6 Theory Appendix

6.1 Equation (8)

To simplify notation, we drop the superscript k. In addition, let $\omega_c = w_c h_c$, $\omega_n = w_n h_n$, $F_c = \frac{\partial F(.)}{\partial \varepsilon_c}$, and $F_{nc} = \frac{\partial^2 F(.)}{\partial \varepsilon_n \partial \varepsilon_c}$. Using the definition of p_n, we have

$$\begin{aligned} p_n &= \Pr(\omega_n \varepsilon_n \geq \omega_c \varepsilon_c) = \int_0^\infty \int_{\frac{\omega_c}{\omega_n}\varepsilon_c}^\infty F_{nc} d\varepsilon_n d\varepsilon_c \\ &= \int_0^\infty [F_c(\varepsilon_c, \varepsilon_n \to \infty) - F_c(\varepsilon_c, \varepsilon_n = \frac{\omega_c}{\omega_n}\varepsilon_c)] d\varepsilon_c \\ &= \int_0^\infty F_c(\varepsilon_c, \varepsilon_n \to \infty) d\varepsilon_c - \int_0^\infty F_c(\varepsilon_c, \varepsilon_n = \frac{\omega_c}{\omega_n}\varepsilon_c) d\varepsilon_c \end{aligned}$$

Using the Frechet distribution (1), we have

$$F_c(\varepsilon_c, \varepsilon_n) = AFT_c \varepsilon_c^{-\theta-1}, A = (1-\rho)\theta(T_n \varepsilon_n^{-\theta} + T_c \varepsilon_c^{-\theta})^{-\rho}$$

(1) When $\varepsilon_n \to \infty$, $A = (1-\rho)\theta(T_c \varepsilon_c^{-\theta})^{-\rho}$ and $F = \exp[-(T_c \varepsilon_c^{-\theta})^{1-\rho}]$. Therefore,

$$\begin{aligned} F_c(\varepsilon_c, \varepsilon_n &\to \infty) = (1-\rho)\theta(T_c \varepsilon_c^{-\theta})^{-\rho} \exp[-(T_c \varepsilon_c^{-\theta})^{1-\rho}][T_c \varepsilon_c^{-\theta-1}] \\ &= \theta(1-\rho)(T_c)^{1-\rho} \varepsilon_c^{-\theta(1-\rho)-1} \exp[-(T_c)^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}] \end{aligned}$$

and

$$\begin{aligned} \int_0^\infty F_c(\varepsilon_c, \varepsilon_n &\to \infty) d\varepsilon_c = \int_0^\infty \theta(1-\rho)(T_c)^{1-\rho} \varepsilon_c^{-\theta(1-\rho)-1} \exp[-(T_c)^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}] d\varepsilon_c \\ &= \int_0^\infty \frac{d \exp[-(T_c)^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}]}{d\varepsilon_c} = \left(\exp[-(T_c)^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}]\right)\Big|_0^\infty = 1 \end{aligned}$$

(2) When $\varepsilon_n = \frac{\omega_c}{\omega_n}\varepsilon_c$,

$$A = (1-\rho)\theta[T_n \varepsilon_c^{-\theta}(\frac{\omega_c}{\omega_n})^{-\theta} + T_c \varepsilon_c^{-\theta})^{-\rho} = (1-\rho)\theta(\varepsilon_c^{-\theta})^{-\rho} B^{-\rho}, B = T_n(\frac{\omega_c}{\omega_n})^{-\theta} + T_c$$

and,

$$F(\varepsilon_c, \varepsilon_n = \frac{\omega_c}{\omega_n}\varepsilon_c) = \exp\{-[T_n \varepsilon_c^{-\theta}(\frac{\omega_c}{\omega_n})^{-\theta} + T_c \varepsilon_c^{-\theta}]^{1-\rho}\} = \exp[-B^{1-\rho}(\varepsilon_c^{-\theta})^{1-\rho}]$$

Therefore,

$$\begin{aligned} F_c(\varepsilon_c, \varepsilon_n &= \frac{\omega_c}{\omega_n}\varepsilon_c) = (1-\rho)\theta(\varepsilon_c^{-\theta})^{-\rho} B^{-\rho} \exp[-B^{1-\rho}(\varepsilon_c^{-\theta})^{1-\rho}][T_c \varepsilon_c^{-\theta-1}] \\ &= (1-\rho)\theta T_c \varepsilon_c^{-\theta(1-\rho)-1} B^{-\rho} \exp[-B^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}] \end{aligned}$$

and

$$\begin{aligned}
\int_0^\infty F_c(\varepsilon_c, \varepsilon_n &= \frac{\omega_c}{\omega_n}\varepsilon_c)d\varepsilon_c = \int_0^\infty (1-\rho)\theta T_c \varepsilon_c^{-\theta(1-\rho)-1} B^{-\rho}\exp[-B^{1-\rho}\varepsilon_c^{-\theta(1-\rho)}]d\varepsilon_c \\
&= T_c B^{-1}\int_0^\infty \frac{d\exp[-B^{1-\rho}\varepsilon_c^{-\theta(1-\rho)}]}{d\varepsilon_c} \\
&= \left(T_c B^{-1}\exp[-B^{1-\rho}\varepsilon_c^{-\theta(1-\rho)}]\right)\Big|_0^\infty = T_c B^{-1}
\end{aligned}$$

(3) Using (1) and (2) above we have

$$p_n = 1 - T_c B^{-1} = \frac{T_n(\omega_c)^{-\theta}(\omega_n)^\theta}{T_c + T_n(\omega_c)^{-\theta}(\omega_n)^\theta} = \frac{T_n(\omega_n)^\theta}{T_c(\omega_c)^\theta + T_n(\omega_n)^\theta}$$

This is equation (8).

6.2 Equations (9) and (12)

To simplify notation, we drop the superscript k. We note that the Frechet distribution is max stable; i.e. the max of Frechet variables is still Frechet. To be specific, consider the random variable $\varepsilon^* = \max\{w_c h_c \varepsilon_c, w_n h_n \varepsilon_n\}$. By our discussions in section 2, $\varepsilon^* = w_n h_n \varepsilon_n$ if and only if the individual chooses occupation n.

We now obtain the cdf of the distribution of ε^*

$$\begin{aligned}
\Pr(\varepsilon^* &\leq y) = \Pr(w_c h_c \varepsilon_c \leq y \text{ and } w_n h_n \varepsilon_n \leq y) \\
&= F(\frac{y}{w_c h_c}, \frac{y}{w_n h_n}) \\
&= \exp[-B_1 y^{-\theta(1-\rho)}], B_1 = \left(T_c\left(\frac{w_c}{P}h_c\right)^\theta + T_n\left(\frac{w_n}{P}h_n\right)^\theta\right)^{1-\rho}
\end{aligned}$$

where we have used the Frechet distribution (1) in the second equality.

Consider the mean of soft-skill workers' net income, I_n, conditional on choosing the soft-skill occupation, n. By the expression of I_n, (7), we know that it is proportional to the mean of $(w_n h_n \varepsilon_n)^{\frac{1}{1-\eta}}$, conditional on choosing occupation n. This conditional mean is, by Bayesian rule, the mean of $(w_n h_n \varepsilon_n)^{\frac{1}{1-\eta}}$ for those choosing occupation n, divided by the employment share p_n. The mean of $(w_n h_n \varepsilon_n)^{\frac{1}{1-\eta}}$ for those choosing occupation n, in turn, is the mean of $(\varepsilon^*)^{\frac{1}{1-\eta}}$ for all workers times the employment share p_n. As a result, the conditional mean of I_n is proportional to the mean of $(\varepsilon^*)^{\frac{1}{1-\eta}}$, which equals

$$\int_0^\infty y^{\frac{1}{1-\eta}}\frac{d\exp[-B_1 y^{-\theta(1-\rho)}]}{dy} = \int_0^\infty y^{\frac{1}{1-\eta}}\exp[-B_1 y^{-\theta(1-\rho)}]B_1\theta(1-\rho)y^{-\theta(1-\rho)-1}dy$$

We then use change-of-variables to calculate the value of this expression, because the Gamma function is defined as

$$\Gamma(a+1)=\int_0^\infty t^a e^{-t}dt,$$

where a is a constant. Let $x=B_1y^{-\theta(1-\rho)}$. Then $y=(\frac{x}{B_1})^{-\frac{1}{\theta(1-\rho)}}$, and $dy=-\frac{1}{\theta(1-\rho)}B_1^{\frac{1}{\theta(1-\rho)}}x^{-\frac{1}{\theta(1-\rho)}-1}dx$. In addition, as $y\to 0$, $x\to\infty$; as $y\to\infty$, $x\to 0$. Therefore,

$$\begin{aligned}
& \int_0^\infty y^{\frac{1}{1-\eta}}\frac{d\exp[-B_1y^{-\theta(1-\rho)}]}{dy} \\
= & \int_0^\infty y^{\frac{1}{1-\eta}}\exp[-B_1y^{-\theta(1-\rho)}]B_1\theta(1-\rho)y^{-\theta(1-\rho)-1}dy \\
= & \int_\infty^0 (\frac{x}{B_1})^{-\frac{1}{\theta(1-\rho)(1-\eta)}}e^{-x}B_1\theta(1-\rho)(\frac{x}{B_1})^{\frac{1+\theta(1-\rho)}{\theta(1-\rho)}}[-\frac{1}{\theta(1-\rho)}]B_1^{\frac{1}{\theta(1-\rho)}}x^{-\frac{1}{\theta(1-\rho)}-1}dx \\
= & \int_0^\infty (\frac{x}{B_1})^{-\frac{1}{\theta(1-\rho)(1-\eta)}+\frac{1}{\theta(1-\rho)}+1-\frac{1}{\theta(1-\rho)}-1}e^{-x}dx \\
= & B_1^{\frac{1}{\theta(1-\rho))(1-\eta)}}\int_0^\infty x^{-\frac{1}{\theta(1-\rho)(1-\eta)}}e^{-x}dx=B_1^{\frac{1}{\theta(1-\rho))(1-\eta)}}\Gamma(1-\frac{1}{\theta(1-\rho)(1-\eta)}) \\
= & \gamma[T_c\left(\frac{w_c}{P}h_c\right)^\theta+T_n\left(\frac{w_n}{P}h_n\right)^\theta]^{\frac{1}{\theta(1-\eta)}},\gamma=\Gamma(1-\frac{1}{\theta(1-\rho)(1-\eta)})
\end{aligned}$$

Therefore, the average net income of soft-skill workers, I_n, equals $(1-\eta)\eta^{\frac{\eta}{1-\eta}}\gamma[T_c(\frac{w_c}{P}h_c)^\theta+T_n(\frac{w_n}{P}h_n)^\theta]^{\frac{1}{\theta(1-\eta)}}$.This is equation (9).

