 illustrates the operation of competitive market principles supporting an efficient allocation of licenses within each market and demonstrates bidders act as price-takers in these markets.

Empirically evaluating the overall allocation of licenses across markets and verifying the auction followed the principles used in the theory of its design to reach an efficient general equilibrium presents a challenging exercise. To approach these questions, we model system behavior as driven by excess demand dynamics interpreted through an important principle, “excess demand revealed at the margin.” This excess demand, readily measured from observed bidding behavior and interim allocations, was first observed in the testbed experiments. Section 5 demonstrates that this excess demand becomes exhausted through the auction mechanism’s bid revision process. We describe the dynamic behavior of the auction mechanism in Section 6, characterizing the total revenues and surplus generated as bidding rounds progressed. Section 7 investigates the relationship of price dynamics across markets to the revealed excess demand in all other markets. This analysis verifies the conditions for stability that would lead to an efficient multi-market allocation and general equilibrium across all market segments given the policy constraints. Taking advantage of the rich data available, these novel statistical tests present the first empirical verification of equilibrating dynamics based on the principles of tâtonnement. Section 8 concludes, presenting a summary of the findings from the implementation of an economic mechanism to address the allocation problem at the heart of a complex government policy project.

**Section 2: Auction Structure**

The economics literature on auctions reports several alternative auction designs that were considered but seemed inapplicable given the structure of the policy constraints and scale of the problem. The obvious candidates include sealed bid auctions similar to U.S. Treasury auctions

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4 These testbed experiments are designed to develop intuition about bidder behavior and support judgements about auction procedures. The exploratory nature of this application stands in contrast to traditional applications of experimental economics to evaluate a fully articulated theory against alternative hypotheses after collecting large sample sizes to provide power for classical statistical testing.
and the sequential forms of auctions such as the simultaneous, multi-round, ascending prices auction (SMRA) used to allocate the electromagnetic spectrum (Milgrom, 2000; Bichler and Goeree, 2017). Both types of auctions have the capacity to accommodate large numbers of bidders competing for large numbers of items. However, governmental policy constraint inequalities that limit the allocations of available units to different subsets of markets, together with limitations on markets in which individuals can participate, lead to structural complexities not addressed in traditional applications of those forms of auctions. For example, in the traditional, multi-market (SMRA) auction the number of units for sale in a given market is fixed so movements of units among markets does not occur. Missing from the traditional auctions are market institutions that facilitate efficiency-improving movement of resources among markets, typical of general equilibrium. Arbitrage based resource movements that might take place are prevented by the ascending price structure. Efficiency issues related to the limited bidder feedback of sealed bid processes are exacerbated by the heterogeneous preferences and limited information available to bidders about preferences and numbers of bidders. Time allowed for the auction presents a further challenge, as the Victorian machine auction was permitted only one day. In contrast, simultaneous ascending price auctions can require months from start to finish and how it might be speeded without compromising efficiency involves open questions.

Given the auction scale (numbers of units and markets), human information processing, decision speeds and error correction delays, open questions exist regarding how to make traditional auction systems work while guaranteeing that all governmental constraints would be satisfied. Of course, in the absence of an impossibility theorem, it is conceivable that modified versions of the traditional auction architectures could be developed and used. However, we were (and still are) unaware of appropriate modifications so both classes of auction architectures in their traditional forms were ruled out by scale and policy constraints.

2.1 Policy Constraints

The auction design problem was to simultaneously sell rights, subject to many overlapping policy constraints. Key policy constraints were focused on the nature of the businesses that were allowed to participate in the auction and acquire entitlements. Half of the 27,500 entitlements were to be sold to businesses classified as Hotels, which were larger venues possessing substantially greater value for the gaming machines. The other half was to be sold to smaller venues called Clubs that cater to local populations. This reflected differences between
the economic environment and social purposes of these venues and differing political bases in various Victorian communities.

For purposes of the allocation Victoria was divided into 88 geographic regions, and each region had a maximum number of entitlements based on area population or other regulations. These constraints, established by legislation, placed limits on the saturation of machines relative to population and were motivated, in part, by social, community, and health issues related to gambling. Additional policy concerns regarding the geographic distribution of entitlements resulted in the creation of a single set geographic regions designated as metropolitan and maximum number of entitlements that could be allocated to the set.\(^5\)

Accounting for these constraints, and neglecting the metropolitan designation, each entitlement has two characteristics: the type of venue (Club or Hotel) and the geographical region (88 distinct areas) in which that establishment is allowed to operate. This required 176 simultaneous markets. While the underlying resource in all markets is an “entitlement”, the policy restrictions differ across clubs, hotels and areas differ. So, from an economic and modeling perspective the items sold in these different markets are completely different commodities even though all are “entitlements”.

2.2 Determining Allocations and Prices in a Continuous Model

The basic auction design can best be understood in the context of a continuous model that assumes away complexities created by underlying integers. Such complexities will be addressed in the later sections that analyze the data from the auction.

As introductory notation, let \( i \in I \) index each establishment and let \( a_i \in \mathcal{A} \) denote the area in which the establishment seeks to obtain licenses.\(^6\) The indicator variable \( h_i \in \{0,1\} \)

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\(^5\) In particular, geographical areas designated as “metropolitan” were limited to obtain no more than 80 percent of all entitlements. Since this constraint did not bind at any point during the auction, we do not discuss the features implemented to accommodate the constraint should it be binding.

\(^6\) Each establishment corresponds to a policy-defined “venue” meaning the location where the machines would be housed and operated. Bids are submitted by the venue and the entitlement is issued to the venue where the machine must be located and counts against the area constraints. A business might own more than one venue and employ a representative bidder authorized to submit bids for more than one venue that the business might own. Auction rules were designed to minimize coordination between bidders. All bidders were located in a large convention hall with cubicles from which other bidders could not be viewed (see Figure 7). To constrain the information that bidders receive, external communication devices (e.g., mobile phones) were prohibited and monitors were tasked with ensuring no unauthorized communication occurred amongst the bidders. While a representative bidder might tender bids for all of the venues from the same terminal each bid is attached to a specific venue and recorded as made by that venue. As such, even though a single business might own more than one venue even in one particular area, the
identifies the $i^{th}$ establishment type, equaling 1 if establishment $i$ is a hotel. The model begins with the classical economic postulate that bidders have well-formed preferences and objectives that are revealed only through their actions taken in the market. Assume that each bidder submits a continuous valuation schedule $V_i(x)$ reporting their total willingness to pay for an allocation of $x$ entitlements. Assume that $D_i(x) = \frac{\partial V_i(x)}{\partial x}$, representing the bidder’s marginal willingness to pay is non-negative, monotonic, and (weakly) decreasing, so that the representation of bidders’ demand schedules for licenses is continuous and monotonically weakly decreasing. While bids are observable, the underlying preferences that would produce a valuation function are not observable and exist as a basic postulate of economic theory. All inquiries that might have yielded such information were prevented by the government probity policies that conversations with bidders were strictly prohibited.

The system allocates entitlements to bidders to maximize the total cumulative reported value of the allocation. Define $X$ as a vector of allocations with the $i^{th}$ entry $x_i$ representing the allocation to establishment $i$. The total market value, $V(X)$, is the aggregate value of bidders’ willingness to pay for their given allocations:

$$V(X) = \sum_{i=1}^{l} V_i(x_i)$$

As discussed in the previous subsection, allocations must satisfy the set of policy constraints facilitated by defining the total allocations to each area by venue type:

$$x_{ac} = \sum_{i\in I} x_i (1-h_i) 1\{a_i = a\} \quad x_{ah} = \sum_{i\in I} x_i h_i 1\{a_i = a\} \quad x_a = x_{ac} + x_{ah}$$

Imposing these definitions as equality constraints on the maximization problem identifies the shadow costs for allocating a marginal license to each area and bidder type. The total allocation to an area, $x_a$, must satisfy the constraint defined by the Victoria government, denoted $\bar{x}_a$:

$$x_a \leq \bar{x}_a, \quad \forall a \in A.$$  

Administering Victoria-wide constraints on the total allocations to Hotels and Clubs, respectively denoted $\bar{x}_{H}$ and $\bar{x}_{C}$, is facilitated by similar constraints:

$$x_H = \sum_{a \in A} x_{ah} \quad x_C = \sum_{a \in A} x_{ac}$$

The system records and treats each venue as a separate entity. Special rules and monitoring were imposed for multiple venues operating under the same ownership.
Each of these aggregated allocations must satisfy government-imposed inequality constraints, \( x_H \leq \overline{x}_H \) and \( x_C \leq \overline{x}_C \).

The Lagrangian for the constrained allocation problem is:

\[
\mathcal{L}(X) = V(X)
- \sum_{a \in A} \lambda_{ac}(x_{ac} - \sum_{i \in I} x_i (1 - h_i) 1\{a_i = a\})
- \sum_{a \in A} \lambda_{alt}(x_{alt} - \sum_{i \in I} x_i h_i 1\{a_i = a\})
- \sum_{a \in A} \lambda_a(x_a - x_{ac} - x_{alt})
- \sum_{a \in A} \mu_a(\overline{x}_a - x_a)
- \lambda_H(x_H - \sum_{a \in A} x_{alt}) - \lambda_C(x_C - \sum_{a \in A} x_{ac})
- \mu_H(\overline{x}_H - x_H) - \mu_C(\overline{x}_C - x_C)
- \mu_O(\overline{x} - x_H - x_C)
\]

Here, the shadow costs denoted by \( \mu \) impose binding equality constraints for aggregating allocations within different market segments and Kuhn-Tucker shadow costs denoted by \( \lambda \) correspond to non-negative inequality constraints that may or may not bind on the final allocation.

### Table 1: Lagrangian Shadow Costs and Approximate Prices

<table>
<thead>
<tr>
<th>Type of License</th>
<th>Binding Constraints</th>
<th>Shadow Costs</th>
<th>Approximate Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Club License in Unconstrained Area</td>
<td>Total Allocation</td>
<td>( \mu_O )</td>
<td>( \mu_O )</td>
</tr>
<tr>
<td>Hotel License in Unconstrained Area</td>
<td>Total Allocation</td>
<td>( \mu_O )</td>
<td>( \mu_O + \mu_H )</td>
</tr>
<tr>
<td></td>
<td>Hotel Allocation</td>
<td>( \mu_H )</td>
<td></td>
</tr>
<tr>
<td>Club License in Constrained Area ( a )</td>
<td>Total Allocation</td>
<td>( \mu_O )</td>
<td>( \mu_O + \mu_a )</td>
</tr>
<tr>
<td></td>
<td>Area Allocation</td>
<td>( \mu_a )</td>
<td></td>
</tr>
<tr>
<td>Hotel License in Constrained Area ( a )</td>
<td>Total Allocation</td>
<td>( \mu_O )</td>
<td>( \mu_O + \mu_H + \mu_a )</td>
</tr>
<tr>
<td></td>
<td>Hotel Allocation</td>
<td>( \mu_H )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area Allocation</td>
<td>( \mu_a )</td>
<td></td>
</tr>
</tbody>
</table>

Given sufficient regularity conditions, the optimization problem (1) solves for a Pareto efficient allocation of licenses and the second welfare theorem states that this allocation can be supported as the competitive equilibrium of a market mechanism with associated prices. Those
prices are approximated by the shadow costs for the binding constraints from the optimization problem. In practice, the constraint on aggregate Club allocations was not binding whereas the constraint for aggregate Hotel allocations did bind. Further, not all areas’ allocation constraints were binding, so these constraints only affected the prices paid for licenses within those areas facing binding constraints. Accounting for these binding constraints, the relationship between the shadow costs and approximate prices for different types of licenses is summarized in Table 1.