Meanwhile, the average real income of a worker in occupation i is I, so the total real income of workers in occupation i is Lp_iI. The real wage of a unit of effective labor of type i is w_i/P and the number of effective units is L_i but we must net out expenditure on education. Hence, we must have

$$\frac{w_i}{P}L_i(1-\eta)=Lp_iI.$$

Substituting using (7), we obtain

$$\begin{aligned}
L_i &= \frac{Lp_i}{w_i}P\gamma\eta^{\frac{\eta}{1-\eta}}\left[T_c\left(\frac{w_c^k}{P^k}h_c^k\right)^\theta+T_n\left(\frac{w_n^k}{P^k}h_n^k\right)^\theta\right]^{\frac{1}{\theta(1-\eta)}} \\
&= \frac{Lp_i}{w_i}\gamma\eta^{\frac{\eta}{1-\eta}}\left[P^{1-\eta}\left(T_c\left(\frac{w_c^k}{P^k}h_c^k\right)^\theta+T_n\left(\frac{w_n^k}{P^k}h_n^k\right)^\theta\right)^{\frac{1}{\theta}}\right]^{\frac{1}{1-\eta}}.
\end{aligned}$$

6.3 Equations (19) and (20)

We normalize $P^k = 1$. By equation (4),

$$
\begin{aligned}
Y^k &= \Theta^k \left(A_c \left(L_c^k\right)^{\frac{\alpha-1}{\alpha}} + A_n \left(L_n^k\right)^{\frac{\alpha-1}{\alpha}}\right)^{\frac{\alpha}{\alpha-1}} \\
&= \Theta^k L_c^k \left(A_c + A_n \left(\frac{L_n^k}{L_c^k}\right)^{\frac{\alpha-1}{\alpha}}\right)^{\frac{\alpha}{\alpha-1}}
\end{aligned}
$$

By equation (16),

$$
\frac{L_c^k}{L_n^k} = \left(\frac{p_c^k}{p_n^k}\frac{A_n}{A_c}\right)^{\frac{\alpha}{\alpha-1}}.
$$

Substituting this expression into the output equation yields

$$
Y^k = \Theta^k L_c^k \left(\frac{A_c}{p_c^k}\right)^{\frac{\alpha}{\alpha-1}}
$$

Substituting out p_c^k using equation (13), we obtain

$$
w_c^k = \Theta^k \left(p_c^k\right)^{-\frac{1}{\alpha-1}} \left(A_c\right)^{\frac{\alpha}{\alpha-1}} \tag{37}
$$

Rearranging equation (10),

$$
E_c^k = \left(\eta w_c^k h_c^k\right)^{\frac{1}{1-\eta}} \left(\frac{T_c}{p_c^k}\right)^{\frac{1}{\theta(1-\eta)}} \gamma,
$$

Given that $E^k = \eta Y^k / L^k$, we can substitute w_c^k in equation (37) to obtain, after rearranging

$$
\frac{Y^k}{L^k} = \left(\Theta^k h_c^k \left(p_c^k\right)^{-\frac{\phi}{\theta(\alpha-1)}} \left(A_c\right)^{\frac{\alpha}{\alpha-1}} \left(T_c\right)^{\frac{1}{\theta}} \eta\right)^{\frac{1}{1-\eta}} \frac{\gamma}{\eta},
$$

where we have defined $\phi \equiv \alpha + \theta - 1$. Substituting out p_c^k using its definition, we obtain

$$
\frac{Y^k}{L^k} = \left(\Theta^k h_c^k \left(1 + \frac{T_n \left(h_n^k\right)^\theta}{T_c \left(h_c^k\right)^\theta} \left(\frac{w_n^k}{w_c^k}\right)^\theta\right)^{\frac{\phi}{\theta(\alpha-1)}} \left(A_c\right)^{\frac{\alpha}{\alpha-1}} \left(T_c\right)^{\frac{1}{\theta}} \eta\right)^{\frac{1}{1-\eta}} \frac{\gamma}{\eta}.
$$

We then substitute equation (18) into this expression, to obtain an expression with no endogenous variables

$$
\frac{Y^k}{L^k} = \left(\Theta^k h_c^k \left(1 + \left(\frac{T_n \left(h_n^k\right)^\theta}{T_c \left(h_c^k\right)^\theta}\right)^{\frac{\alpha-1}{\phi}} \left(\frac{A_n}{A_c}\right)^{\alpha \frac{\theta}{\phi}}\right)^{\frac{\phi}{\theta(\alpha-1)}} \left(A_c\right)^{\frac{\alpha}{\alpha-1}} \left(T_c\right)^{\frac{1}{\theta}} \eta\right)^{\frac{1}{1-\eta}} \frac{\gamma}{\eta}.
$$

Therefore,

$$\frac{Y^k/L^k}{Y^0/L^0} = \left(\frac{\Theta^k}{\Theta^0} \frac{h_c^k \left(1 + \left(\frac{T_n \left(h_n^k\right)^\theta}{T_c \left(h_c^k\right)^\theta} \right)^{\frac{\alpha-1}{\phi}} \left(\frac{A_n}{A_c} \right)^{\alpha \frac{\theta}{\phi}} \right)^{\frac{\phi}{\theta(\alpha-1)}}}{h_c^0 \left(1 + \left(\frac{T_n (h_n^0)^\theta}{T_c (h_c^0)^\theta} \right)^{\frac{\alpha-1}{\phi}} \left(\frac{A_n}{A_c} \right)^{\alpha \frac{\theta}{\phi}} \right)^{\frac{\phi}{\theta(\alpha-1)}}} \right)^{\frac{1}{1-\eta}}.$$

Combining equations (8) and (18) for the base country, we have

$$\left(\frac{A_n}{A_c}\right)^{\frac{\theta\alpha}{\phi}} \left(\frac{T_n}{T_c}\right)^{\frac{\alpha-1}{\phi}} = \left(\frac{(h_c^0)^\theta}{(h_n^0)^\theta}\right)^{\frac{\alpha-1}{\phi}} \frac{p_n^0}{p_c^0}.$$

Substituting this expression into the expression for $\frac{Y^k/L^k}{Y^0/L^0}$, and simplifying, we arrive at our equations (19) and (20):

$$\frac{Y^k/L^k}{Y^0/L^0} = \left(\frac{\Theta^k}{\Theta^0} \left(p_c^0 \left(\frac{h_c^k}{h_c^0} \right)^{\frac{\theta(\alpha-1)}{\phi}} + p_n^0 \left(\frac{h_n^k}{h_n^0} \right)^{\frac{\theta(\alpha-1)}{\phi}} \right)^{\frac{\phi}{\theta(\alpha-1)}} \right)^{\frac{1}{1-\eta}}.$$

6.4 Hard and Soft-Skill Intensive Occupations

In this appendix, we prove the main equation in sub-section 2.5; i.e.

$$HCAP^k = \left(p_1^0 \left(\frac{\widetilde{h}_1^k}{\widetilde{h}_1^0} \right)^{\frac{\theta(\alpha-1)}{\phi}} + p_2^0 \left(\frac{\widetilde{h}_2^k}{\widetilde{h}_2^0} \right)^{\frac{\theta(\alpha-1)}{\phi}} \right)^{\frac{\phi}{\theta(\alpha-1)} \frac{1}{1-\eta}}.$$

The intuition of our results is as follows. Both occupations now use hard-skill and soft-skill human capital, and so both occupations are packages of skills. What matters for human-capital productivity of country k, then, is the productivities of these packages, $\widetilde{h}_1^k$ and $\widetilde{h}_2^k$. On the other hand, the structure and mechanism of our model remain unchanged, and so all the results there hold for $\widetilde{h}_1^k$ and $\widetilde{h}_2^k$.

We now show Proposition 3. The worker who chooses occupation i solves the following optimization problem

$$w_i \widetilde{h}_i^k e^\eta \epsilon_i - P^k e$$

which is similar to section 2. The solution is similar to equation (6):

$$e(\epsilon_i) = \left(\eta \frac{w_i^k}{P^k} \widetilde{h}_i^k \epsilon_i \right)^{\frac{1}{1-\eta}}.$$

Net income for occupation i is similar to equation (7)

$$I_i(\varepsilon_i) = (1-\eta)\eta^{\frac{\eta}{1-\eta}} \left(\frac{w_i^k}{P^k} \widetilde{h}_i^k \varepsilon_i^k \right)^{\frac{1}{1-\eta}}$$

So we chose occupation 1 if $w_2^k \widetilde{h}_2^k \varepsilon_2^k \leq w_1^k \widetilde{h}_1^k \varepsilon_1^k$. From this, the share of workers who choose occupation i is, similar to equation (8),

$$p_i^k = \frac{T_i \left(\widetilde{h}_i^k w_i \right)^\theta}{\Delta^k}, \Delta^k = T_1 \left(\widetilde{h}_1^k w_1^k \right)^\theta + T_2 \left(\widetilde{h}_2^k w_2^k \right)^\theta, \tag{38}$$

The aggregate supply of human capital in occupation i is similar to equation (12)

$$L_i^{kS} = \frac{L^k p_i^k}{w_i^k} \left(\eta^\eta (P^k)^{1-\eta} \left(T_c \left(\frac{w_c^k}{P^k} \widetilde{h}_1^k \right)^\theta + T_n \left(\frac{w_n^k}{P^k} \widetilde{h}_2^k \right)^\theta \right)^{\frac{1}{\theta}} \right)^{\frac{1}{1-\eta}} \gamma = \gamma \frac{L^k p_i^k}{w_i^k} \left(\eta^\eta (P^k)^{-\eta} \left(\Delta^k \right)^{\frac{1}{\theta}} \right)^{\frac{1}{1-\eta}}$$

and average spending is similar to equation (10).

$$E = \gamma \eta^{\frac{1}{1-\eta}} \left(\Delta^k \right)^{\frac{1}{\theta(1-\eta)}} (\frac{1}{P^k})^{\frac{1}{1-\eta}} \tag{39}$$

The relative supply of occupation 1 is similar to equation (14)

$$\frac{L_1^{kS}}{L_2^{kS}} = \frac{T_1}{T_2} \left(\frac{\widetilde{h}_1^k}{\widetilde{h}_2^k} \right)^\theta \left(\frac{w_1^k}{w_2^k} \right)^{\theta-1}$$

and relative demand is the same as equation (16). So the relative return is similar to equation (18),

$$\frac{w_1^k}{w_2^k} = \left(\frac{T_2}{T_1} \left(\frac{\widetilde{h}_2^k}{\widetilde{h}_1^k} \right)^\theta \left(\frac{A_1}{A_2} \right)^\alpha \right)^{\frac{1}{\theta+\alpha-1}}$$

and comparative advantage is similar to equation (28).

$$\frac{\widetilde{h}_1^k / \widetilde{h}_2^k}{\widetilde{h}_1^0 / \widetilde{h}_2^0} = \left(\frac{p_1^k / p_2^k}{p_1^0 / p_2^0} \right)^{\frac{\phi}{\theta(\alpha-1)}}.$$

The derivation of the HCAP index follows similar steps as Theory Appendix 6.3.

6.5 Additional Equations and Derivations: Open Economy

6.5.1 Additional Equilibrium Conditions and Equilibrium Definition

International equilibrium requires that countries' exports of hard skills must be equal to other countries' imports of soft skills. Defining M_c as the set of countries that import hard-skill labor (i.e. $x_c^k < 0$) and X_c as the set of countries that export hard-skill labor (i.e.