**Section 3: Auction Procedures to Determine Allocations and Prices**

In practice, bidders submit schedules reporting their willingness to pay for different allocations through a bidding mechanism described in Section 5. Here, we describe the submitted bid schedules under the simplifying assumption that reported bid schedules reveal bidders’ true valuations to illustrate properties of the allocation, recognizing that bidders’ reported willingness to pay and valuations may diverge in practice.

### 3.1 Reported Bid Schedules and Accumulated Bid Functions

Each establishment submits a bid schedule containing \(L_i\) entries specifying its willingness to pay for each marginal unit. The lists’ entries are sorted by descending bid and the entry at the \(j^{th}\) level in the bid schedule is denoted \(B_{ij} = (b_{ij}, x_{ij})\). The bid price \(b_{ij}\) reports the price the bidder is willing to pay and the quantity \(x_{ij}\) reports the number of marginal units the bidder demands at that price in addition to the units they would receive from any higher-priced bids. Bidder \(i\)’s cumulative bid schedule, denoted \(X_i(p)\), reports the total quantity of bids with reported value weakly greater than \(p\), computed by summing \(X_i(p) = \sum_{d=1}^{L_i} x_{id} 1\{b_{id} \geq p\}\).

From the reported bid schedule, let \(\hat{V}_i(x)\) denote bidder \(i\)’s cumulative reported valuation for an allocation of \(x\) licenses. We calculate \(\hat{V}_i(x)\) by summing the area under the bidder’s reported bid function up to the quantity of \(x\).7 Since \(\hat{V}_i(x)\) can be evaluated at any quantity, it can also be stated as a function of price evaluated at bidder \(i\)’s cumulative bid schedule. Denoted

\[\hat{V}_i(x) = \sum_{j=1}^{L_i} b_{ij} \left[ x_j \left( \sum_{y=1}^{j} x_y < x \right) + \left( x - \sum_{y=1}^{j} x_y \right) 1\left\{ \sum_{y=1}^{j} x_y \geq x \right\} 1\left\{ \sum_{y=1}^{j} x_y < x \right\} \right] \]

---

7 The formula for \(\hat{V}_i(x)\) is a bit convoluted, due to the discrete nature of bids, but can be calculated as:
\( \hat{v}_i(X_i(p)) \), this represents the total valuation bidder \( i \) assigns to the licenses they would bid for if the price were \( p \).

### Table 2: Sample Individual and Cumulative Bid Schedules

<table>
<thead>
<tr>
<th>List Entry [( l )]</th>
<th>Bid [( b_i )]</th>
<th>Bid Quantity [( x_i )]</th>
<th>Price [( p )]</th>
<th>Cumulative Bid [( X(p) )]</th>
<th>Cumulative Reported Value [( \hat{v}_i(X_i(p)) )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>5</td>
<td>100</td>
<td>5</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
<td>5</td>
<td>96</td>
<td>5</td>
<td>500</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>10</td>
<td>95</td>
<td>10</td>
<td>975</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>5</td>
<td>90</td>
<td>20</td>
<td>1,875</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>80</td>
<td>25</td>
<td>2,275</td>
</tr>
</tbody>
</table>

Table 2 provides a hypothetical example of an individual bid schedule and its translation into cumulative bids and reported valuations. Panel A presents a schedule with four entries at four different price points for an establishment that bids for up to 25 units if the price is no greater than 80. The Cumulative Bid Schedule in Panel B demonstrates how different prices translate into total quantity desired by that bidder at each price.

### 3.2 Implemented Allocation Rule

The auction allocation system approximates the continuous model presented in section 2.2 with the elicited valuations as reported in Section 2.3. Measuring total welfare by the aggregated reported license valuations, \( \hat{v}(X) = \sum_{i \in A} \hat{v}_i(x_i) \), and maintaining all the relevant constraints, the system determines the allocation by optimizing:

\[
\hat{L}(X) = \hat{v}(X) \\
- \sum_{a \in A} \lambda_{ac}(x_{ac} - \sum_{i \in A} x_i (1-h_i) 1\{a_i = a\}) \quad \text{Constraints on area Club and Hotel allocations} \\
- \sum_{a \in A} \lambda_{alt}(x_{alt} - \sum_{i \in I} x_i h_i 1\{a_i = a\}) \\
- \sum_{a \in A} \lambda_{ac}(x_{ac} - x_{alt}) \\
- \sum_{a \in A} \mu_H(\bar{x}_H - x_H) - \mu_C(\bar{x}_C - x_C) \\
- \mu_H(\bar{x}_H - x_H) \quad \text{Constraints on aggregate Club and Hotel allocations} \\
- \mu_C(\bar{x}_C - x_C)
\]

The discrete nature of the problem introduces a number of complexities in solving the optimization problem from equation (2). These are well known features of integer programming.
optimization, requiring tie-breaking rules, non-uniqueness of shadow costs, and potential for multiple solutions due to overlapping constraints. We resolved tied bids through a first come, first serve rule. All bids are time-stamped, so if multiple bids are submitted at the market price allocations are made to the bids according to their arrival time. The market clearing prices need not be unique if the quantity demanded at a price exactly equals the supply to that market. This was addressed by adding a very small quantity to every bid (essentially picking the best price for the seller) so the quantity demanded at a price is always slightly above the integer parts. Finally, although multiple constraints could bind and thus create multiple price solutions, such multiplicity could only arise from relationships between metropolitan constraints and area constraints. The problem never surfaced because the metropolitan constraint was never binding.

Given the adjustments to the optimization problem necessary to address these practical considerations, the shadow costs from the optimization only approximate the shadow costs from the continuous Lagrangian in equation (1). As we will see in Section 4, these approximations do not induce disequilibrium in any individual market’s allocation.

3.3 Preview of Dynamic Auction Features and Bidding Revisions

The system arrived at its final allocation after progressing through a series of 63 bidding rounds. Each round began with establishments submitting provisional bid functions. The auction algorithm computes the allocation and prices based on these bid functions. These provisional prices and allocations are then announced at the conclusion of the round. Thus bidders observe the quantity of entitlements that they would purchase, and the price they would pay per unit, if the auction were to stop in that round.

Given this information, bidders are aware of the prices revised bids need to meet or beat in order to obtain additional licenses. Before the next round begins, bidders can use this information to revise their submitted bid functions, subject to the restriction that they increase their original bid by at least a specified minimal increment. This restriction induces an ascending auction format with excluded bid pricing as bidding progresses from round to round with the quantity demanded at the announced pricing revealed to the auction system but not published.

While individual bidders cannot decrease leading bids, some market prices can go down while the prices in other markets go up due to the interdependence of the markets and shifting supplies. A simple example illustrates this unintuitive feature. Suppose there are three units that are to be allocated in two markets, {commodity 1 in mkt1 and commodity 2 in mkt2} and there
are six agents, \( \{a,b,c,d,e,f\} \), each wants only one unit. Suppose agents \( a,b,c \) have assessed values \((4, 0.05, 0.05)\), respectively, for commodity 1 and agents \( d,e,f \) have assessed values \((1,1,1)\) for commodity 2 and all agents reveal their value in their bids. Prices, which equal the bid of the last accepted unit in a market, are respectively, \((4,1)\) for markets mkt1 and mkt 2 and respective volumes are \((1,2)\). Total revenue is \( 4+2 = 6 \) and (maximized) total “utility” is \( 6 = 4+1+1 \). Now, suppose agent \( b \) increases assessed demand value from 0.05 to 2 and raises her bid to 2. As a result of this demand and bidding increase, the surplus maximizing feature of the process automatically shifts an entitlement unit from market 2 to market 1. Prices as determined by the marginal bid, decrease to 2 in mkt 1 and remain 1 in mkt 2 while volume increases to 2 in mkt 1 and decreases to 1 in mkt 2. Total revenue is \( 5 = 2+2+1 \) and total “utility” is \( 7 = 4+2+1 \). Notice that prices and total revenue have both decreased but total “utility” has increased. Bids increased but the prices decreased. Price decreases can occur in other circumstances that involve multiple market coordination and derived demands. Notice also that the system response need not be related to revealed excess demands at existing prices.

The system initiates an ending process based on the number of significant revisions in individually submitted bid schedules and the resulting patterns of market price changes. When the ending process is initiated, bidders are notified about the number of potential rounds the market will remain open. This process terminates with an announcement that the market may close in the subsequent round and bidders are given a final opportunity to revise their bid schedules. Absent any significant revisions in this next round, the auction closes. After this last round, the bidders pay the announced price for their market for each entitlement awarded.

3.4 Design, Design Decisions and Testbed Methods

The auction design was influenced by testbed experiments outlined in Appendix A. These methods are especially useful in unique applications when little is known about a new form of institution to be implemented and/or the environment in which it might be implemented. The objectives of testbed experiments are to inform judgements and make predictions regarding behavior and outcomes in a wide range of environments and stress-test those predictions in very

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8 Such instances are apparent later in the paper when we present the time-series of market prices and area price premia in Figure 8. Between Period 59 and 60, the Hotel Base Price increases by $914 from $31,261 to $32,175, while the Area 185 Price Premium correspondingly decreased by $914 from $38,739 to $37,825. The allocation of licenses to hotels in Area 185 and the price paid by those hotels ($70,000) didn’t change during these periods. The price paid for club licenses in Area 185, however, declined from $44,239 to $43,325, while the allocation remained constant since there were no inframarginal bids.
unlikely or extreme events. Rather than seeking to test specific statistical hypotheses, the focus of testbed experiments can be broad ranging in order to explore what might happen and how problems might be avoided.

While the policy objectives of the machines auction were clearly stated, the operational design and the process itself faced many challenges aside from the size and technical complexity of the allocation problem. First, probity concerns about information advantages among bidders prevented all inquiry, feedback, discussions with or exercises involving potential bidders or the circumstances they faced. Second, the time of the auction was limited to one day. Third, no comparable auction processes existed in practice or in theory to provide any useful history of applications.

The design exercise began with a theoretical sketch of an auction where: (i) bidder preferences were limited to a single establishment; (ii) agents submit “truthful” demand functions; (iii) the auction winner is chosen by maximizing the revealed value of the allocation; and (iv) policy limits regarding multiple markets exist as constraints on final allocations. This sketch suggested the theory of general equilibrium as a possible source of principles that might guide implementation and auction behavior. The subsequent and evolving design judgements informed by principles drawn from the theory of general equilibrium and by testbed experiments are discussed further in the appendix.