$x_c^k > 0$). International factor market clearing requires that

$$\sum_{k \in X_c} x_c^k L_c^{kS} + \sum_{k \in M_c} x_c^k L_c^{kS} \tau^k = 0, \tag{40}$$

where L_c^{kS} must satisfy (12). Let w_c and w_n denote the prices of soft and hard skills on the international factor market clearinghouse. Then factor prices in country k are given by

$$\begin{aligned} w_c^k &= w_c,\, w_n^k = w_n \tau^k \text{ if } k \in X_c, \\ w_c^k &= w_c \tau^k,\, w_n^k = w_n \text{ if } k \in X_n. \end{aligned} \tag{41}$$

We are now in a position to define the equilibrium of our model.

Definition 2 *An equilibrium to our model is a set of international factor prices w_c and w_n that imply local factor prices via (41) and that imply quantities of factors supplied locally, given by (14), and factors demanded, given by (16). These quantities clear domestic factor markets, and the associated factor trades clear the international market for hard skills, given by (40) in conjunction with (12).*

6.5.2 Equations (23) and (24)

We start by substituting equation (12) into equation (13), to obtain

$$\frac{Y^k}{L^k} = \frac{1}{P^k}\left(\frac{\eta}{P^k}\right)^{\frac{\eta}{1-\eta}} \left[T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)^\theta\right]^{\frac{1}{\theta(1-\eta)}} \gamma$$

and so relative to the base country we have

$$\frac{Y^k/L^k}{Y^0/L^0} = \left(\frac{P^0}{P^k}\right)^{\frac{\eta}{1-\eta}} \left[p_c^0\left(\frac{w_c^k h_c^k}{w_c^0 h_c^0}\right)^\theta + p_n^0\left(\frac{w_n^k h_n^k}{w_n^0 h_n^0}\right)^\theta\right]^{\frac{1}{\theta(1-\eta)}}$$

rearranging

$$\frac{Y^k/L^k}{Y^0/L^0} = \left(\frac{w_c^k/P^k}{w_c^0/P^0}\right)^{\frac{1}{1-\eta}} \left[p_c^0\left(\frac{h_c^k}{h_c^0}\right)^\theta + p_n^0\left(\frac{w_c^0}{w_c^k}\frac{w_n^k h_n^k}{w_n^0 h_n^0}\right)^\theta\right]^{\frac{1}{\theta(1-\eta)}} \tag{42}$$

Let $nx_i^k = p_i^k x_i^k$, $i = n, c$. We now use equations (14), (16) and (17) to show that

$$\frac{w_c^k}{w_n^k} = \left(\frac{A_c}{A_n}\right)^{\frac{\alpha}{\alpha-1}} \left(\frac{p_n^k - nx_n^k}{p_c^k - nx_c^k}\right)^{\frac{1}{\alpha-1}} \tag{43}$$

From the price index we have

$$\begin{aligned} P^k &= \frac{1}{\Theta^k}\left((A_c)^\alpha (w_c^k)^{1-\alpha} + (A_n)^\alpha (w_n^k)^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \\ &= \frac{w_c^k}{\Theta^k}\left((A_c)^\alpha + (A_n)^\alpha\left(\frac{w_c^k}{w_n^k}\right)^{\alpha-1}\right)^{\frac{1}{1-\alpha}} \end{aligned} \tag{44}$$

Combine this expression with equation (43), we have

$$\begin{aligned}
\frac{w_c^k}{P^k} &= \Theta^k \left((A_c)^\alpha + (A_n)^\alpha \left(\left(\frac{A_c}{A_n} \right)^{\frac{\alpha}{\alpha-1}} \left(\frac{p_n^k - nx_n^k}{p_c^k - nx_c^k} \right)^{\frac{1}{\alpha-1}} \right)^{\alpha-1} \right)^{\frac{1}{\alpha-1}} \\
&= \Theta^k \left((A_c)^\alpha + (A_c)^\alpha \frac{p_n^k - nx_n^k}{p_c^k - nx_c^k} \right)^{\frac{1}{\alpha-1}} \\
&= \Theta^k (A_c)^{\frac{\alpha}{\alpha-1}} \left(p_c^k - nx_c^k \right)^{-\frac{1}{\alpha-1}}
\end{aligned}$$

where the last equality uses $p_n^k - nx_n^k + p_c^k - nx_c^k = 1$, which is implied by equation (8) and balance of trade. so relative real wage is

$$\frac{w_c^k/P^k}{w_c^0/P^0} = \frac{\Theta^k}{\Theta^0} \left(\frac{p_c^k - nx_c^k}{p_c^0 - nx_c^0} \right)^{-\frac{1}{\alpha-1}} \tag{45}$$

We substitute equations (43) and (45) into (42):

$$\begin{aligned}
\frac{Y^k/L^k}{Y^0/L^0} &= \left(\frac{\Theta^k}{\Theta^0} \left(\frac{p_c^k - nx_c^k}{p_c^0 - nx_c^0} \right)^{-\frac{1}{\alpha-1}} \right)^{\frac{1}{1-\eta}} \left[p_c^0 \left(\frac{h_c^k}{h_c^0} \right)^\theta + p_n^0 \left(\frac{\left(\frac{p_n^0 - nx_n^0}{p_c^0 - nx_c^0} \right)^{\frac{1}{\alpha-1}}}{\left(\frac{p_n^k - nx_n^k}{p_c^k - nx_c^k} \right)^{\frac{1}{\alpha-1}}} \frac{h_n^k}{h_n^0} \right)^\theta \right]^{\frac{1}{\theta(1-\eta)}} \\
&= \left[\frac{\Theta^k}{\Theta^0} \left(p_c^0 \left(\left(\frac{p_c^k - nx_c^k}{p_c^0 - nx_c^0} \right)^{-\frac{1}{\alpha-1}} \frac{h_c^k}{h_c^0} \right)^\theta + p_n^0 \left(\left(\frac{p_c^k - nx_c^k}{p_c^0 - nx_c^0} \right)^{-\frac{1}{\alpha-1}} \frac{\left(\frac{p_n^0 - nx_n^0}{p_c^0 - nx_c^0} \right)^{\frac{1}{\alpha-1}}}{\left(\frac{p_n^k - nx_n^k}{p_c^k - nx_c^k} \right)^{\frac{1}{\alpha-1}}} \frac{h_n^k}{h_n^0} \right)^\theta \right)^{\frac{1}{\theta}} \right]^{\frac{1}{1-\eta}} \\
&= \left[\frac{\Theta^k}{\Theta^0} \left(p_c^0 \left(\left(\frac{p_c^0 - nx_c^0}{p_c^k - nx_c^k} \right)^{\frac{1}{\alpha-1}} \frac{h_c^k}{h_c^0} \right)^\theta + p_n^0 \left(\left(\frac{p_n^0 - nx_n^0}{p_n^k - nx_n^k} \right)^{\frac{1}{\alpha-1}} \frac{h_n^k}{h_n^0} \right)^\theta \right)^{\frac{1}{\theta}} \right]^{\frac{1}{1-\eta}}
\end{aligned}$$

This is equation (23). Meanwhile, under free trade, $P^k = (\Theta^k)^{-1}$, and so equation (12) implies

$$L_i^k = \frac{L^k p_i^k}{w_i} \left((\eta \Theta^k)^\eta \left(T_c \left(w_c h_c^k \right)^\theta + T_n \left(w_n h_n^k \right)^\theta \right)^{1/\theta} \right)^{1/(1-\eta)} \gamma$$

Combine this expression with equation (13), we can write real output per capita in country k relative to a base country 0 as

$$\frac{Y^k/L^k}{Y^0/L^0} = \left(\frac{\Theta^k}{\Theta^0} \left(\frac{T_c \left(w_c h_c^k \right)^\theta + T_n \left(w_n h_n^k \right)^\theta}{T_c \left(w_c h_c^0 \right)^\theta + T_n \left(w_n h_n^0 \right)^\theta} \right)^{\frac{1}{\theta}} \right)^{\frac{1}{1-\eta}}$$

Rearranging, we obtain

$$\frac{Y^k/L^k}{Y^0/L^0} = \left(\frac{\Theta^k}{\Theta^0}\left(\frac{T_c\left(w_c h_c^0\right)^\theta}{T_c\left(w_c h_c^0\right)^\theta + T_n\left(w_n h_n^0\right)^\theta}\left(\frac{T_c\left(w_c h_c^k\right)^\theta}{T_c\left(w_c h_c^0\right)^\theta}\right) + \frac{T_n\left(w_n h_n^0\right)^\theta}{T_c\left(w_c h_c^0\right)^\theta + T_n\left(w_n h_n^0\right)^\theta}\left(\frac{T_n\left(w_n h_n^k\right)^\theta}{T_n\left(w_n h_n^0\right)^\theta}\right)\right)^{\frac{1}{\theta}}\right)^{\frac{1}{1-\eta}}$$

Replacing the expressions with occupation shares from the base country, we obtain

$$\frac{Y^k/L^k}{Y^0/L^0} = \left(\frac{\Theta^k}{\Theta^0}\left(p_c^0\left(\frac{h_c^k}{h_c^0}\right)^\theta + p_n^0\left(\frac{h_n^k}{h_n^0}\right)^\theta\right)^{\frac{1}{\theta}}\right)^{\frac{1}{1-\eta}}.$$

This is equation (24).

6.6 Equation (25)

Using equations (27), (10) and (12), we can show that

$$\begin{aligned}
\frac{L_c^{kS}}{L^k} &= \frac{p_c^k}{w_c^k}\left((\eta(P^k)^{-1})^\eta\left(T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)\right)^{1/\theta}\right)^{1/(1-\eta)}\gamma = \frac{p_c^k}{w_c^k}\frac{E^k}{\eta(P^k)^{-1}} \\
\iff\ w_c^k &= \frac{p_c^k}{L_c^{kS}/L^k}\frac{E^k}{\eta(P^k)^{-1}} = \frac{p_c^k}{L_c^{kS}/L^k}\frac{E^k}{\eta}\frac{1}{(P^k)^{-1}}
\end{aligned}$$

We now use equation (8) to obtain that $T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right) = \frac{T_i(w_i^k h_i^k)^\theta}{p_i}$. This expression allows us to substitute out the term $T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)$ in equation (12), giving us, together with equation (27), that

$$\begin{aligned}
\frac{L_c^{kS}}{L^k} &= p_c^k\left(h_c^k(\eta(P^k)^{-1}w_c^k)^\eta\left(\frac{T_c}{p_c^k}\right)^{1/\theta}\right)^{1/(1-\eta)}\gamma \\
&= (p_c^k)^{1-\frac{1}{\theta(1-\eta)}}\gamma\eta^{\frac{\eta}{1-\eta}}(T_c)^{\frac{1}{\theta(1-\eta)}}((P^k)^{-1}w_c^k)^{\frac{\eta}{1-\eta}}(h_c^k)^{\frac{1}{1-\eta}}
\end{aligned}$$

We then substitute out w_c^k using $b\frac{p_c^k}{L_c^{kS}/L^k}\frac{E^k}{\eta}\frac{1}{(P^k)^{-1}}$ to obtain

$$\begin{aligned}
\frac{L_c^{kS}}{L^k} &= (p_c^k)^{1-\frac{1}{\theta(1-\eta)}}\gamma\eta^{\frac{\eta}{1-\eta}}(T_c)^{\frac{1}{\theta(1-\eta)}}((P^k)^{-1}\frac{p_c^k}{L_c^{kS}/L^k}\frac{E^k}{\eta}\frac{1}{(P^k)^{-1}})^{\frac{\eta}{1-\eta}}(h_c^k)^{\frac{1}{1-\eta}} \\
&= (\frac{1}{L_c^{kS}/L^k})^{\frac{\eta}{1-\eta}}\gamma\eta^{\frac{\eta}{1-\eta}}(T_c)^{\frac{1}{\theta(1-\eta)}}(p_c^k)^{1-\frac{1}{\theta(1-\eta)}+\frac{\eta}{1-\eta}}(\frac{E^k}{\eta})^{\frac{\eta}{1-\eta}}(h_c^k)^{\frac{1}{1-\eta}} \\
\Leftrightarrow\ \frac{L_c^{kS}}{L^k} &= \gamma^{1-\eta}\eta^\eta(T_c)^{\frac{1}{(1-\eta)}}(p_c^k)^{1-\frac{1}{\theta}}(E^k)^\eta h_c^k
\end{aligned}$$

Taking the ratio of this expression with respect to country 0, and substituting out E^k using equation (11), we get equation (25).