Laboratory experimental studies of multiple markets lend strong support to the basic principle that market dynamics are driven by excess demands which guide the system to a general equilibrium. In spite of theoretical discussions that raise doubts about applicability of general equilibrium, such as Ackerman (2002) or papers in Bridel (2011) and references found in Mukherji (2019), the design decisions rested on the fact that the convergence principles are evident in a wide range of experiments organized with both buyers and sellers operating in single and multiple markets. This is found in the multiple markets of international trade in Noussair, et al. (1995), disequilibrium dynamics in Gillen, et al. (2021), Anderson, et al. (2004), and Hatfield, et al. (2016), and finance in Asparouhova, et al. (2003). Importantly, the fundamental role of excess demand in driving price adjustments exists when bidding is expressed as demand functions (Goeree and Lindsay, 2016). Convergence is also seen in call markets (Plott and Pogorelskiy, 2017), and complex auctions exhibit the same tendency. A prime example is the simultaneous, ascending price auction used for the auction of the broadcast spectrum that was
successfully demonstrated in the U.S. and other countries with convergence properties clearly demonstrated in experiments (Plott and Salmon, 2004). The principle is observed in complex networks such as power grids (Chao and Plott, 2009) and combinatorial auctions (Lee, et al., 2014). However, experiments have demonstrated the unreliability of the pure tâtonnement institution (Plott, 1988; 2001) and served as a warning about the incomplete nature of theory as a model of actual behavior. Still, in other experiments exploring institutional variations a key behavioral feature, demand revelation at the margin, had been identified in an unpublished working paper on fuel efficiency under the CAFE constraint (Katz and Plott, 2008).

The use of repeated rounds as a design foundation, as opposed to a one shot, sealed bid computation, is based on findings from general equilibrium related experiments that consistently reflect increasing efficiency, dynamic equilibration and convergence over time. The scale of the gaming machines auction and nature of the multi-unit demand for licenses required analyzing functions (inverse demand functions) as opposed to separate bids on (thousands of) individual units. Competitive theory as applied to smooth demand functions identified theoretical and technical relationships among bids, prices (as Lagrangian multipliers), equilibrium (as a competitive equilibrium), allocations and efficiency. The actual auction required solving an integer-constrained, linear program for the allocation problem each round.

The scale of the design problem derives from the size of markets and the number of markets, units, and bidders. The testbed exercises outlined in Appendix A established the economic performance and technical control of the system in relation to scale. Two performance measures were useful tools to refine the rules of the auction. The first was market efficiency measured as consumer surplus as developed by Plott and Smith (1978). Since the cost of the entitlements is ignored in this government allocation problem, this measure is simply the sum of observed willingness to pay divided by the maximum sum of willingness to pay given experimentally induced preferences. The second was speed of convergence measured in terms of number of rounds required for equilibration.

The scale testing experiments started small and were scaled up. Over 40 different experiments were conducted and some were repeated to explore problems they exposed. The largest testbed employing human subject participants operated with 50 markets and 160 participants at a subject payments cost of $8,866. Larger scales were simulated with multiple
computers programmed to place bids to test network configurations, processing speeds, and computation reliability and speed.

Experiments revealed the auction could coordinate convergence for prices and allocations across multiple markets, equilibrating derived demand with the available supply. Allocations and prices typically ended near the predictions of the general competitive equilibrium and thus efficiency tended to be in the high 90% levels and often near 100%. Such high performance occurred at all tested levels of scale.

While each experiment examined multiple dimensions of performance, many focused on two specific areas. The first related to real time control of price movement and procedures for ending the auction. Previous experimental work revealed that bidding incentives and stopping rules are important for performance. The second broad area included market performance, efficiency and reliability in both a software and bidding behavior.

The timing of the auction rounds needed adjustments to account for the reaction speed of bidders. Bidder behavior reflected their own information and preferences as well as an important strategic dimension affecting the degree to which bidders should “reveal” their willingness to pay by increasing bids. An increment requirement defining the minimal allowable increase had the obvious role of ensuring revisions were economically meaningful. We adopted a two-clock methodology for controlling bidder behavior. Requiring only a single bid revision for auction continuation is not practical because randomness in bidding and bid timing. The practical question becomes “how many” bids or price changes in a round justifies keeping markets open for additional rounds. Testing in experiments with different controls led to a decision to use the number of bidders that attempted to increase their allocation as the controlling measurement for the first clock and number of markets that changed prices as the controlling measure for the second clock. Time was measured in number of rounds required before a change in these thresholds. New bid increment requirements were announced as a percentage over existing prices and these were enforced beginning in a specified round in the future.

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9 In continuous time actions one clock counts down in seconds and resets with the submission of a new bid in any market. In the absence of additional bids, the clock counts down to zero and the auction ends. A second clock is employed in auctions with complex bids, such as bid functions, because new bids need not result in price changes and can become cheap talk that simply keeps the auction open. We avoid such possibilities by using a second clock that counts down and resets if a bid results in new winners, thus exerting pressure to place bids that affect prices.
Experiments provided substantial experience with how the auction would respond to the chosen parameters. Observations from testbed experiments were the primary source of information about the likely auction ending time, which was important given the government’s decision to limit it to one day. Experiments demonstrated that bidding followed a principle of revelation at the margin. Announced prices were accompanied by the increment requirement. All new bids or changed bids had to be no less than the existing price plus increments. Unchanged bids remained in the system. The upcoming price was not known and price often remain unchanged in many individual markets. New bids were automatically integrated with the bidder’s existing bids to form a revised bid function.

The new bid function is a type of “revealed demand” but it is not fully revealing. The revealed demand function always falls short of the limit values (demand prices) of infra-marginal units. While the revealed demand curve falls under the actual demand curve, the experiments indicated that an important element – demand at the margin – was accurately revealed. As illustrated in the appendix, the quantity demanded at the announced price was typically very close to the quantity demanded according to the induced preferences. Given the rules, the bidder could adjust the bid price to ensure the purchase of the marginal unit given the announced price and do so without directly influencing that price. To express a preference for an additional unit at the stated price the bidder merely needed to express a willingness to pay for it by tendering a bid price for the unit above the stated market price. Thus, the value of the marginal demand is revealed. The demand function becomes traced out as price moves up following the required bid increments. In particular, the slope is useful information revealing the state of demand relative to prices and thus when the auction is near a competitive equilibrium. This key feature is discussed in more detail in Section 5 and illustrated in Figures 4 and 5.

Section 4: Partial Equilibrium Properties of the Final Allocation

This section describes the system’s algorithm for determining the allocation and prices to a competitive model of demand under inelastic supply with segmented markets. We present three results establishing partial equilibrium for the final allocations in each market segment. We begin by defining the different market segments based on the prices paid for different licenses and aggregate the derived demand for establishments competing to obtain these licenses based on firms’ reported bid schedules. Throughout this analysis, we assume three stylized facts for all allocations in the system to identify these segments. First, we partition the set of areas
\[ A = A_c \cup A_u \] into constrained areas \( A_c \) (where \( x_a = \bar{x}_a, \forall a \in A_c \)) and unconstrained areas \( A_u \) (where \( x_a < \bar{x}_a, \forall a \in A_u \)). Second, hotel establishments were allocated the maximum number of licenses available based on their venue type (\( x_H = \bar{x}_H \)) while club establishments’ allocation did not meet this maximum (\( x_C < \bar{x}_C \)). Third, all licenses available to the system were allocated (i.e., \( \bar{x} = x_H + x_C \)).

These assumptions give rise to two large market segments for unconstrained clubs and unconstrained hotels and a number of smaller market segments for each constrained area in \( A_c \) commanding a local price premium. Within each of the market segments, the derived supply is price-inelastic at a fixed quantity. The price at which derived demand equals derived supply in each market is approximated by the price premia for different types of licenses from the constrained optimization problem determining the allocation. As such, the final allocation and prices in each market segment are consistent with an ex-post partial equilibrium given bidders’ unwillingness to submit revisions to their reported bid schedules.

4.1 Partial Equilibrium in Derived Demand and Supply for Club Licenses Unconstrained Areas

Consider the market for club licenses in unconstrained areas under an allocation in which \( x_C < \bar{x}_C \). Bidders in these markets compete with each other for the pool of licenses that are not allocated to hotels or to any of the constrained areas. Collectively, the aggregated bid schedules for bidders in these markets identify the Derived Demand for Unconstrained Clubs calculated as:

\[
D_{UC}(p) = \sum_{a \in A_c} \sum_{i \in I} X_i(p)(1-h_i)1\{a_i = a\}.
\]

The supply available to these bidders is determined after all constrained area markets for both clubs and hotels, and the unconstrained hotel market have already cleared:

\[
S_{UC}'(p) = \bar{x} - x_H - \sum_{a \in A_c} x_{aC}.
\]

Finally, the “Club Base Price,” \( p_{UC}^* \), is the market clearing price where \( D_{UC}(p_{UC}^*) = S_{UC}(p_{UC}^*) \).

---

10 The derived quantity supplied may depend on allocations to other market segments, though this feature of the market system doesn’t impact the partial equilibrium analysis in this section. We return to discuss the general equilibrium properties of the market system below when analyzing stability of excess demand functions and a model of convergence to an equilibrium.
Figure 1 presents the derived demand and supply for unconstrained clubs in the final round of the auction relative to the minimum permissible club price of $5,500. The residual supply available to this market consisted of 3,693 units, which matched Derived Demand to set the Club Base Price of $5,500. Due to a large mass of demand at exactly the minimum price of $5,500, the quantity demanded at this price exceeded the available supply. The proportion of demand met at this price was determined by priority based on when establishments submitted their bids. The mass at this price point has two important implications for the system. First, any establishment that did not receive an allocation at the $5,500 price could have increased their bid to receive additional licenses with no impact on price. Given the quantity supplied at this margin, an establishment could have obtained up to 61 additional licenses without having any impact on price. This number of licenses was greater than the number acquired by any individual club. Second, the unmet demand of 64 units at the external margin suggests that no establishment would have any power to reduce prices by lowering their bids while still receiving the same allocation. If any bidder were to reduce their stated value for licenses allocated at this external margin, they would lose those licenses to the unmet demand.

Finally, we connect the price for this market segment to the optimization problem (1). The allocation of all other license types were at their constrained maxima, either due to individual area constraints or the aggregate hotel maximum license constraint. Consequently, any additional licenses made available to the market in total (i.e., an increase in \( \bar{x} \)) would be sold in the unconstrained club market at price \( p_{uc}^* \). As the potential increase in the value function from
relaxing the total license quantity constraint, this price must then equal the shadow cost of the
constraint so that \( p^*_UC = \mu_0 = 5,500 \).

The analysis demonstrates that the market for unconstrained club licenses cleared in a
classical sense where revealed bid schedules are interpreted as demand functions. Bidders for
unconstrained club licenses could unilaterally revise their bid schedules to increase their
provisional allocation without changing the prices they would pay for the allocated licenses.
Bidders could cancel bids that were not provisional winners and avoid the possibility that the
bids would be filled on subsequent rounds. Bidders chose not to take advantage of such revision
opportunities, which demonstrates the allocation of licenses among bidders for unconstrained
cub licenses represents an ex-post equilibrium as summarized in Result 1.

**Result 1: Partial Equilibrium Model in the Market for Unconstrained Club Licenses**

a. The Derived Demand Curve model for Club licenses in unconstrained areas presented
in Figure 1 aggregates the bid schedules for participants seeking club licenses in all
unconstrained areas at any given price:

\[
D_{UC}(p) = \sum_{a \in A_c} \sum_{i \in I} X_i(p)(1-h_i)1\{a_i = a\}
\]

b. The Derived Supply Curve model for Club licenses in unconstrained areas presented
in Figure 1 is inelastic and determined by the quantity of licenses that are not allocated
to hotels or to clubs in constrained areas.

\[
S_{UC}'(p) = \bar{x} - x_h - \sum_{a \in A_c} x_{ac}
\]

c. The Club Base Price, \( p^*_UC = $5,500 \), is the market clearing price:

\[
D_{UC}(p^*_UC) = 3,693 = S_{UC}(p^*_UC)
\]

d. Bidders act as competitive price takers. Any bidder could have obtained up to 61
additional licenses by increasing bids without affecting the market clearing price and
would have lost at least 64 units by lowering bids without affecting the price paid.

**4.2 Partial Equilibrium in the Market for Unconstrained Hotels**

We now extend the analysis of the previous subsection to other types of licenses,
identifying the different market segments by the inequality constraints in the optimization
problem that bind on any given allocation. Consider the market for licenses for hotel
establishments competing in unconstrained regions. Similar to the market for unconstrained
clubs, these bidders compete solely with each other for the pool of hotel licenses that are not
allocated to any of the constrained areas. Again, the expressed bid schedules are interpreted as
demand functions. Define the $i^{th}$ round Derived Demand and Supply, respectively, for Unconstrained Hotels as:

\[
D_{UH}(p) = \sum_{a \in A_h} \sum_{i \in I} D_i(p) h_i 1\{a_i = a\}, \text{ and, } S_{UH}(p) = X_H - \sum_{a \in A_c} x_{ah}.
\]

The “Hotel Base Price” represents the market clearing price, $p_{UH}^*$, that balances derived demand and supply so that $D_{UH}(p_{UH}^*) = S_{UH}(p_{UH}^*)$.

Figure 2 presents the derived demand and supply for unconstrained hotels in the final round of the system’s operation. The 10,356 units of supply available in this market matched Derived Demand to set the Hotel Base Price of $33,350. Derived demand for hotel licenses appears relatively elastic compared to demand for unconstrained club licenses, likely as a consequence of greater heterogeneity in values for establishments in this market segment. Still, market power and market impact for bidders is quite limited, with fourteen units allocated out of twenty demanded at the Hotel Base Price. Consequently, a bidder in the unconstrained hotel market would be able to obtain an additional fourteen licenses by stating a higher willingness to pay without impacting their actual price paid. Bidders could also reduce their stated willingness to pay for the non provisional winning bids and thus avoid acquiring units in subsequent rounds.

**Figure 2: Derived Demand and Supply for Unconstrained Hotels**

*Panel A: Full Theoretical Demand and Supply*  
*Panel B: Equilibrium Detail*

From the optimization problem (1), the Hotel Base Price reflects the marginal revenue available from selling one additional license to a hotel, effectively increasing $x_H$ by one unit. However, holding $x$ constant means this unconstrained hotel license must come from the supply of unconstrained club licenses. Combined, the Hotel Base Price equals the shadow cost of the
constraint on the supply of hotel licenses plus the shadow constraint on the total supply of licenses, i.e., $P_{UH}^* = \mu_o + \mu_H = P_{UC}^* + \mu_H$. We refer to the margin between the Base Hotel Cost and Base Club Cost, $\mu_H = 27,850$, as the Hotel Price Premium.

As is the case with the market for unconstrained club licenses, the analysis demonstrates that the market for unconstrained hotel licenses cleared in a classical sense. Bidders in this market retain the unilateral ability to revise their bid schedules and alter their provisional allocation without changing the prices paid and thus supports the interpretation of the bid schedules as demand functions. Consequently, bidders’ revealed unwillingness to make such revisions demonstrates the ex-post equilibrium nature of the allocation among bidders for unconstrained hotel licenses.

Result 2: Partial Equilibrium Model in the Market for Unconstrained Hotel Licenses

a. The Derived Demand Curve model for Hotel licenses in unconstrained areas presented in Figure 2 aggregates the bid schedules interpreted as demand functions for participants seeking hotel licenses in all unconstrained areas at any given price:

$$D_{UH}(p) = \sum_{a \in A_c} \sum_{i \in I} D_i(p)h_i1\{a_i = a\}$$

b. The Derived Supply Curve model for Hotel licenses in unconstrained areas presented in Figure 2 is inelastic and determined by the quantity of hotel licenses available that are not allocated to hotels in constrained areas.

$$S_{UH}(p) = \bar{x}_H - \sum_{a \in A_c} x_{ah}$$

c. The Hotel Base Price, $P_{UH}^* = 33,350$, is the market clearing price where:

$$D_{UH}(P_{UH}^*) = 10,356 = S_{UH}(P_{UH}^*)$$

d. Bidders act as price takers, as any bidder could have obtained up to 5 additional licenses by increasing their bids without affecting the market clearing price and would have lost at least 14 units by lowering their bids before affecting the price.

4.3 Partial Equilibrium in the Markets for Licenses in Constrained Areas

In geographic areas where the quantity of licenses is constrained, Clubs and Hotels compete with each other to determine the Area Price Premium. Again, bid schedules are interpreted as the demand functions found in the general theory. The derived demand for licenses in these areas aggregates the demand schedules for bidders in the area in excess of the base price determined by the unconstrained markets for each venue type:
\[ D_a(p) = \sum_{i \in \mathcal{I}} \left[ X_i (p + p_{UH}) h_i + X_i (p + p_{UC}^*) (1-h_i) \right] 1\{a_i = a\}. \]

Impounding the venue base price into the demand schedule translates each establishment’s bid in terms of the premium realized by not allocating the license to a bidder in an unconstrained market. In constrained markets, the inelastic derived supply is fixed at the maximum constraint so \( S_a(p) = \bar{x}_a \). The “Area Price Premium,” denoted \( p_a^* \), represents the price that clears the market, so that \( D_a(p_a^*) = S_a(p_a^*) \).

Based on observed bid schedules, Figure 3 presents the derived demand and supply for the final allocation in area number 110, with the schedules for all other areas appearing collectively in Appendix C. The total allocation of 494 licenses for this area is much smaller than for the unconstrained markets, with the market clearing at an area price premium of $26,900. Total demand at this external margin was 160 units, leaving 19 units of unmet demand at this price. As in the unconstrained markets, this atom of demand demonstrates bidders’ ability to increase their allocations by revising their bid schedules without impacting prices and their unwillingness to cancel non-provisional winners to avoid acquiring additional units at the stated market price.

**Figure 3: Derived Demand and Supply for Constrained Area 110**

Panel A: Full Theoretical Demand and Supply
Panel B: Equilibrium Detail

**Result 3:** Partial Equilibrium and Price Premium for Licenses in Constrained Areas
a. The **Derived Demand Curve** model for licenses in constrained area 110 presented in Figure 3 aggregates the bid schedules for participants seeking hotel or club licenses in that area according to the premium above their respective base prices:
\[ D_a(p) = \sum_{i=1}^{n} \left[ X_i \left( p + p_{UH}^* \right) h_i + X_i \left( p + p_{WC}^* \right) (1 - h_i) \right] \mathbb{1}\{a_i = a\} \]

b. The **Derived Supply Curve** model for licenses in constrained area 110 presented in Figure 3 is inelastic and determined by the policy constraint.

\[ S_a(p) = x_a = 494 \]

c. The **Area Price Premium**, \( p_a^* = 26,900 \), is the market clearing price that recipients of licenses in area \( a \) must pay in addition to the hotel or club base price where:

\[ D_a(p_a^*) = 494 = S_a(p_a^*) \]

d. Bidders act as price takers, as any bidder could have obtained up to 141 additional licenses by adjusting their bids without affecting the market clearing price and would have lost at least 19 units by lowering their bids before affecting the price.

**Section 5: Bidding Dynamics, Excess Demand Revelation, and Equilibrium Convergence**

We now present the auction’s dynamic features across rounds. At the conclusion of each bidding round, provisional prices are announced for each market so bidders are aware of the prices they need to meet or beat to obtain a different allocation of entitlements. To make the price determination more transparent we adopted last-accepted bid rules for the uniform price, rather than first-rejected bid rules. Although these uniform price rules do not make value revelation incentive compatible, as described earlier, theoretically they nevertheless encourage value revelation on the margin as the market price rises through successive bidding rounds.

Identifying the demand revealed at the margin as a measure of unobserved excess demand for licenses suggests prices adjusted following a tâtonnement-like process. Using the rich bidding data, we estimate the parameters of this price adjustment process to empirically verify it satisfies well-known stability conditions of general equilibrium theory. The theoretical and empirically verified properties of the system demonstrates the final allocation achieved general equilibrium conditions to represent an efficient allocation across markets.

**5.1 Demand Revelation and Incentives at the Margin**

To track revisions in bid schedules across rounds, superscript bids, prices, and demand calculations with their associated round. To illustrate, \( B_{il}^t = (b_{il}^t, x_{il}^t) \) represents the \( t \)th entry from the \( i \)th establishment’s bid schedule submitted in the \( t \)th round of the mechanism with associated cumulative bid schedule \( X_i^t(p) \). Similarly, let \( x_i^t \) denote the \( t \)th round’s provisional allocation to
establishment $i$, with $p_{dH}^t$ and $p_{dC}^t$ identifying the market clearing prices for this allocation. Given the uniform pricing rule, these market clearing prices identify the point in the demand schedule at which a bidder’s incentives for truthful demand have a binding property in the theoretical sense that the individual would choose the quantity demanded as measured by the demand function.\textsuperscript{11} For bid schedule entries with prices above or below this margin, a bidder could respectively inflate or deflate their stated willingness to pay without changing their allocation or the payment required to receive that allocation.

Round $t$ opens after announcement of provisional prices and quantities allocated based on the bid functions submitted by all bidders in round $t-1$. As the solution to the optimization problem, these provisional allocations and prices each satisfy the partial equilibrium conditions balancing derived supply and reported demand demonstrated in Section 4. Bidders respond by submitting round $t$’s bid schedules. If the bidder is satisfied with their $t-1$ round allocation at the $t-1$ round ending price, they have no incentive to revise their reported bid schedule. If all bidders are satisfied with their $t-1$ round allocation at the $t-1$ round’s ending price, then no bidders would revise their reported bid schedules and the market would close due to inactivity, determining the final allocations and prices.\textsuperscript{12} Given their potential as final auction outcomes, the provisional allocations and prices offer incentives for further revelation due to threat of closing.

However, if a bidder wanted to increase their $t-1$ round allocation, they could do so by increasing the bid price for some entries in their schedule to be above the $t-1$ round ending price. Bid revisions arise when bidders decide they want a greater allocation at the announced price, with the system prompting a bidder to consider whether they want more licenses at the announced price. The system incentivizes bidders to report the quantity they wish to buying at (or slightly above) the publicly announced prices for the market. At each newly announced price,

\textsuperscript{11} As demonstrated in Section 4, no individual bidder has the power to influence prices, so truthfully reporting their demand ensures they obtain exactly the quantity they want regardless of the final price. As advice to bidders, market designers called attention to this possibility when submitting bids. While some bidders followed this strategy, the pilot experiments and testbed studies demonstrated the vast majority of bidders responded to announced prices and provisional allocations in a marginal manner.