6.7 Notes for sub-sections 3.3 and 3.5

6.7.1 Relative Employment Share for Country-k's U.S. Emigrants

Assume that there is no variation in hard- and soft-skill productivities across countries, and that occupational employment shares, p_c^k and p_n^k, are completely driven by relative demand, A_c^k/A_n^k. In this case, we can normalize $h_c^k = h_n^k = 1$. Then equations (8) and (18) imply that

$$\frac{p_n^k}{p_c^k} = (\frac{T_n}{T_c})^{\frac{\alpha-1}{\phi}} \left[\left(\frac{A_n^k}{A_c^k} \right)^{\alpha\theta} \right]^{\frac{1}{\phi}}$$

Assume, in addition, that country k's emigrants to the U.S. are randomly selected from country k's labor force. Then these emigrants' occupational choices in the U.S. are described in a way similar to equation (8), with country-k wages replaced by U.S. wages. We then have

$$\frac{p_n^{k,0}}{p_c^{k,0}} = \frac{T_n (w_n^0)^\theta}{T_c (w_c^0)^\theta}.$$

This expression says that the emigrants' relative occupational employment share, $p_n^{k,0}/p_c^{k,0}$, has no cross-country variation, and so is uncorrelated with p_n^k/p_c^k.

Now suppose that U.S. immigration policy replaces restrictions on country k's emigrants. The variation of the relative restrictiveness of this policy across k then introduces variation in $p_n^{k,0}/p_c^{k,0}$. If the relative restrictiveness is uncorrelated with relative demand, $p_n^{k,0}/p_c^{k,0}$ is uncorrelated with p_n^k/p_c^k.

6.7.2 Iso-PISA Score Curve: Theoretical Approach

First, we examine how average hard-skill human capital, L_c^{kS}/L^k, changes in response to changes in hard-skill accumulation productivity, h_c^k, and soft-skill accumulation productivity, h_n^k, and perform comparative statics. We keep the labor-force size, L^k, and output TFP, Θ^k, fixed. Equations (14) and (17) imply that $\frac{L_n^{kS}}{L_c^{kS}} = \left(\frac{A_c}{A_n} \frac{p_n^k}{p_c^k} \right)^{\frac{\alpha}{\alpha-1}}$. We then substitute out the term L_n^{kD} in (4) using this expression and L_c^{kS}, to obtain

$$Y^k = \Theta^k \{ A_c (L_c^{kS})^{\frac{\alpha-1}{\alpha}} + A_n [L_c^{kS} (\frac{p_n^k}{p_c^k} \frac{A_c}{A_n})^{\frac{\alpha}{\alpha-1}}]^{\frac{\alpha-1}{\alpha}} \}^{\frac{\alpha}{\alpha-1}} = \Theta^k L_c^{kS} (A_c)^{\frac{\alpha}{\alpha-1}} (1 + \frac{p_n^k}{p_c^k})^{\frac{\alpha}{\alpha-1}}$$

This expression, the identity $p_n^k + p_c^k = 1$, and equations (28) and (25) imply that

$$d \ln p_n^k = -(d \ln p_c^k) \frac{p_c^k}{p_n^k}$$

$$(d \ln p_c^k) - d \ln p_n^k = \frac{\theta(\alpha - 1)}{\theta + \alpha - 1} (d \ln h_c^k - d \ln h_n^k)$$

$$d\ln L_c^{kS} - \eta d\ln Y^k = (1-\frac{1}{\theta})d\ln p_c^k + d\ln h_c^k$$

$$d\ln Y^k - d\ln L_c^{kS} = -\frac{\alpha}{\alpha-1}d\ln p_c^k$$

These four equations allow us to solve $d\ln L_c^{kS}$ in terms of $d\ln h_c^k$ and $d\ln h_n^k$.

Next, we map $d\ln L_c^{kS}$ to changes in test score by using equation (27) and metric 1 for hard skills, which has $g(.) = \exp(0.002t^k)$. Since we focus on test score, we assume that the number of schooling years remains unchanged. We thus obtain $0.002dt^k = d\ln L_c^{kS}$. As a result, we have

$$B_1 dt^k = (1+B_2 p_c^k)d\ln h_c^k - (B_2 p_n^k)d\ln h_n^k, B_1 = 0.002(1-\eta), B_2 = \frac{(\theta-1)(\alpha-1)-\alpha\eta}{\theta+\alpha-1}, \quad (46)$$

where $B_2 = 0.11$ according to our parameter values. The iso-PISA score curve that equation (46) implies is similar to the one in Figure 5.

6.7.3 Computation of the Free-Trade Equilibrium: Outline

As we know from equations (20) and (24), a country's HCAP index depends on the extent of openness to factor service trade. We show, below, that the international equilibrium condition (40) can be rewritten as

$$\sum_k \frac{H\left(\frac{h_c^k}{h_c^0}\widetilde{\omega}\right)^\theta}{H\left(\frac{h_c^k}{h_c^0}\widetilde{\omega}\right)^\theta + \left(\frac{h_n^k}{h_n^0}\right)^\theta} \frac{\left(\Theta^k\right)^{\frac{\eta}{1-\eta}} L^k \left(H\left(\frac{h_c^k}{h_c^0}\widetilde{\omega}\right)^\theta + \left(\frac{h_n^k}{h_n^0}\right)^\theta\right)^{\frac{1}{\theta}\frac{1}{1-\eta}}}{\sum_{k'}\left(\Theta^{k'}\right)^{\frac{\eta}{1-\eta}} L^{k'} \left(H\left(\frac{h_c^{k'}}{h_c^0}\widetilde{\omega}\right)^\theta + \left(\frac{h_n^{k'}}{h_n^0}\right)^\theta\right)^{\frac{1}{\theta}\frac{1}{1-\eta}}} = \frac{\left(\widetilde{\omega}\right)^{1-\alpha}}{\left(\widetilde{\omega}\right)^{1-\alpha}+1}, \quad (47)$$

where

$$H = \left(\frac{p_c^0}{p_n^0}\right)^{\frac{\theta+\alpha-1}{\alpha-1}}, \widetilde{\omega} = \left(\frac{A_c}{A_n}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{w_c}{w_n}\right).$$

In equation (47), the superscript "0" denotes the same base country as used in equation (24) (which is the U.S. in our computation). The only unknown variable in (47) is $\widetilde{\omega}$; all the other variables are known, either data or parameters. This means we can solve equation (47) for $\widetilde{\omega}$, recover relative demand from the expression (where the superscript "T" denotes the free-trade equilibrium)

$$s_c^{kT} = \frac{\left(\widetilde{\omega}\right)^{1-\alpha}}{\left(\widetilde{\omega}\right)^{1-\alpha}+1}$$

and equation (16), and then recover relative supply using

$$p_c^{kT} = \frac{H\left(\frac{h_c^k}{h_c^0}\widetilde{\omega}\right)^\theta}{H\left(\frac{h_c^k}{h_c^0}\widetilde{\omega}\right)^\theta + \left(\frac{h_n^k}{h_n^0}\right)^\theta}$$

and equation (14). Finally, factor service trade can be recovered from the expression $s_c^{kT} = p_c^{kT}(1 - x_c^k)$.

We now derive equation (47). With free trade, (40) simplifies to $\sum_k L_c^{kS} = \sum_k L_c^{kD}$.Using equation (13), the first-order condition for cost minimization, and factor price equalization,this equilibrium condition becomes

$$\sum_k p_c^k \frac{P^k Y^k}{\sum_{k'} P^{k'} Y^{k'}} = s_c$$

where a country's output weight can be written

$$\frac{P^k Y^k}{\sum_{k'} P^{k'} Y^{k'}} = \frac{\left(\Theta^k\right)^{\frac{\eta}{1-\eta}} L^k \left(T_c \left(\frac{w_c}{w_n} h_c^k\right)^\theta + T_n \left(h_n^k\right)^\theta\right)^{\frac{1}{\theta}\frac{1}{1-\eta}}}{\sum_{k'} \left(\Theta^{k'}\right)^{\frac{\eta}{1-\eta}} L^{k'} \left(T_c \left(\frac{w_c}{w_n} h_c^{k'}\right)^\theta + T_n \left(h_n^{k'}\right)^\theta\right)^{\frac{1}{\theta}\frac{1}{1-\eta}}}.$$

Substituting for factor supplies, factor demands, and for income weights, the international equilibrium condition becomes

$$\sum_k \frac{\frac{T_c}{T_n}(h_c^k \frac{w_c}{w_n})^\theta}{\frac{T_c}{T_n}(h_c^k \frac{w_c}{w_n})^\theta + (h_n^k)^\theta} \frac{\left(\Theta^k\right)^{\frac{\eta}{1-\eta}} L^k \left(\frac{T_c}{T_n} \left(\frac{w_c}{w_n} h_c^k\right)^\theta + \left(h_n^k\right)^\theta\right)^{\frac{1}{\theta}\frac{1}{1-\eta}}}{\sum_{k'} \left(\Theta^{k'}\right)^{\frac{\eta}{1-\eta}} L^{k'} \left(\frac{T_c}{T_n} \left(\frac{w_c}{w_n} h_c^{k'}\right)^\theta + \left(h_n^{k'}\right)^\theta\right)^{\frac{1}{\theta}\frac{1}{1-\eta}}}$$

$$= \frac{\left(\frac{A_c}{A_n}\right)^\alpha \left(\frac{w_c}{w_n}\right)^{1-\alpha}}{\left(\frac{A_c}{A_n}\right)^\alpha \left(\frac{w_c}{w_n}\right)^{1-\alpha} + 1}.$$

Defining $\widetilde{\omega} = \left(\frac{A_c}{A_n}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{w_c}{w_n}\right)$, we can substitute out $\frac{w_c}{w_n}$ using $\frac{w_c}{w_n} = \widetilde{\omega} \left(\frac{A_n}{A_c}\right)^{\frac{\alpha}{1-\alpha}}$, to obtain

$$\sum_k \frac{\frac{T_c}{T_n}\left(\left(\frac{A_n}{A_c}\right)^{\frac{\alpha}{1-\alpha}} h_c^k \widetilde{\omega}\right)^\theta}{\frac{T_c}{T_n}\left(\left(\frac{A_n}{A_c}\right)^{\frac{\alpha}{1-\alpha}} h_c^k \widetilde{\omega}\right)^\theta + (h_n^k)^\theta} \frac{\left(\Theta^k\right)^{\frac{\eta}{1-\eta}} L^k \left(\frac{T_c}{T_n} \left(\widetilde{\omega} \left(\frac{A_n}{A_c}\right)^{\frac{\alpha}{1-\alpha}} h_c^k\right)^\theta + \left(h_n^k\right)^\theta\right)^{\frac{1}{\theta}\frac{1}{1-\eta}}}{\sum_{k'} \left(\Theta^{k'}\right)^{\frac{\eta}{1-\eta}} L^{k'} \left(\frac{T_c}{T_n} \left(\left(\frac{A_n}{A_c}\right)^{\frac{\alpha}{1-\alpha}} \widetilde{\omega} h_c^{k'}\right)^\theta + \left(h_n^{k'}\right)^\theta\right)^{\frac{1}{\theta}\frac{1}{1-\eta}}}$$

$$= \frac{\left(\widetilde{\omega}\right)^{1-\alpha}}{\left(\widetilde{\omega}\right)^{1-\alpha} + 1}$$

Defining $\Psi \equiv \frac{T_c}{T_n} \left(\frac{A_n}{A_c}\right)^{\frac{\theta\alpha}{1-\alpha}}$, this expression becomes

$$\sum_k \frac{\Psi \left(h_c^k \widetilde{\omega}\right)^\theta}{\Psi \left(h_c^k \widetilde{\omega}\right)^\theta + \left(h_n^k\right)^\theta} \frac{\left(\Theta^k\right)^{\frac{\eta}{1-\eta}} L^k \left(\Psi \left(h_c^k \widetilde{\omega}\right)^\theta + \left(h_n^k\right)^\theta\right)^{\frac{1}{\theta}\frac{1}{1-\eta}}}{\sum_{k'} \left(\Theta^{k'}\right)^{\frac{\eta}{1-\eta}} L^{k'} \left(\Psi \left(h_c^{k'} \widetilde{\omega}\right)^\theta + \left(h_n^{k'}\right)^\theta\right)^{\frac{1}{\theta}\frac{1}{1-\eta}}} = \frac{\left(\widetilde{\omega}\right)^{1-\alpha}}{\left(\widetilde{\omega}\right)^{1-\alpha} + 1}$$

Rearranging this expression so that all human capital productivities appear as ratios, we obtain

$$\frac{(\widetilde{\omega})^{1-\alpha}}{(\widetilde{\omega})^{1-\alpha}+1}=\sum_k \frac{\Psi\left(\frac{h_c^0}{h_n^0}\right)^\theta\left(\frac{h_c^k}{h_c^0}\widetilde{\omega}\right)^\theta}{\Psi\left(\frac{h_c^0}{h_n^0}\right)^\theta\left(\frac{h_c^k}{h_c^0}\widetilde{\omega}\right)^\theta+\left(\frac{h_n^k}{h_n^0}\right)^\theta}\frac{\left(\Theta^k\right)^{\frac{\eta}{1-\eta}}L^k\left(\Psi\left(\frac{h_c^0}{h_n^0}\right)^\theta\left(\frac{h_c^k}{h_c^0}\widetilde{\omega}\right)^\theta+\left(\frac{h_n^k}{h_n^0}\right)^\theta\right)^{\frac{1}{\theta}\frac{1}{1-\eta}}}{\sum_{k'}\left(\Theta^{k'}\right)^{\frac{\eta}{1-\eta}}L^{k'}\left(\Psi\left(\frac{h_c^0}{h_n^0}\right)^\theta\left(\frac{h_c^{k'}}{h_c^0}\widetilde{\omega}\right)^\theta+\left(\frac{h_n^{k'}}{h_n^0}\right)^\theta\right)^{\frac{1}{\theta}\frac{1}{1-\eta}}}$$

Now, let us assume that the data we observe can be well approximated by the closed economy equilibrium. We have, by equation (8),

$$\frac{p_c^0}{p_c^0}=\frac{T_c}{T_n}\left(\frac{h_c^0}{h_n^0}\frac{w_c^0}{w_n^0}\right)^\theta .$$

Substituting for the autarky equilibrium wages in the base country using equation (18), we obtain

$$\Psi\left(\frac{h_c^0}{h_n^0}\right)^\theta=\left(\frac{p_c^0}{p_n^0}\right)^{\frac{\theta+\alpha-1}{\alpha-1}} .$$

Substituting this expression back into the labor market clearing condition, we obtain equation (47).

6.8 Notes for sub-section 4.2

From the skilled labor composite input producer's cost minimization problem, we have

$$\frac{L_c^k}{L_n^k}=\left(\frac{w_c^k}{w_n^k}\frac{A_n}{A_c}\right)^{-\alpha}$$

This implies demand for soft skills of

$$L_n^k=\frac{\left(A_n\right)^\alpha\left(w_n^k\right)^{-\alpha}}{\left(\left(A_c\right)^\alpha\left(w_c^k\right)^{1-\alpha}+\left(A_n\right)^\alpha\left(w_n^k\right)^{1-\alpha}\right)^{\frac{\alpha}{\alpha-1}}}$$

Combined with hard skill demand, it follows from the production function that the cost to produce one unit of the skilled composite is

$$w_s^k=\left(\left(A_c\right)^\alpha\left(w_c^k\right)^{1-\alpha}+\left(A_n\right)^\alpha\left(w_n^k\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha}} . \tag{48}$$

The same calculation done for the final good's producer yields, the price index for output of

$$P^k=\frac{1}{\Theta^k}\left(\left(B_u\right)^\rho\left(w_u^k\right)^{1-\rho}+\left(B_s\right)^\rho\left(w_s^k\right)^{1-\rho}\right)^{\frac{1}{1-\rho}}$$

Occupational Choice, Educational Investment, and Skilled Earnings Workers receive ε_c, ε_n draws if they have access to education, and they then consider their options. If they study skill i and they invest e units of the composite skilled good, then their human capital in this occupation will be $h_i^k e^{\eta}\varepsilon_i$. In making this decision, they take into account the effective wage of that occupation w_i^k and the cost of an effective unit of composite skilled labor, w_s^k, given by equation (48). Their net income after education is

$$\max_{e}\{w_i^k h_i^k e^{\eta}\varepsilon_i - w_s^k e\}. \tag{49}$$

Equation (49) has the first order condition of $\eta w_i^k h_i^k e^{\eta-1}\varepsilon_i = w_s^k$, and so workers spend

$$e(\varepsilon_i) = \left(\eta\frac{w_i^k}{w_s^k}h_i^k\varepsilon_i\right)^{\frac{1}{1-\eta}} \tag{50}$$

and they will earn a lifetime income of

$$I_i(\varepsilon_i) = w_i^k h_i^k e(\varepsilon_i)^{\eta}\varepsilon_i - w_s^k e(\varepsilon_i) = w_s^k e(\varepsilon_i)\frac{1-\eta}{\eta}.$$

Given the symmetry of the income expressions, a worker will choose occupation c if $w_c^k h_c^k\varepsilon_c > w_n^k h_n^k\varepsilon_n$. The probability that a random worker chooses occupation c is

$$\Pr\left(\varepsilon_n < \frac{w_c^k h_c^k}{w_n^k h_n^k}\varepsilon_c\right) = \int_0^{\infty}\frac{\partial F(\varepsilon_c', \varepsilon_n)}{\partial\varepsilon_c'}\Big|_0^{\frac{w_c^k h_c^k}{w_n^k h_n^k}\varepsilon_c'}d\varepsilon_c'$$

This calculation is the same as in Theory Appendix 6.1 and is not be repeated here. As a result, the expressions for p_c^k and p_n^k are the same as equation (8). Note that p_c^k and p_n^k are now conditional on skilled labor.

Now, we solve for the total nominal educational expenditure per capita. As can seen from equation (7), the average income of skilled workers is proportional to the average nominal educational spending per capita. To keep the notation simple, we temporarily drop the country subscript k. Defining $\xi \equiv 1/(1-\eta)$, we wish to solve for $E[\varepsilon_c^{\xi}|$ choose $c]$ knowing that the properties of the Frechet guarantee that it will be the same for n. We have

$$E[\varepsilon_c^{\lambda}|\text{choose } c] = \frac{1}{p_c}\int_0^{\infty}\varepsilon_c^{\xi}\frac{\partial F(\varepsilon_c', \varepsilon_n)}{\partial\varepsilon_c'}\Big|_0^{\frac{w_c^k h_c^k}{w_n^k h_n^k}\varepsilon_c'}d\varepsilon_c'$$

From the analysis above, we know that

$$\frac{1}{p_c}\frac{\partial F(\varepsilon_c', \varepsilon_n)}{\partial\varepsilon_c'}\Big|_0^{\frac{w_c^k h_c^k}{w_n^k h_n^k}\varepsilon_c'} = \frac{1}{p_c}\theta(1-\rho)T_c\left(T_c + T_n\left(\frac{w_c^k h_c^k}{w_n^k h_n^k}\right)^{-\theta}\right)^{-\rho}\varepsilon_c'^{-\theta(1-\rho)-1}\exp\left(-\left(T_c + T_n\left(\frac{w_c^k h_c^k}{w_n^k h_n^k}\right)^{-\theta}\right)^{1-\rho}\varepsilon\right.$$

After some simple derivation, we obtain

$$\frac{1}{p_c}\frac{\partial F(\varepsilon_c', \varepsilon_n)}{\partial \varepsilon_c'}|_0^{\frac{w_c^k h_c^k}{w_n^k h_n^k}\varepsilon_c'} = \theta(1-\rho)\left(\frac{T_c}{p_c}\right)^{1-\rho} \varepsilon_c'^{-\theta(1-\rho)-1} \exp\left(-\left(\frac{T_c}{p_c}\right)^{1-\rho} \varepsilon_c'^{-\theta(1-\rho)}\right).$$