\textsuperscript{12} When there was an insufficient amount of bidding activity in a round, the auctioneer would publicly announce that the auction will close if there is insufficient activity in one more round of bidding. If that tentatively “last” round of bidding featured significant revision activity, the auction would again proceed until revision activity ceased again. The auctioneer would then repeat their announcement and the process would continue until there is insufficient revision activity in that “last” round of bidding. In practice, the auctioneer announced only two tentative ends to the bidding process, with the second corresponding to the close of the auction.
bidders reveal the maximum quantity they want at that price, a process we refer to as “demand revelation at the margin.”

Through demand revelation at the margin, rising prices across auction rounds “trace out” points on the market’s aggregate demand curve as illustrated in Figure 4. The market opens with an initially announced price of $P_1$, leading bidders to reveal (approximately) the actual quantity demanded at this price in the subsequent round, but with potential under-revelation of demand at higher prices as illustrated in aggregated bid function $B_1$. The reported bids generating bid function $B_1$ lead to an announced price of $P_2$. The round 2 bid revisions generate a new revealed demand curve, with the point $B_2$ indicating the true quantity desired at this price. The reported bids generating aggregated schedule $B_2$ lead to a market clearing price of $P_3$, which then becomes the operative point of demand revelation for the round 3 bid revisions. According to the model, as rounds progress and prices continue to rise, demand revelation at the margin will trace out more points very close to the true demand curve until the system converges.

**Figure 4: Illustrating Dynamics of Demand Revelation**

Figure 5 contains the time path of revealed demand for the unconstrained hotels, discussed above in Section 4.2. The total number of units allocated to constrained hotels changes very little over periods so the supply of entitlements to unconstrained hotels remains relatively constant (as illustrated in the figure). Shown for each period is the aggregate demand revealed at the market price of for unconstrained hotels. Total demand revealed at the margin appears
somewhat inelastic and with the difference between total excess demand at the margin and supply slowly shrinking to zero where the auction terminates. The inset panels at the upper right show the details in terms of units of excess demand at the end of the auction \([\text{quantity demanded at the margin} – \text{supply} = 10509 - 10332]\) and also the entire demand curve as revealed at end of the auction. We formalize these measures of excess demand in the next section.

This multi-round bidding process thus induces a price adjustment process analogous to tâtonnement with the uncertainty in how newly submitted bid functions collectively lead to further price adjustments. As prices converge over time, bidders can eventually develop confidence that they will become provisional winners for quantities bid at prices that exceed previous round provisional prices.

Figure 5: Marginal Demand Revelation for Unconstrained Hotel Licenses

5.2 Measuring Excess Demand Revealed at the Margin

Given the implicit assumption that bidders have a stable preference, we now empirically evaluate the extent to which demand is revealed at the margin as the system progresses through rounds. To begin, define the round \(t\) Revealed Demand at any price \(p\) as the difference between the round \(t\) demand and the round \(t-1\) demand:
\[ \Delta X_i^t(p) = X_i^t(p) - X_i^{t-1}(p) \quad (3) \]

Note that if a bidder submits an identical bid schedule in two subsequent rounds, \( \Delta X_i^t(p) \) will be zero for all values of \( p \).

Clearly, both the Demand and Revealed Demand measures can be aggregated to the market (area-venue) level by summing over bidders within each market. Define the aggregated (market-level) demand schedules as:

\[
X_{alt}^t(p) = \sum_{i \in I} X_i^t(p) h_i 1\{a_i = a\}, \text{ and, } X_{ac}^t(p) = \sum_{i \in I} X_i^t(p)(1-h_i) 1\{a_i = a\}
\]

\[
\Delta X_{alt}^t(p) = \sum_{i \in I} \Delta X_i^t(p) h_i 1\{a_i = a\}, \text{ and, } \Delta X_{ac}^t(p) = \sum_{i \in I} \Delta X_i^t(p)(1-h_i) 1\{a_i = a\}
\]

These measures characterize the aggregate revealed demand in a market as rounds of the mechanism progress, with the higher-level aggregates characterizing total Hotel, Club, and system-wide revealed demand for licenses at any given price.

Given binding incentives at the market clearing price based on the last round’s submitted bid schedules, this “opening” price provides the most relevant measure of revealed demand. As the price bidders would have to exceed in order to obtain additional units relative to their provisional allocation, this price sets bidders’ expectations for what bid values are likely to be awarded additional licenses. We adopt this measure, \( \Delta X_i^t(p_{t-1}^-) \), as the empirical measure of Revealed Excess Demand at the Margin.

**Definition 1:** The **Revealed Demand at the Margin** for licenses represents the increase in quantity demanded for a license during a bidding round due to revisions in participants bid schedules, valued at the closing price of the previous round. For hotels and clubs in Area a, respectively, this is calculated as:

\[
\Delta X_{alt}^t(p_{t-1}^-) = \sum_{i \in I} \Delta X_i^t(p_{t-1}^-) h_i 1\{a_i = a\}, \text{ and, }
\]

\[
\Delta X_{ac}^t(p_{t-1}^-) = \sum_{i \in I} \Delta X_i^t(p_{t-1}^-)(1-h_i) 1\{a_i = a\}
\]

Figure 6 presents the time series of the total Revealed Excess Demand at the Margin from round 2 forward, disaggregated by the type of venue revealing demand. Panel A presents the number of units of revealed excess demand while Panel B reports the number of bidders whose changes in bid schedules revealed demand. As is apparent from the graph, all revealed excess
demand is generated by revisions for unconstrained Hotels and bidders in constrained areas, with all revealed demand from Clubs associated with constrained areas. Turning toward the dynamics of aggregate Revealed Excess Demand at the Margin, note the downward trend and convergence toward zero in the later rounds of the market after a few minor bumps in later rounds.

The revealed excess demand in the last round of bid revisions is attributed to the activity of a small number of bidders in only three markets for licenses in constrained areas. The isolated nature of these revelations suggest the system had largely converged by this point. Since these markets’ allocations met the constrained maximum for these areas, any additional demand revelation would do nothing to change either the area-level allocation or the allocations in unconstrained markets. Rather, any changes to the allocation would only affect the allocation of licenses among those establishments operating within these three markets along with their associated prices.

**Figure 6: Revealed Excess Demand at the Margin**

*Panel A: Revealed Demand Quantity*  
*Panel B: Number of Active Bidders*

**Result 4: Exhaustion of Revealed Demand at the Margin.** Through the rounds of bidding, the auction mechanism exhausted the Revealed Demand at the Margin, demonstrating convergence to an allocation with negligible excess demand.

5.3 Gross Substitutes, Stability, and Equilibrium Convergence

We now argue that, if bidders have well-behaved preferences then by satisfying and subsequently reducing revealed excess demand at the margin, the system guides the allocation to a general equilibrium across market segments. First, we demonstrate that the licenses in different markets represent gross substitutes. Second, the evolution of allocations as bidders revise their bid schedules corresponds to adjustments in prices that reduces excess demand. Combined, these suggest the price adjustment mechanism follows the principles of tâtonnement. Since
tâtonnement processes converge to the unique, stable, general equilibrium for gross substitutes, we conclude that the market system’s final allocation matches that of a general equilibrium.

From a theoretical perspective, the gross substitute property of preferences, i.e., that excess demand weakly increases in response to an increase in the price for another good, is a sufficient condition for stability of a general equilibrium system. To establish the gross substitute property for licenses in different markets, consider the response of derived supply and demand for licenses in market \( i \) as the price for licenses in market \( j \) increases. On the supply side, the price increase in market \( j \) could increase the number of licenses allocated to market \( j \), weakly reducing the number of licenses available in market \( i \).

On the demand side, bidders are not allowed to reduce the prices in their submitted bid schedules but are allowed to increase their prices. As a result, the demand schedule is weakly increasing by construction. Using the notation of derived supply and demand in round \( t \) from the previous section, these arguments establish \( \frac{\partial D_i(p'_t)}{\partial p'_j} \leq 0 \) and \( \frac{\partial E_i(p'_t)}{\partial p'_j} \geq 0 \). Combining these inequalities, the excess demand in market \( i \) is weakly increasing in response to changes in the price of other markets, establishing the gross substitute property:

\[
\frac{\partial}{\partial p'_j} \left[ D_i(p'_t) - S_i(p'_t) \right] \geq 0.
\]

Having established the gross substitutes property, we now relate price adjustments to a tâtonnement-like process driven by excess demand. As in the model of the hypothetical Walrasian auctioneer, all provisional allocations and prices are announced based on reported demand with no actual trading taking place, a key feature of tâtonnement. Through revisions to the submitted bid schedules, prices mechanically increase in response to excess demand revealed at the margin pushing up against an inelastic supply curve. This price increase is directly attributable to excess demand in the market for that license.

---

13 This classical economic result goes back to Arrow and Hurwicz (1958) and Hahn (1958) and is commonly presented in core textbooks such as Arrow and Hahn (1971), Mas-Colell, Winston, and Green (1995), and McKenzie (2002). Analyzing markets with indivisibility is considered by Kelso and Crawford (1982), who identify the gross substitutes property as a sufficient condition for the existence of Walrasian equilibria with indivisible goods. Gul and Stacchetti (1999) study the efficiency of Walrasian equilibrium in economies with indivisibilities satisfying gross substitutes.

14 Technically speaking, the supply of licenses in the mechanism as determined by the vector of prices is defined as a correspondence. The uniquely realized supply from this correspondence minimizes the total value of excess demand in all markets. As such, while realized supply is endogenously determined within the mechanism, excess demand is well-defined and its first order dynamics satisfy the gross substitutes property.
Section 6 presents an overview of the time-series properties of the bidding data, presenting summary statistics relating to the revenue generated and prices in each market. Section 7 estimates a structural model of tâtonnemont to characterize the adjustment process for prices in response to excess demand. We test for cross-market effects in how prices for a specific license respond to excess demand for other types of gambling licenses, demonstrating the estimated adjustment matrix satisfies stability conditions for convergence. These empirical findings concerning stability of the price adjustment process further support theoretical arguments of system stability. Our demonstration of general equilibrium stability for the system, both theoretically and empirically, establishes that the system attains an efficient allocation of licenses across markets.

Result 5: Gross Substitutes, Tâtonnemont, and General Equilibrium Stability.
Licenses in different market segments represent gross substitutes. Since the auction’s price adjustment mechanism corresponds to a tâtonnemont process, theory suggests its convergence leads to a stable general equilibrium allocation.

Section 6: Implementation, Revenue, and System Convergence

We now discuss the system’s performance in terms of the total revenue generated from distributing licenses, the prices realized for licenses sold in different market segments, and the reported dynamics of demand. The government’s original plan was to allocate all 27,500 entitlements in the auction, to take place in early 2010. The government later decided, after the auction rules had been largely designed, to give existing clubs an opportunity to buy a capped number of entitlements at a set price. Eligible clubs could buy an entitlement for each gaming machine currently operating at their venue, up to a cap of 40 entitlements per venue. The offer price was based on a percentage of the individual venue’s historic gaming revenue, and thus differed across clubs. Most clubs (236 out of 247 eligible) bought at least one entitlement in this phase, and in aggregate they purchased 8,712 of the 13,750 entitlements available to clubs (63 percent). Following this pre-auction sale, which took place in October and November of 2009, 5,038 club entitlements remained for sale in the auction along with the original 13,750 entitlements available for hotels.