We then define $x = \left(\frac{T_c}{p_c}\right)^{1-\rho} \varepsilon_c'^{-\theta(1-\rho)}$ and this implies $dx = -\left(\frac{T_c}{p_c}\right)^{1-\rho} \varepsilon_c'^{-\theta(1-\rho)-1} d\varepsilon_c'$ and $\varepsilon_c^{\lambda} = x^{-\frac{\lambda}{\theta(1-\rho)}}\left(\frac{T_c}{p_c}\right)^{\frac{\lambda}{\theta}}$. Noting that the change of variables reverses the limits of the integration, we then have

$$E[\varepsilon_c^{\frac{1}{1-\eta}}|\text{choose } c] = \left(\frac{T_c}{p_c}\right)^{\frac{\lambda}{\theta}}\gamma,$$

where $\gamma \equiv \Gamma(1 - \frac{1}{\theta(1-\rho)(1-\eta)})$. Using (50), and the expression we just derived, the observation that the symmetry implied by Frechet means that n workers have the same average wage,applying the country superscripts, we obtain the average educational spending per educated worker:

$$\begin{aligned} w_s^k E[e^k|\text{choose } i] &= w_s^k \left(\eta \frac{w_i^k}{w_s^k} h_i^k\right)^{\frac{1}{1-\eta}} \left(\frac{T_i}{p_i}\right)^{\frac{1}{\theta(1-\eta)}} \gamma \\ &= w_s^k \gamma \left[\eta \left(T_c \left(h_c^k \frac{w_c^k}{w_s^k}\right)^{\theta} + T_n \left(h_n^k \frac{w_n^k}{w_s^k}\right)^{\theta}\right)^{\frac{1}{\theta}}\right]^{\frac{1}{1-\eta}}. \end{aligned} \tag{51}$$

Finally, using equation (7), which shows that income is proportional to educational spending, we have the average income of a skilled worker:

$$I^k = I_c^k = I_n^k = \frac{1-\eta}{\eta} w_s^k \gamma \left[\eta \left(T_c \left(h_c^k \frac{w_c^k}{w_s^k}\right)^{\theta} + T_n \left(h_n^k \frac{w_n^k}{w_s^k}\right)^{\theta}\right)^{\frac{1}{\theta}}\right]^{\frac{1}{1-\eta}}. \tag{52}$$

National Accounting The total income of workers in occupation i is the number of workers in that occupation multiplied by the average income or $(1-\lambda^k)L^k p_i^k I^k$. The real wage of a unit of effective labor of type i is w_i^k/P^k and the number of (unobserved) effective units is L_i^k. Hence, the real aggregate income of i labor after netting out educational expenditure is

$$\frac{w_i^k}{P^k} L_i^k (1-\eta) = (1-\lambda^k) L^k p_i^k \frac{I^k}{P^k},\ i \in \{c, n\} \tag{53}$$

Let U^k denote total educational spending; i.e. $U^k = E^k(1-\lambda^k)L^k$. It also follows from these equations and from (7) that nominal aggregate educational expenditure is equal to

$$w_s^k U^k = \frac{\eta}{1-\eta}(1-\lambda^k) L^k I^k \tag{54}$$

This spending would appear in the national accounts as GDP even if it is not final output for consumption, because educational spending is like investment spending (this is similar to the 2-sector models used in the amplification literature, such as Erosa et al. 2010 and Manuelli and Seshadri 2014). Looking at (7) we see that spending on education is netted out of income. It follows then that $I^k/(1-\eta)$ is the gross amount that skilled workers are paid in the aggregate. Hence, the appropriate accounting identity is

$$P^kY^k + w_s^kU^k = L^k\left(w_u^k\lambda^k + (1-\lambda^k)\frac{I^k}{1-\eta}\right). \tag{55}$$

Substituting for $w_s^kU^k$ using (54), we obtain the equilibrium condition that supply equals demand for the final good:

$$\begin{aligned} P^kY^k + \frac{\eta}{1-\eta}(1-\lambda^k)L^k\cdot I^k &= L^k\left(w_u^k\lambda^k + (1-\lambda^k)\frac{I^k}{1-\eta}\right) \\ P^kY^k &= L^k\left(w_u^k\lambda^k + (1-\lambda^k)I^k\right). \end{aligned} \tag{56}$$

Factor Market Clearing: Skilled Composite Sector The immediate employers of skilled labor are the producers of the composite skilled good, H^k. This composite, some of which is used as an investment good and some of which is used as an input into final output, is created through the use of effective labor supplies L_c^{kS} and L_n^{kS}. Using (53) we can isolate the (gross) supply of type $i \in \{c, n\}$ labor:

$$\begin{aligned} L_i^{kS} &= (1-\lambda^k)L^k\frac{p_i^k}{w_i^k}\frac{\eta I^k}{1-\eta}, \\ &= (1-\lambda^k)L^k\frac{1}{w_i^k}\frac{T_i\left(h_i^k\frac{w_i^k}{w_s^k}\right)^\theta}{T_c\left(h_c^k\frac{w_c^k}{w_s^k}\right)^\theta + T_n\left(h_n^k\frac{w_n^k}{w_s^k}\right)^\theta}w_s^k\gamma\left[\eta\left(T_c\left(h_c^k\frac{w_c^k}{w_s^k}\right)^\theta + T_n\left(h_n^k\frac{w_n^k}{w_s^k}\right)^\theta\right)^{\frac{1}{\theta}}\right]^{\frac{1}{1-\eta}} \\ &= \gamma\eta^{\frac{1}{1-\eta}}(1-\lambda^k)L^kT_i\left(h_i^k\right)^\theta\left(\frac{w_i^k}{w_s^k}\right)^{\theta-1}\left(T_c\left(h_c^k\frac{w_c^k}{w_s^k}\right)^\theta + T_n\left(h_n^k\frac{w_n^k}{w_s^k}\right)^\theta\right)^{\frac{1}{\theta(1-\eta)}-1}. \end{aligned} \tag{57}$$

The relative supply of hard skills is similar to equation (14),

$$\frac{L_c^{kS}}{L_n^{kS}} = \frac{T_c}{T_n}\left(\frac{h_c^k}{h_n^k}\right)^\theta\left(\frac{w_c^k}{w_n^k}\right)^{\theta-1}. \tag{58}$$

Now consider the demand side of the skilled labor market. The CES skilled labor aggregator implies that, for skilled labor of occupation $i \in \{c, n\}$,

$$w_i^{kD}L_i^{kD} = s_i^kw_s^kH^k, \tag{59}$$

where the cost share of occupation $i \in \{c, n\}$ is given by

$$s_i^k = \frac{A_i^\alpha (w_i^k)^{1-\alpha}}{A_c^\alpha (w_c^k)^{1-\alpha} + A_n^\alpha (w_n^k)^{1-\alpha}}.$$

This implies that the relative demand for hard skills is

$$\frac{L_c^{kD}}{L_n^{kD}} = \left(\frac{A_c}{A_n}\right)^\alpha \left(\frac{w_c^k}{w_n^k}\right)^{-\alpha}. \tag{60}$$

Combining equations (58) and (60), we show that the expression for relative prices, $\frac{w_c^k}{w_n^k}$, is the same as equation (18). We then plug this expression into equation (8) to obtain

$$\begin{aligned} p_c^k &= \frac{T_c (w_c^k h_c^k)^\theta}{T_c (w_c^k h_c^k)^\theta + T_n (w_n^k h_n^k)^\theta} = \frac{\frac{T_c}{T_n}\left(\frac{h_c^k}{h_n^k}\right)^\theta \left(\frac{w_c^k}{w_n^k}\right)^\theta}{\frac{T_c}{T_n}\left(\frac{h_c^k}{h_n^k}\right)^\theta \left(\frac{w_c^k}{w_n^k}\right)^\theta + 1} \\ &= \frac{\left((A_c)^{\alpha\theta}\left(T_c\left(h_c^k\right)^\theta\right)^{\alpha-1}\right)^{\frac{1}{\theta+\alpha-1}}}{\left((A_c)^{\alpha\theta}\left(T_c\left(h_c^k\right)^\theta\right)^{\alpha-1}\right)^{\frac{1}{\theta+\alpha-1}} + \left((A_n)^{\alpha\theta}\left(T_n\left(h_n^k\right)^\theta\right)^{\alpha-1}\right)^{\frac{1}{\theta+\alpha-1}}}. \end{aligned} \tag{61}$$

Because this is a closed economy, the cost shares must be equal to the income shares: $s_c^k = p_c^k$.

Human Capital Index Substituting (53) into equation (59) and using $s_i^k = p_i^k$, and then substituting in (52) implies

$$\begin{aligned} w_s^k H^k &= \frac{w_i^{kD} L_i^{kD}}{s_i^k} = \left(1-\lambda^k\right) L^k \frac{I^k}{1-\eta} \text{ so} \\ H^k &= \left(1-\lambda^k\right) L^k \frac{\gamma}{\eta}\left[\eta\left(T_c\left(h_c^k \frac{w_c^k}{w_s^k}\right)^\theta + T_n\left(h_n^k \frac{w_n^k}{w_s^k}\right)^\theta\right)^{\frac{1}{\theta}}\right]^{\frac{1}{1-\eta}}. \end{aligned}$$

Pulling out $\frac{w_n^k}{w_s^k}$ of the parentheses and substituting using (**??**), this becomes

$$H^k = \left(1-\lambda^k\right) L^k \frac{\gamma}{\eta}\left[\frac{w_n^k}{w_s^k}\eta\left(T_c\left(h_c^k\left[\left(\frac{A_c}{A_n}\right)^\alpha \frac{T_n}{T_c}\left(\frac{h_n^k}{h_c^k}\right)^\theta\right]^{\frac{1}{\theta+\alpha-1}}\right)^\theta + T_n\left(h_n^k\right)^\theta\right)^{\frac{1}{\theta}}\right]^{\frac{1}{1-\eta}}.$$

Now substitute out w_s^k using the price index (48) to obtain

$$\frac{H^k}{\left(1-\lambda^k\right)L^k}=\frac{\gamma}{\eta}\left[\eta\frac{\left(T_c\left(h_c^k\left[\left(\frac{A_c}{A_n}\right)^{\alpha}\frac{T_n}{T_c}\left(\frac{h_n^k}{h_c^k}\right)^{\theta}\right]^{\frac{1}{\theta+\alpha-1}}\right)^{\theta}+T_n\left(h_n^k\right)^{\theta}\right)^{\frac{1}{\theta}}}{\left(\left(A_c\right)^{\alpha}\left(\left[\left(\frac{A_c}{A_n}\right)^{\alpha}\frac{T_n}{T_c}\left(\frac{h_n^k}{h_c^k}\right)^{\theta}\right]^{\frac{1}{\theta+\alpha-1}}\right)^{1-\alpha}+\left(A_n\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}}\right]^{\frac{1}{1-\eta}}.$$

Simplifying,

$$\frac{H^k}{\left(1-\lambda^k\right)L^k}=\frac{\gamma}{\eta}\left[\eta\left(\left(A_c\right)^{\frac{\alpha\theta}{\theta+\alpha-1}}\left(T_c\left(h_c^k\right)^{\theta}\right)^{\frac{\alpha-1}{\theta+\alpha-1}}+\left(A_n\right)^{\frac{\alpha\theta}{\theta+\alpha-1}}\left(T_n\left(h_n^k\right)^{\theta}\right)^{\frac{\alpha-1}{\theta+\alpha-1}}\right)^{\frac{\theta+\alpha-1}{\theta(\alpha-1)}}\right]^{\frac{1}{1-\eta}}.$$

Relative to a base country 0, we have

$$\frac{H^k/L^k\left(1-\lambda^k\right)}{H^0/L^0\left(1-\lambda^0\right)}=\left[\frac{\left(A_c\right)^{\frac{\theta\alpha}{\phi}}\left[T_c(h_c^k)^{\theta}\right]^{\frac{\alpha-1}{\phi}}+\left(A_n\right)^{\frac{\theta\alpha}{\phi}}\left[T_n(h_n^k)^{\theta}\right]^{\frac{\alpha-1}{\phi}}}{\left(A_c\right)^{\frac{\theta\alpha}{\phi}}\left[T_c(h_c^0)^{\theta}\right]^{\frac{\alpha-1}{\phi}}+\left(A_n\right)^{\frac{\theta\alpha}{\phi}}\left[T_n(h_n^0)^{\theta}\right]^{\frac{\alpha-1}{\phi}}}\right]^{\frac{1}{1-\eta}\frac{\phi}{\theta(\alpha-1)}}$$

where $\phi\equiv\theta+\alpha-1$. Reorganizing, we have

$$\frac{H^k/L^k\left(1-\lambda^k\right)}{H^0/L^0\left(1-\lambda^0\right)}=\left[\frac{\left(A_c\right)^{\frac{\theta\alpha}{\phi}}\left[T_c(h_c^0)^{\theta}\right]^{\frac{\alpha-1}{\phi}}\left[\frac{h_c^k}{h_c^0}\right]^{\frac{\theta(\alpha-1)}{\phi}}+\left(A_n\right)^{\frac{\theta\alpha}{\phi}}\left[T_n(h_n^0)^{\theta}\right]^{\frac{\alpha-1}{\phi}}\left[\frac{h_n^k}{h_n^0}\right]^{\frac{\theta(\alpha-1)}{\phi}}}{\left(A_c\right)^{\frac{\theta\alpha}{\phi}}\left[T_c(h_c^0)^{\theta}\right]^{\frac{\alpha-1}{\phi}}+\left(A_n\right)^{\frac{\theta\alpha}{\phi}}\left[T_n(h_n^0)^{\theta}\right]^{\frac{\alpha-1}{\phi}}}\right]^{\frac{1}{1-\eta}\frac{\phi}{\theta(\alpha-1)}}$$

$$\frac{H^k/L^k}{H^0/L^0}=\frac{\left(1-\lambda^k\right)}{\left(1-\lambda^0\right)}HCAP^k,\ HCAP^k=\left(p_c^0\left(\frac{h_c^k}{h_c^0}\right)^{\frac{\theta(\alpha-1)}{\phi}}+p_n^0\left(\frac{h_n^k}{h_n^0}\right)^{\frac{\theta(\alpha-1)}{\phi}}\right)^{\frac{1}{1-\eta}\frac{\phi}{\theta(\alpha-1)}}$$

This expression has two terms. The first is relative access to education on the left-hand side. The second is conditional on having access to an education how well does the country provide human capital.

Factor Market Clearing: Final Good Sector Factor market clearing for the composite skilled good and unskilled labor must satisfy

$$\begin{aligned} w_s^k H^{kS} &= \pi^k P^k Y^k + w_s^k U^k \\ w_u^k L_u^{kS} &= \left(1-\pi^k\right) P^k Y^k \end{aligned}$$

where H^{kS} is the supply for the skilled labor composite and its cost share in the final good sector is given by

$$\pi^k = \frac{B_s^\rho (w_s^k)^{1-\rho}}{B_u^\rho (w_u^k)^{1-\rho} + B_s^\rho (w_s^k)^{1-\rho}}$$

The demand for the skilled labor composite is partly from the education sector ($w_s^k E^k$). We know that $w_s^k H^{kS}$ is the total income of the skilled labor sector and that η of this income is spent on education so that $w_s^k U^k = \eta w_s^k H^{kS}$. Substituting this back into the equilibrium conditions, we have

$$\begin{aligned} w_s^k H^k (1-\eta) &= \pi^k P^k Y^k \\ w_u^k L_u^{kS} &= \left(1-\pi^k\right) P^k Y^k \end{aligned} \tag{62}$$

Using these expressions we obtain:

$$\frac{w_s^k H^k (1-\eta)}{w_u^k L_u^{kS}} = \frac{\pi^k}{1-\pi^k} = \frac{B_s^\rho (w_s^k)^{1-\rho}}{B_u^\rho (w_u^k)^{1-\rho}}$$

Hence, relative prices for the skill versus unskilled inputs are

$$\frac{w_s^k}{w_u^k} = \frac{B_s}{B_u} \left(\frac{\lambda^k L^k}{(1-\eta) H^k} \right)^{\frac{1}{\rho}}, \tag{63}$$

where we have used the assumption that $L_u^k = \lambda L^k$. This has the implication that the cost share of the skilled labor composite used in the final good is

$$\pi^k = \frac{B_s \left(\frac{(1-\eta)H^k}{L^k}\right)^{\frac{1-\rho}{\rho}}}{B_u \left(\lambda^k\right)^{\frac{\rho-1}{\rho}} + B_s \left(\frac{(1-\eta)H^k}{L^k}\right)^{\frac{\rho-1}{\rho}}}. \tag{64}$$

Going back to the equation (62), we have

$$\frac{Y^k}{L^k} = (1-\eta) \frac{w_s^k H^k}{P^k L^k} \frac{1}{\pi^k}.$$

From the definitions of the price index and of the composite skilled good's cost share in final production we have

$$\begin{aligned} \frac{Y^k}{L^k} &= (1-\eta) \frac{w_s^k}{\frac{1}{\Theta^k} \left((B_u)^\rho (w_u^k)^{1-\rho} + (B_s)^\rho (w_s^k)^{1-\rho} \right)^{\frac{1}{1-\rho}}} \frac{B_u^\rho (w_u^k)^{1-\rho} + B_s^\rho (w_s^k)^{1-\rho}}{B_s^\rho (w_s^k)^{1-\rho}} \frac{H^k}{L^k} \\ &= \frac{1-\eta}{B_s^\rho} \Theta^k \left((B_u)^\rho \left(\frac{w_u^k}{w_s^k} \right)^{1-\rho} + (B_s)^\rho \right)^{\frac{\rho}{\rho-1}} \frac{H^k}{L^k} \end{aligned}$$

Substitute using (63), $H^k/(L^k B_s^\rho)$ into the parentheses, this expression becomes

$$\frac{Y^k}{L^k} = (1-\eta)\Theta^k \left(B_u \left(\frac{\lambda^k}{1-\eta} \right)^{\frac{\rho-1}{\rho}} + B_s \left(\left(1-\lambda^k\right) \frac{H^k}{L^k \left(1-\lambda^k\right)} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$$

or relative to a base country, we have

$$\frac{Y^k/L^k}{Y^0/L^0} = \frac{\Theta^k}{\Theta^0}\left(\frac{B_u\left(\lambda^k\right)^{\frac{\rho-1}{\rho}} + B_s\left(\left(1-\lambda^k\right)\frac{(1-\eta)H^k}{L^k\left(1-\lambda^k\right)}\right)^{\frac{\rho-1}{\rho}}}{B_u\left(\lambda^0\right)^{\frac{\rho-1}{\rho}} + B_s\left(\left(1-\lambda^0\right)\frac{(1-\eta)H^0}{L^0\left(1-\lambda^0\right)}\right)^{\frac{\rho-1}{\rho}}}\right)^{\frac{\rho}{\rho-1}}$$

Finally, using (64) the decomposition becomes

$$\frac{Y^k/L^k}{Y^0/L^0} = \frac{\Theta^k}{\Theta^0}\left(\left(1-\pi^0\right)\left(\frac{\lambda^k}{\lambda^0}\right)^{\frac{\rho-1}{\rho}} + \left(\pi^0\right)\left(\frac{1-\lambda^k}{1-\lambda^0}HCAP^k\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{65}$$

We now relate the nominal value of final goods, P^kY^k, to the nominal value of total output, G^k. We start with the observation that

$$G^k = w_s^k H^{kS} + w_u^k L_u^{kS} = P^kY^k + w_s^k E^k(1-\lambda^k)L^k = P^kY^k + \eta w_s^k H^{kS}$$

where we have used equation (62). We next use equation (62) to obtain

$$\beta_s^k = \frac{\pi^k P^kY^k/(1-\eta)}{\pi^k P^kY^k/(1-\eta) + (1-\pi^k)P^kY^k} = \frac{\pi^k/(1-\eta)}{\pi^k/(1-\eta) + (1-\pi^k)}$$

Solving for π^k, we have

$$\pi^k = \frac{1-\eta}{1/\beta_s^k - \eta} \tag{66}$$

Finally, noting that $w_s^k H^{kS} = \beta_s^k G^k$, we have

$$G^k = \frac{1}{1-\eta\beta_s^k}P^kY^k = P^kY^k(1 + \frac{\eta}{1-\eta}\pi^k) \tag{67}$$

where, in the second equality, we have used equation (66). Equations (65) and (67) imply equations (31) and (32).

6.9 Notes for sub-section 4.3

In this appendix, we provide the derivations for the equations in sub-section 4.3.

6.9.1 Approach by CC

We translate the notation used in CC to our notation. CC's rich country, R, is our country k. CC's poor country, $P =$ our country 0. CC's body count of low skilled, $L_1 =$ our λ. CC's body count of high skilled $L_2 =$ our $1-\lambda$. CC's sub elasticity $\varepsilon =$ our ρ. We follow CC's notation for average wages, W_1 = low skill, W_2 = high skill. Finally, let HC^k denote the contribution of human capital to output per capita.

We first sub out the relative efficiency terms in CC's equation (3) using their equation (5). This equation reads, with the above translation, (the first equality)

$$\frac{HC^k_{CC}}{HC^0_{CC}} = \frac{\lambda^k}{\lambda^0}\frac{[1+\frac{W_2^0}{W_1^0}(\frac{1-\lambda^0}{\lambda^0})^{\frac{1}{\rho}}\left(\frac{1-\lambda^k}{\lambda^k}\right)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}}{[1+\frac{W_2^0}{W_1^0}(\frac{1-\lambda^0}{\lambda^0})^{\frac{1}{\rho}}\left(\frac{1-\lambda^0}{\lambda^0}\right)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}} = \frac{[\left(\lambda^k\right)^{\frac{\rho-1}{\rho}}+\frac{W_2^0}{W_1^0}(\frac{1-\lambda^0}{\lambda^0})^{\frac{1}{\rho}}\left(1-\lambda^k\right)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}}{[\left(\lambda^0\right)^{\frac{\rho-1}{\rho}}+\frac{W_2^0}{W_1^0}(\frac{1-\lambda^0}{\lambda^0})^{\frac{1}{\rho}}\left(1-\lambda^0\right)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}} \tag{68}$$

$$= \left(\left(1-\pi^0\right)\left(\frac{\lambda^k}{\lambda^0}\right)^{\frac{\rho-1}{\rho}}+\left(\pi^0\right)\left(\frac{1-\lambda^k}{1-\lambda^0}\right)^{\frac{\rho-1}{\rho}}RE^k_{CC}\right)^{\frac{\rho}{\rho-1}},\ RE^k_{CC}=1$$

where

$$\pi^0 = \frac{\frac{W_2^0}{W_1^0}(\frac{1-\lambda^0}{\lambda^0})^{\frac{1}{\rho}}\left(1-\lambda^0\right)^{\frac{\rho-1}{\rho}}}{\left(\lambda^0\right)^{\frac{\rho-1}{\rho}}+\frac{W_2^0}{W_1^0}(\frac{1-\lambda^0}{\lambda^0})^{\frac{1}{\rho}}\left(1-\lambda^0\right)^{\frac{\rho-1}{\rho}}} = \frac{W_2^0(1-\lambda^0)}{W_1^0\lambda^0+W_2^0(1-\lambda^0)}$$

is skilled-labor's share in income, as in our model. In this expression, RE^k represents the relative efficiency of human capital of country k vs. the benchmark country 0. Equation (68) says that $RE^k_{CC} = 1$. This is intuitive, because the point of CC 2019 is to fix the relative efficiency at the level of the base country, which is a poor country.