The auction took place in two phases. First was an initial round of bid submissions without price or bid revelations that was open for two weeks. During this period an individual bidder could examine or change their own bids as practice with the bidding and information
interface. Prices were computed and revealed at the beginning of a one-day, one-site auction that lasted for about 10 hours and an additional 62 rounds. Bidders were required to submit a bid in the initial round in order to be eligible to participate in the later one-day auction that finalized the entitlement allocation. The initial two-week time period for bid submission was similar in all respects to the bidding features of later rounds, but was open for many days to ensure that interested bidders had an opportunity to become familiar with the auction interface, consult with advisors when constructing their initial round bid, and communicate with “coaches” who provided technical assistance with bid preparation and submission. Prices and allocations were announced for the first time at the opening of the auction day.

After the two-week initial round was completed, bidders were required to bid on-site at a secured convention center in Melbourne for phase two. Upon check-in they were assigned to a bidding station, which contained a visually isolated computer workstation and seating for a bidding “team” of up to two individuals as illustrated in Figure 7. A total of 363 bidding teams participated in the on-site auction. Bidder cell phones were collected and they had no access to public phones. Bidders could not walk through the bidding area unmonitored. They could talk to the bidder on their own team but not bidders on other teams. Coaches were assigned and available for assistance or to interface with auctioneers should problems arise.

Figure 7: Bidding Stations in Melbourne Convention Center
Bidders were registered according to the markets in which they wanted to place bids. As explained earlier, bidders submitted a schedule that indicated the marginal quantities that they were willing to pay at various market prices of their choosing. At the close of each round provisional prices were calculated and publicly announced for each market, and bidders learned their provisional allocation and prices per entitlement won. Market volumes were not announced publicly. Between rounds bidders could revise upwards any bids subject to a minimum price increment. They could also cancel bids that were not provisionally winning. Rounds continued as long as the closing was not triggered by insufficient new bids or winning bids as explained previously.

The auction performed smoothly and ended as planned, and there were no technical difficulties. Price discovery was slower than seen in the testbed experiments. Unlike in an experiment with induced values, of course the underlying entitlement values are not observable. Nevertheless, based on an analysis of the bidding behavior we conclude that prices and allocation quantities appear to reflect the conditions of an efficient allocation. The auction permitted entry by new venues as anticipated, since bidders other than existing venues were successful in acquiring entitlements.

<table>
<thead>
<tr>
<th>Clubs</th>
<th>Number of Bidders</th>
<th>Number of Areas</th>
<th>Total Bid Quantity</th>
<th>Total Bid Value</th>
<th>Awarded Quantity</th>
<th>Average Price</th>
<th>Total Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>103</td>
<td>72</td>
<td>3,757</td>
<td>58,688,745</td>
<td>3,693</td>
<td>5,500</td>
<td>20,311,500</td>
</tr>
<tr>
<td>Constrained</td>
<td>64</td>
<td>16</td>
<td>1,617</td>
<td>105,545,885</td>
<td>1,345</td>
<td>36,558</td>
<td>49,171,047</td>
</tr>
<tr>
<td>Clubs Total</td>
<td>167</td>
<td>88</td>
<td>5,374</td>
<td>164,234,630</td>
<td>5,038</td>
<td>13,792</td>
<td>69,482,547</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hotels</th>
<th>Number of Bidders</th>
<th>Number of Areas</th>
<th>Total Bid Quantity</th>
<th>Total Bid Value</th>
<th>Awarded Quantity</th>
<th>Average Price</th>
<th>Total Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>184</td>
<td>72</td>
<td>12,751</td>
<td>498,270,158</td>
<td>10,356</td>
<td>33,350</td>
<td>345,372,600</td>
</tr>
<tr>
<td>Constrained</td>
<td>77</td>
<td>16</td>
<td>4,479</td>
<td>280,848,522</td>
<td>3,394</td>
<td>59,019</td>
<td>200,311,424</td>
</tr>
<tr>
<td>Hotels Total</td>
<td>261</td>
<td>88</td>
<td>17,230</td>
<td>779,118,680</td>
<td>13,750</td>
<td>39,686</td>
<td>545,684,024</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Overall Total</th>
<th>Number of Bidders</th>
<th>Number of Areas</th>
<th>Total Bid Quantity</th>
<th>Total Bid Value</th>
<th>Awarded Quantity</th>
<th>Average Price</th>
<th>Total Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>428</td>
<td>176</td>
<td>22,604</td>
<td>943,353,310</td>
<td>18,788</td>
<td>32,743</td>
<td>615,166,571</td>
<td></td>
</tr>
</tbody>
</table>

6.1 Price Dynamics and Revenue Convergence

In its final allocation, the system allocated 18,788 licenses to 428 establishments, generating a total revenue of AU$615M, or about US$555M based on prevailing exchange rates at the time. Table 3 presents summary statistics on the number of bidders in each market, the number of areas assigned their maximal allocations, and the components of the system’s total revenue. Representing more bidders submitting higher total valuations, Hotel establishment
licenses were capped at their maximal total allocation of 13,750 units. The remaining licenses were allocated to clubs, which collectively generated 12% of the system’s revenue. Only sixteen areas’ allocations (out of 88 total) reached the constrained maximum allowance, shown in Panel B of Figure 8. Due to the price premium for these licenses, these areas represented nearly 41% of system-wide revenue.

Figure 8: Time Series of Prices in Unconstrained and Constrained Markets

Figure 9 presents the time series of total revenue realized by the system’s provisional allocations, broken down by market segments (unconstrained clubs, unconstrained hotels, constrained clubs, and constrained hotels). The solid black line plots the total reported surplus, based on the total stated willingness to pay from submitted bid schedules for acquired entitlements.
Section 7: Estimating Price Dynamics and Testing Stability

This section provides an empirical counterpart to stability arguments presented in section 5.3 using the gross substitutes property for different licenses to provide theoretical conditions for general equilibrium stability of the auction across markets. As suggested earlier, the multi-round bidding format gives rise to a price adjustment process analogous to tâtonnement with the demand revealed at the margin providing a measure of excess demand. Exploring the dynamic relationship between price changes and the revealed demand at the margin characterizes the equilibration process in these markets. Here, we demonstrate the observed process of price adjustment satisfies conditions for stability and general equilibrium convergence to prices supporting an efficient allocation across markets.

Classical analysis of general equilibrium systems\textsuperscript{15} characterizes the forces driving price equilibration by differentiating excess demand functions $z_t(p)$ with respect to prices:

$$\frac{\delta}{\delta t} z_t(p) = \nabla_p z_t(p) \cdot \frac{\delta}{\delta t} p_t + \varepsilon_t = B^{-1} \Delta p_t + \varepsilon_t$$  \hspace{1cm} (4)

\textsuperscript{15} For brief outlines, see Negishi (1989) for a discussion of tâtonnement and its theoretical background. Reviews of general equilibrium theory can also be found in McKenzie (2002) and Fisher (1989). More extensive discussions of the gross substitute case are found in Arrow and Hahn (1971), Negishi (1972, 1989), Mukerji (1995, 2002), and Mukerji and Guha (2011). Experimental work on stability of general equilibrium can be found in, Gillen, et al. (2021), which demonstrates that double auction markets that do not satisfy general equilibrium stability conditions follow excess demands and as predicted by the dynamic model, diverge from an interior competitive equilibrium.
Because $B^{-1} = \nabla_p z_i(p)$ is the gradient of current excess demand with respect to prices, the matrix $B$ is sometimes referred to as the Inverse Jacobian of the excess demand function. Solving for $\Delta p_i$ in equation (4) and letting $u_i = B\varepsilon_i$ represents the price adjustment process as a linear function of excess demand that resembles a regression equation.

$$\Delta p_i = Bz_i(p) + u_i$$  \hspace{1cm} (5)  

**Definition 2:** The **Empirical Inverse Jacobian** represents the expected change in a price adjustment process predicted by excess demands for goods under a set of disequilibrium prices. Given a time series of price changes and measurements of excess demand in each market, the Empirical Inverse Jacobian can be estimated directly by the Seemingly Unrelated Regression model in equation (5).

McFadden (1968) provides sufficient conditions for the matrix $B$ to characterize a stable system. These conditions impose two restrictions on the coefficients that predict price movements in response to excess demand. First, the diagonal entries in $B$ must be positive, so that $\beta_{mm} > 0$ for all $m$. Second, the determinant of the $(m, m)$ principal minor for the matrix $B$ must be weakly negative. We can thus evaluate whether the system’s response to excess demand is stable by testing these two conditions.

Here, we pose two questions. First, as an empirical characterization of equilibration dynamics, do observed prices adjust according to measures of excess demand? We approach this question by directly estimating the price adjustment process in equation (5). In so doing, we characterize the “Empirical Inverse Jacobian” and test the significance of its estimated coefficients. Second, does this Empirical Inverse Jacobian satisfy the conditions for stability that would lead to price convergence to equilibrium according to classical analysis of multi-market economic systems? These conditions are well understood from a purely theoretical perspective from which we derive novel econometric tests. The rich data available in the Victoria Gaming Auction provides novel empirical insight into these basic and important theoretical properties.

**7.1 Estimating Price Response to Excess Demand**

We begin by considering the first classical condition for stability of the system presented in equation (5). This condition requires the price for a given type of license respond positively to excess demand for that license. Following the analysis from Section 4, the constraints on the system effectively generated eighteen (18) different market segments or types of licenses:
Unconstrained Clubs, Unconstrained Hotels, and sixteen (16) Constrained Areas with Area Price Premia. Let $N$ denote the set of derived markets,\textsuperscript{16} and let $z_{nt}(p_{t-1})$ denote the excess demand in market $n$ and round $t$. Consider the price adjustment process for market $m$, $\Delta p_{mt} = p_m - p_{m,t-1}$:

$$\Delta p_{mt} = \beta_{m1}z_{t1}(p_{t-1}) + \cdots + \beta_{mn}z_{nt}(p_{t-1}) + \cdots + \beta_{mN}z_{N,t}(p_{t-1}) + u_{mt}$$  \hfill (6)

The regression equation (6) relates the expected price changes for market $m$ to the level of disequilibrium in each of the individual markets including the excess demand in $m$ itself, providing a predictive model for price changes driven by excess demand.

**Hypothesis 1: Prices Adjust Positively in Response to Excess Demand.** Consistent with a tâtonnement price adjustment process, auction prices are predicted to adjust positively (negatively) in response to excess demand (supply) for licenses.

Estimating the model requires first specifying a measure of excess demand for each market in each round. “Closing” prices, or the prices associated with any given allocation, are determined to clear markets. Therefore, zero excess demand exists based on stated bid functions at closing prices and provisional allocations. The Revealed Excess Demand at the Margin introduced in section 5.2, however, provides a natural proxy of excess demand at the opening price for a round. The measure takes advantage of reported demand at two different price points where bidders’ incentives are binding, at least in determining provisional allocations. As Revealed Demand is reported in terms of licenses for each specific market, these measures need to be translated into a common numeraire for evaluation across markets. Consequently, we measure excess demand by valuing revealed demand according to the opening price of a round:

$$z_{mt}(p_{t-1}) = p_{m,t-1} \sum_{a,v=m} \Delta D'_{av}(p_{t-1})$$

Table 4 reports the estimated values for the coefficients $\beta_{mn}$ along with their standard errors, with estimates for all parameters appearing in Appendix D.\textsuperscript{17} The estimated coefficients

---

\textsuperscript{16} Here, and henceforth, we will use “market” to refer to a derived market as defined in Section 4. That section identified Unconstrained Clubs as representing one market that determines the Unconstrained Club Base Price, Unconstrained Hotels as another that determines Hotel Premium, and fifteen Constrained Area Markets that determine the Area Premia paid by all bidders within each of these areas. Since no variation in price or excess demand exists for the Unconstrained Clubs, we exclude that market from the analysis.

\textsuperscript{17} As discussed in the next section, this regression equation represents a system of Seemingly Unrelated Regressions. We estimate these regressions on an equation-by-equation basis in this section to focus on each individual market’s price response to excess demand within that market. In the next section, when we consider cross-market restrictions necessary for stability, we exploit the SUR structure to establish joint asymptotic normality of all estimates and justify a parametric bootstrap as a device for calculating standard errors for inference.
suggest that prices typically respond positively to excess demand, given 12 of the 17 estimated \( \beta_{nm} \) coefficients are positive and none of the negative estimates are significantly negative. These results suggest an affirmative answer to our first question of whether prices respond to excess demand. Further, the direction of the price response is consistent with theory suggesting positive excess demand leads to rising prices and prices drop in response to negative excess demand.

**Table 4: Price Adjustment in Response to Excess Demand**

<table>
<thead>
<tr>
<th>Area</th>
<th>Area 105</th>
<th>Area 106</th>
<th>Area 107</th>
<th>Area 110</th>
<th>Area 112</th>
<th>Area 118</th>
<th>Area 121</th>
<th>Area 123</th>
<th>Area 134</th>
<th>Area 159</th>
<th>Area 167</th>
<th>Area 171</th>
<th>Area 176</th>
<th>Area 178</th>
<th>Area 185</th>
<th>Area 186</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\beta}_{nm} )</td>
<td>1.87E-05</td>
<td>1.57E-02</td>
<td>-1.81E-04</td>
<td>1.19E-04</td>
<td>-2.72E-04</td>
<td>2.09E-04</td>
<td>6.83E-04</td>
<td>2.55E-04</td>
<td>-3.92E-04</td>
<td>2.69E-05</td>
<td>-1.53E-02</td>
<td>2.20E-03</td>
<td>9.23E-05</td>
<td>-2.65E-04</td>
<td>1.45E-04</td>
<td>2.76E-04</td>
</tr>
<tr>
<td>t-Stat</td>
<td>4.60**</td>
<td>3.01**</td>
<td>-1.39</td>
<td>0.50</td>
<td>-0.24</td>
<td>0.31</td>
<td>1.11</td>
<td>1.00</td>
<td>-1.60</td>
<td>1.25</td>
<td>-0.55</td>
<td>2.35**</td>
<td>0.54</td>
<td>-0.35</td>
<td>0.30</td>
<td>1.29</td>
</tr>
</tbody>
</table>

A practical challenge arises in analyzing the regression specification in (6) due to the need to fit 17 coefficients with only 62 rounds of data for each of the 17 market segments (recalling the unconstrained club market is excluded due to lack of variation in price and excess demand). Given the number of free parameters in regression equation (6), one might be concerned that the model is over parameterized as evidenced by the relatively large standard errors and limited significance of estimates in Table 4. To reduce the dimensionality of the problem, we evaluate price changes with respect to the excess demand for licenses in a single market and the total excess demand for licenses in all other markets. That is, define the composite measure of excess demand for other markets as \( z_{(-j),p} (p_{t-1}) = \sum_{x \neq j} z_{mt} (p_{t-1}) \) and consider the simplified regression:

\[
\Delta p_{mt} = \beta_{m1} z_{mt} (p_{t-1}) + \beta_{m2} z_{(-m),p} (p_{t-1}) + \varepsilon_{mt} \tag{7}
\]

This specification concentrates the influence of excess demand across other markets, reducing the dimensionality of the regression to enable more precise estimates.
Table 5 represents the results for estimating regression specification (7) using a maximum likelihood mixed effects model for heterogeneous coefficients across markets (Panel A) and OLS fixed effects model (Panel B). For ease of interpreting the coefficients, all independent and dependent variables are standardized to mean zero and unit variance by market. Overall, the results demonstrate that positive excess demand is associated with an expected increase in prices in both the mixed-effects and fixed-effects specifications. Prices respond positively to market-specific excess demand in 11 of the 17 markets featuring positive estimates for $\beta_{m1}$ with t-Statistics exceeding the traditional threshold for 5% significance in eight of these markets. The mixed effects model consolidates these results, further verifying the expected positive sign of $\beta_i$ consistent with theoretical restrictions of stability. This robustness result further affirms the results demonstrated in Table 4: the empirical evidence shows that prices change in response to excess demand as posited by theoretical analysis.

### Table 5: Excess Demand and Price Dynamics

**Panel A: Random Effects Pooled Results Across Markets**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>Std Error</th>
<th>t-Stat</th>
<th>RE Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}_1$</td>
<td>0.205</td>
<td>0.066</td>
<td>3.12</td>
<td>0.24</td>
</tr>
<tr>
<td>$\hat{\beta}_2$</td>
<td>0.178</td>
<td>0.046</td>
<td>3.88</td>
<td>0.14</td>
</tr>
</tbody>
</table>

**Panel B: Fixed Effects Individual Market Results**

<table>
<thead>
<tr>
<th>Market</th>
<th>$\hat{\beta}_1$</th>
<th>t-Stat</th>
<th>$\hat{\beta}_2$</th>
<th>t-Stat</th>
<th>Market</th>
<th>$\hat{\beta}_1$</th>
<th>t-Stat</th>
<th>$\hat{\beta}_2$</th>
<th>t-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC Hotel</td>
<td>0.335</td>
<td>2.32</td>
<td>0.025</td>
<td>0.17</td>
<td>Area 134</td>
<td>0.595</td>
<td>5.30</td>
<td>-0.081</td>
<td>-0.72</td>
</tr>
<tr>
<td>Area 105</td>
<td>0.516</td>
<td>4.17</td>
<td>0.131</td>
<td>1.06</td>
<td>Area 159</td>
<td>-0.073</td>
<td>-0.50</td>
<td>0.322</td>
<td>2.18</td>
</tr>
<tr>
<td>Area 106</td>
<td>-0.054</td>
<td>-0.42</td>
<td>0.200</td>
<td>1.55</td>
<td>Area 167</td>
<td>-0.023</td>
<td>-0.19</td>
<td>0.507</td>
<td>4.17</td>
</tr>
<tr>
<td>Area 107</td>
<td>0.261</td>
<td>1.90</td>
<td>0.160</td>
<td>1.17</td>
<td>Area 171</td>
<td>-0.021</td>
<td>-0.13</td>
<td>0.297</td>
<td>1.89</td>
</tr>
<tr>
<td>Area 110</td>
<td>0.289</td>
<td>2.31</td>
<td>-0.004</td>
<td>-0.03</td>
<td>Area 176</td>
<td>-0.301</td>
<td>-1.98</td>
<td>0.261</td>
<td>1.71</td>
</tr>
<tr>
<td>Area 112</td>
<td>0.587</td>
<td>5.23</td>
<td>0.096</td>
<td>0.86</td>
<td>Area 178</td>
<td>-0.123</td>
<td>-0.87</td>
<td>0.014</td>
<td>0.10</td>
</tr>
<tr>
<td>Area 118</td>
<td>0.512</td>
<td>5.82</td>
<td>0.440</td>
<td>5.00</td>
<td>Area 185</td>
<td>0.288</td>
<td>2.55</td>
<td>0.372</td>
<td>3.29</td>
</tr>
<tr>
<td>Area 121</td>
<td>0.000</td>
<td>0.00</td>
<td>0.151</td>
<td>0.99</td>
<td>Area 186</td>
<td>0.077</td>
<td>0.61</td>
<td>-0.171</td>
<td>-1.35</td>
</tr>
<tr>
<td>Area 123</td>
<td>0.532</td>
<td>5.72</td>
<td>0.371</td>
<td>3.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Interestingly, Table 5 also demonstrates significant sensitivity for prices to respond to excess demand in other markets. Four of the individual-market regressions demonstrate a significant price response to aggregate excess demand in other markets. Further, the estimated value of $\beta_2$ in the random effects model is positive and highly significant. These estimates provide evidence of general equilibrium dynamics in the system that cause prices to shift in
response to excess demand in other markets and raise the question of whether these effects might lead to instability in the price adjustment process.

**Result 6: Prices Adjust Positively in Response to Excess Demand.** Consistent with the prediction of Hypothesis 1, the auction adjusts prices positively in response to excess demand as measured by the Revealed Demand at the Margin.

- In the Empirical Inverse Jacobian, twelve of seventeen markets predict a positive price adjustment to demand revealed at the margin, three of which are statistically significant.
- Pooling excess demand for other markets to create a more parsimonious model yields similar results, with a random-effects specification yielding a positive and statistically significant relationship.
- Estimating the parsimonious model using fixed-effects, eleven of the seventeen markets predict a positive price adjustment to demand revealed at the margin with eight of these demonstrating statistical significance.

### 7.2 Testing Stability of Price Adjustment Process

Given our measures of excess demand and observed interim prices, we can treat tâtonnement as an empirical process rather than a purely theoretical construct. That is, treating the dynamic price adjustment process specified in equation (5) as a regression equation allows us to estimate the Empirical Inverse Jacobian for the price adjustment process. By estimating the rates of adjustment for prices in response to observed excess demand, we can estimate and test the hypothesis that observed price adjustments are consistent with classical restrictions of stability under which prices converge to equilibrium.

The regression specifications (6) and (7) also provide a device for evaluating the stability of the observed price dynamics. A variety of classical models for disequilibrium price dynamics characterize the system of price dynamics in response to prevailing excess demand in multiple markets. These models consider the full system of price changes:

\[
\begin{bmatrix}
\Delta p_{1t} \\
\vdots \\
\Delta p_{Nt}
\end{bmatrix} = \Delta p_t = B z_t \left( p_{t-1} \right) + \epsilon_t = 
\begin{bmatrix}
\beta_{11} & \cdots & \beta_{1N} \\
\vdots & \ddots & \vdots \\
\beta_{N1} & \cdots & \beta_{NN}
\end{bmatrix}
\begin{bmatrix}
z_{1t} \\
\vdots \\
z_{Nt}
\end{bmatrix} + 
\begin{bmatrix}
\epsilon_{1t} \\
\vdots \\
\epsilon_{Nt}
\end{bmatrix}
\]  

(8)

Recalling McFadden’s (1968) two conditions for the matrix \( B \) to characterize a stable system, we test if (1) the diagonal entries in \( B \) are positive, so that \( \beta_{mm} > 0 \) and (2) the determinant of the \((m, m)\) principal minor for the matrix \( B \) is weakly negative.

We have already evaluated condition (1) in the previous subsection, with the results in Tables 4 and 5 providing supportive evidence that prices respond positively to excess demand.
within a market. Indeed, results for the mixed effects model suggests fewer than 20% of samples will yield negative estimates for $\beta_{m1}$, which corresponds to the $\beta_{mn}$ coefficient in model (6).

Considering the regression estimates presented in Table 5, we note positive coefficient estimates suggest that the prices increase with excess demand in twelve of the seventeen markets. While five markets have negative coefficient estimates, suggesting that prices actually fall in the presence of significant excess demand, none of these are statistically significant.

**Hypothesis 2: Price Adjustment Stability is Robust to Cross-Market Influences.** The Empirical Inverse Jacobian satisfies classical tâtonnement conditions for stability:

1) Its diagonal entries are positive (as stated in Hypothesis 1), and,

2) The determinant of the principal minors along the diagonal of the Empirical Inverse Jacobian are weakly negative.

The second condition for determining stability is somewhat more challenging to test and, to our knowledge, has not yet been performed in a multiple market economic system. From an econometric perspective the test is facilitated by the Seemingly Unrelated Regression (SUR) representation implied by (8). Given coefficient estimates in the matrix $B$, computing the determinant of the principal minors for each of its diagonal elements is straightforward. Since the determinant is a continuous function, a delta-method approximation will establish asymptotic normality and a parametric bootstrap is available to construct confidence intervals. The details of this inference technique, including the parametric bootstrap, are reported in Appendix B.

Table 6 presents the estimates of the relevant determinants for each market, along with their 95% confidence intervals. Panel A characterizes confidence intervals for the maximum of the determinants across all principal minors of the $B$ matrix from Regression (8). Though the bootstrapped mean of this statistic is positive, it is quite close to zero and an order of magnitude smaller than the bootstrapped standard error of the estimate so that the 95% confidence interval clearly includes zero. An asymptotic approximation assigns a p-value of only 0.34 for the null hypothesis that the maximum determinant is weakly negative, which does not support rejecting the second condition of stability at any reasonable level.

Panel B of Table 6 presents the estimated principal minor determinants of the Empirical Inverse Jacobian for each market along with their 95% confidence intervals. Though many markets’ point estimates for these determinants are positive, the 95% confidence interval always includes zero in every case and their magnitudes are extremely small. One challenge associated with the test relates to the relatively large number of parameters involved relative to the number
of rounds for which we have bidding data, and the large standard errors suggest the test features
limited power in this sample.

Table 6: Estimates and Bootstrap Confidence Intervals for Determinant Test of Stability

Panel A: Bootstrap Confidence Interval for
Maximum Principal Minor Determinant across Markets

<table>
<thead>
<tr>
<th>Lower 95%</th>
<th>Expectation</th>
<th>Upper 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.14E-42</td>
<td>3.05E-43</td>
<td>1.75E-42</td>
</tr>
</tbody>
</table>

Panel B: Bootstrap Confidence Interval for Principal Minor Determinants by Market

<table>
<thead>
<tr>
<th>Market</th>
<th>Lower</th>
<th>Estimate</th>
<th>Upper</th>
<th>Market</th>
<th>Lower</th>
<th>Estimate</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncon Hotel</td>
<td>-2.17E-42</td>
<td>1.26E-44</td>
<td>2.19E-42</td>
<td>Area 134</td>
<td>-2.32E-44</td>
<td>-7.17E-47</td>
<td>2.31E-44</td>
</tr>
<tr>
<td>Area 105</td>
<td>-1.77E-44</td>
<td>1.55E-48</td>
<td>1.77E-44</td>
<td>Area 159</td>
<td>-1.43E-45</td>
<td>9.70E-48</td>
<td>1.45E-45</td>
</tr>
<tr>
<td>Area 106</td>
<td>-4.59E-44</td>
<td>-1.14E-46</td>
<td>4.56E-44</td>
<td>Area 167</td>
<td>-1.07E-45</td>
<td>-1.33E-48</td>
<td>1.07E-45</td>
</tr>
<tr>
<td>Area 107</td>
<td>-3.73E-45</td>
<td>4.38E-48</td>
<td>3.73E-45</td>
<td>Area 171</td>
<td>-6.80E-44</td>
<td>-4.09E-47</td>
<td>6.80E-44</td>
</tr>
<tr>
<td>Area 110</td>
<td>-2.31E-44</td>
<td>-1.37E-47</td>
<td>2.31E-44</td>
<td>Area 176</td>
<td>-4.88E-45</td>
<td>-1.92E-48</td>
<td>4.88E-45</td>
</tr>
<tr>
<td>Area 112</td>
<td>-1.25E-44</td>
<td>2.31E-47</td>
<td>1.26E-44</td>
<td>Area 178</td>
<td>-1.60E-44</td>
<td>-4.43E-47</td>
<td>1.59E-44</td>
</tr>
<tr>
<td>Area 118</td>
<td>-1.70E-44</td>
<td>2.60E-47</td>
<td>1.71E-44</td>
<td>Area 185</td>
<td>-3.02E-44</td>
<td>4.26E-47</td>
<td>3.03E-44</td>
</tr>
<tr>
<td>Area 121</td>
<td>-1.46E-44</td>
<td>8.51E-48</td>
<td>1.46E-44</td>
<td>Area 186</td>
<td>-1.10E-43</td>
<td>3.29E-46</td>
<td>1.10E-43</td>
</tr>
<tr>
<td>Area 123</td>
<td>-6.05E-44</td>
<td>-6.56E-47</td>
<td>6.03E-44</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Overall, our analysis of the Empirical Inverse Jacobian of excess demand demonstrates that the price dynamics observed in the Victoria Gaming Auction are consistent with a stable equilibration process. First, prices in a market shift positively in response to excess demand, increasing when excess demand in the market is positive and decreasing when excess demand is negative. Second, excess demand in other markets does not generate unstable general equilibrium effects in the price adjustment process. Showing the Empirical Inverse Jacobian satisfies these stability conditions provides further evidence that the price adjustment process leads to a stable equilibrium in prices. Beyond informing the scientific interest in empirically investigating conditions for tâtonnemont convergence, this stable equilibrium further supports the efficiency of the final allocation achieved by the auction mechanism.


We estimate and test the determinants for the diagonal principal minors of the Empirical Inverse Jacobian using a delta method approximation, finding that:

- The 95% confidence intervals for all seventeen determinants include zero.
- Eight of the seventeen principal minors have negative estimated determinants.
- The 95% confidence interval for the maximal determinant of the seventeen principal minors includes zero. The one-sided hypothesis test that the maximal determinant is weakly negative receives a p-value of only 0.34 and is not rejected.
Section 8: Conclusion and After Market Evaluation

This paper reviews the design and implementation of a Victoria Gaming Auction mechanism to solve a complex allocation problem that involved 176 interdependent markets and prices, 18,788 entitlements and 363 bidders. The allocation problem was dictated by social policies that led to constraints on the distribution of gambling activities in a highly regulated industry. Addressing these government concerns presented a challenging policy design problem. The analysis approached this problem by starting with theoretical properties of an efficient allocation, identified as the solution to a constrained surplus maximization problem in which participants have well formed, but unobserved, preferences. The performance of the auction mechanism, including practical elements of its function, rules, and technical issues were determined and refined through extensive testbedding in an experimental setting. The successful transition from the lab to the field is supported by theoretical principles, and verified by the empirical properties from the time-series of observed price dynamics and underlying excess demand. The mechanism itself is based on competitive economic theory of markets with practical features suggested by some of the prominent features of the classical tâtonnement theory of price adjustment and refined through an experimental testbedding process.

In the end, we demonstrate that the mechanism achieved the basic design and assessment goals. First, it did what is was designed to do. The resulting allocation satisfied basic properties of efficiency subject to the fact that the complex legal, political and social goals were met. Secondly, the data demonstrate that the design success can be attributed to the underlying principles from which the design was constructed. The results were not a consequence of luck or some arbitrary random events.

A competitive theory of general equilibrium related behavior underlies the principles determining allocations within the mechanism. If individuals prefer to buy more entitlements at a stated price, they can attempt to obtain additional licenses by simply increasing their bids. The decisions result in measurements naturally interpreted as market demand functions. The response of the auction to such revealed demand functions and resulting excess demands was to adjust allocations and thus prices to reflect an efficient allocation and equilibrium supporting prices given participants’ values revelation. In the last round of the auction’s operation, only five bidders made small adjustments to their bid schedules, suggesting that bidders were satisfied
with their allocations at the prevailing prices. In effect, participants’ unwillingness to revise their bid schedules suggests an ex post efficiency to the final allocation.

The bidding process also provided new insights into the dynamics by which demand and excess demand are revealed through the auction mechanism. Since bidders’ incentives bind only at the interim prices and allocations announced between rounds, it is at these prices and allocations where bidders’ demand is truthfully revealed. This principle of “demand revelation at the margin” allows us to measure excess demand and demonstrate that excess demand diminishes as bidding rounds progress and prices increase. The property is closely related to a general principle—treated as a theoretical axiom since Walras—that if excess demand of a commodity is negative then other things being equal, its price will fall.\(^{18}\)

Finally, we observe how relative prices evolve across bidding rounds in response to these revealed excess demands. This provides a unique opportunity to evaluate classical properties of multi-market, equilibrating dynamics. We define the Empirical Inverse Jacobian as the empirical counterpart to the inverse Jacobian of excess demand governing price dynamics under tâtonnement price adjustments. We estimate this Empirical Inverse Jacobian using the Victoria Gaming Auction interim prices, bids, and allocations to compute observed price changes and imputed excess demand. We derive tests to show the Empirical Inverse Jacobian satisfies classical conditions for stability, with results supporting the hypothesis that prices converge toward their equilibrium values as revealed by the model. Though abstract, theoretical considerations based on the Sonnenshein-Mantel-Debreu Theorem (Mas-Colell, Whinston, Green, 1995) can be interpreted as calling into question theoretical micro foundations of general equilibrium dynamics, our focus on observed system behavior renders such considerations irrelevant for the application we study. Because empirical evidence suggests the system satisfies conditions necessary for convergence to a stable general equilibrium, we can conclude that such convergence does occur. These results demonstrate the importance and power of classical general equilibrium theory in addressing real-world market design problems.

In sum, the Victoria Gaming Auction delivered a stable and efficient allocation for licenses across a large number and a variety of markets and to a variety of establishments. This

\(^{18}\) See the discussion in Mukerji (2002), p.74, or in McKenzie (2002), p.54. Walras (1954, p.170) notes the property as fundamental: “If the demand for any one commodity is greater than the offer, the price of that commodity in terms of the numeraire will rise; if the offer is greater than the demand, the price will fall.”
allocation satisfied policy constraints while generating revenues of AU$614 million for the Victoria Government in a ten-hour period plus an additional AU$366 million from the pre-auction offer to existing bidders – a total of AU$980 million. Its success demonstrates the effectiveness of combining economic theory with experimental testbedding in applied mechanism design. It also shows the usefulness of using laboratory experimental techniques for revealing the content and meaning of basic economic principles in the context of a multidisciplinary and politically and legally sensitive policy. Finally, evaluating the mechanism’s performance and analyzing the time series of prices and demand provides new insights into the market and excess demand forces, at the heart of general equilibrium theory, driving price equilibration in multiple market settings.
REFERENCES

Ackerman, F. (2002). Still dead after all these years: Interpreting the failure of general equilibrium theory. *Journal of Economic Methodology*, 9(2), 119-139.