6.9.2 Approach by Jones

We combine CC's equations (4) and (5), to show Jones' approach

$$\frac{HC^k_{Jones}}{HC^0_{Jones}} = \frac{\lambda^k}{\lambda^0}\frac{[1+\frac{W_2^k}{W_1^k}(\frac{1-\lambda^k}{\lambda^k})^{\frac{1}{\rho}}\left(\frac{1-\lambda^k}{\lambda^k}\right)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}}{[1+\frac{W_2^0}{W_1^0}(\frac{1-\lambda^0}{\lambda^0})^{\frac{1}{\rho}}\left(\frac{1-\lambda^0}{\lambda^0}\right)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}} = \frac{[\left(\lambda^k\right)^{\frac{\rho-1}{\rho}}+\frac{W_2^k}{W_1^k}(\frac{1-\lambda^k}{\lambda^k})^{\frac{1}{\rho}}\left(1-\lambda^k\right)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}}{[\left(\lambda^0\right)^{\frac{\rho-1}{\rho}}+\frac{W_2^0}{W_1^0}(\frac{1-\lambda^0}{\lambda^0})^{\frac{1}{\rho}}\left(1-\lambda^0\right)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}} \tag{69}$$

$$= \left(\left(1-\pi^0\right)\left(\frac{\lambda^k}{\lambda^0}\right)^{\frac{\rho-1}{\rho}}+\left(\pi^0\right)\left(\frac{1-\lambda^k}{1-\lambda^0}RE^k_{Jones}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}},RE^k_{Jones}=[\frac{SP^k(\frac{1-\lambda^k}{\lambda^k})^{\frac{1}{\rho}}}{SP^0(\frac{1-\lambda^0}{\lambda^0})^{\frac{1}{\rho}}}]^{\frac{\rho}{\rho-1}}$$

Equation (69) says that the relative efficiency in Jones is backed out using data on body counts, and average earnings.

6.9.3 Our approach

Our approach is described in equation (32), with $RE^k_{XY} = HCAP^k$.

6.10 Notes for Our Preferred Value of ρ

Let W_2^k denote the wage of a skilled worker as predicted by our model; i.e. $\pi^k/(1-\lambda^k) = W_2^k$. W_2^k is distinct from w_s^k, the earning of 1 effective unit of skilled labor. Likewise, W_1^k denotes the wage of one unskilled worker as predicted by our model; i.e. $(1-\pi^k)/\lambda^k = W_1^k$. Let $\widetilde{W}_2^k$ and $\widetilde{W}_1^k$ denote the wages of skilled and unskilled workers in the data (computed in the same way as in CC and Jones). Then the skill premium predicted by our model is $\frac{W_2^k}{W_1^k}$, while that given by data is $\widetilde{W}_2^k/\widetilde{W}_1^k$.

Equation (62) implies that

$$\frac{w_s^k H^k(1-\eta)}{w_u^k \lambda^k L^k} = \frac{\pi^k}{1-\pi^k} \Leftrightarrow \frac{w_s^k}{w_u^k} = \frac{\pi^k}{1-\pi^k}\frac{\lambda^k L^k}{H^k(1-\eta)}$$

and equation (63) implies that

$$1 = \frac{(w_s^k/w_u^k)\left(\frac{(1-\eta)H^k}{\lambda^k L^k}\right)^{\frac{1}{\rho}}}{(w_s^0/w_u^0)\left(\frac{(1-\eta)H^0}{\lambda^0 L^0}\right)^{\frac{1}{\rho}}}$$

Combining these two expressions, and noting that $HCAP^k = \frac{H^k/[(1-\lambda^k)L^k]}{H^0/[(1-\lambda^0)L^0]}$, we have

$$\begin{aligned} 1 &= \frac{\left(\frac{\pi^k}{1-\pi^k}/\frac{1-\lambda^k}{\lambda^k}\right)}{\left(\frac{\pi^0}{1-\pi^0}/\frac{1-\lambda^0}{\lambda^0}\right)}\frac{\left(\frac{1-\lambda^k}{\lambda^k}\right)^{\frac{1}{\rho}}}{\left(\frac{1-\lambda^0}{\lambda^0}\right)^{\frac{1}{\rho}}}/\left(HCAP^k\right)^{\frac{\rho-1}{\rho}} = \frac{(W_2^k/W_1^k)}{(W_2^0/W_1^0)}\frac{\left(\frac{1-\lambda^k}{\lambda^k}\right)^{\frac{1}{\rho}}}{\left(\frac{1-\lambda^0}{\lambda^0}\right)^{\frac{1}{\rho}}}/\left(HCAP^k\right)^{\frac{\rho-1}{\rho}} \\ \Longleftrightarrow \ln\frac{(W_2^k/W_1^k)}{(W_2^0/W_1^0)} &= \ln\left(HCAP^k\right)^{\frac{\rho-1}{\rho}} - \ln\frac{\left(\frac{1-\lambda^k}{\lambda^k}\right)^{\frac{1}{\rho}}}{\left(\frac{1-\lambda^0}{\lambda^0}\right)^{\frac{1}{\rho}}} \end{aligned}$$

Meanwhile, using the definition of RE^k_{Jones}, as given by equation (34), we have

$$\begin{aligned} \left(RE^k_{Jones}\right)^{\frac{\rho-1}{\rho}} &= \frac{(\widetilde{W}_2^k/\widetilde{W}_1^k)(\frac{1-\lambda^k}{\lambda^k})^{\frac{1}{\rho}}}{(\widetilde{W}_2^0/\widetilde{W}_1^0)(\frac{1-\lambda^0}{\lambda^0})^{\frac{1}{\rho}}} \\ &\Leftrightarrow \ln\frac{(\widetilde{W}_2^k/\widetilde{W}_1^k)}{(\widetilde{W}_2^0/\widetilde{W}_1^0)} = \ln\left(RE^k_{Jones}\right)^{\frac{\rho-1}{\rho}} - \ln\frac{\left(\frac{1-\lambda^k}{\lambda^k}\right)^{\frac{1}{\rho}}}{\left(\frac{1-\lambda^0}{\lambda^0}\right)^{\frac{1}{\rho}}} \end{aligned}$$

Combining these two equations, we have

$$\ln\frac{W_2^k/W_1^k}{W_2^0/W_1^0} - \ln\frac{\widetilde{W}_2^k/\widetilde{W}_1^k}{\widetilde{W}_2^0/\widetilde{W}_1^0} = \ln\left(HCAP^k\right)^{\frac{\rho-1}{\rho}} - \ln\left(RE^k_{Jones}\right)^{\frac{\rho-1}{\rho}} \tag{70}$$

Equation (70) shows how our model's prediction of the ratio of skill premium differs from the data, and allows us to compute this deviation by country for a given value of ρ. Our preferred value of ρ produces the smallest standard deviation of this difference. This approach is similar, in some sense, to Hendricks and Schoellman (2018), who use their data and equations to compute model-consistent ρ values.

7 Data Appendix

7.1 Notes for Occupation Classification

7.1.1 O*NET Data for Leadership

We first list the O*NET ID's of the leadership characteristics we discuss in the text. Guiding and directing sub-ordinates is 4.A.4.b.4; leadership in work style = 1.C.2.b; coordinating the work and activities of others = 4.A.4.b.1; developing and building teams = 4.A.4.b.2; coaching and developing others = 4.A.4.b.5; recruiting and promoting employees = 4.A.4.c.2; monitoring and controlling resources and spending = 4.A.4.c.3; and coordinate or lead others in work = 4.C.1.b.1.g.

For each occupation characteristic, O*NET provides two metrics, importance and level. The importance metric ranges from 1 to 5, with 1 = not important, 3 = important, and 5 = extremely important, etc. These interpretations are common to all occupation characteristics. The level metric ranges from 0 to 7, and its interpretation is specific to each characteristic. For example, for guiding and directing subordinates, 2 = "work occasionally as a backup supervisor", 4 = "supervise a small number of subordinates in a well-paid industry", and 7 = "manage a severely downsized unit"; for developing and building teams, 1 = "encourage two coworkers to stick with a tough assignment", 4 = "lead an assembly team in an automobile plant", and 6 = "lead a large team to design and build a new aircraft".

We use the importance metric, not the level metric, because the former has the same intuitive interpretation for all occupation characteristics (e.g. a value of 3 means this characteristic is important). We also recognize that the importance metric is ordinal, not cardinal, and so we use its ordinal properties to classify occupations. To be specific, we rescale the numerical value of the leadership principal component to also range from 1 to 5. The soft-skill occupations have importance metric of 3 or above, and the hard-skill occupations have importance metric below 3. Figure A1 shows the histogram of this metric.

Column (1) of Table A1 recaps column (5) of Table 1, but we have removed the stars indicating statistical significance, and show t-statistics, instead of standard errors. In column (2) of Table A1, we classify occupations using the single O*NET characteristic of Guiding and directing sub-ordinates, 4.A.4.b.4. The interaction between AFQT score and the soft–skill-occupation dummy is negative and significant, similar to column (1).

7.1.2 Sample Cuts for NLSY-79 Data

Following Neal and Johnson (1996) we: (1) use the 1989 version of AFQT and drop the observations with missing AFQT scores; (2) drop the individuals whose wage is above \$75 or below \$1 in 1991; and (3) drop those who are older than 17 when they take the AFQT.

7.1.3 Alternative Occupation Classifications

We first experiment with ratios of occupation characteristics. We specify two types of hard-skill characteristics, math and information processing. For information processing, we use the following characteristics to construct the principal component: processing information (4.A.2.a.2) and evaluating information to determine compliance with standards (4.A.2.a.3). For math, we use the following characteristics to construct the principal component: mathematical reasoning (1.A.1.c.1), number facility (1.A.1.c.2), advanced math (2.D.3.g), mathematics (2.A.1.e and 2.C.4.a), basic math (2.D.3.f), mathematics - entry requirement (3.B.1.e). We then calculate the ratio of the leadership principal component, which we have been using, to the sum of the principal components of leadership, math and information processing.

For the occupation characteristics we examine, the level and importance metrics are highly correlated. So we examine the following 4 specifications. (1) We use the level metrics to compute principal components, and obtain the ratio of the leadership principal component to the sum of the principal components of leadership, math, and information processing. (2) We use the level metric, but drop information processing from the denominator. (3) We use the importance metric, and have all the 3 principal components in the denominator. (4) We use the importance metric, and again drop information processing.

These ratios are highly correlated with the numerical values of our benchmark leadership principal component. The correlation coefficients are, respectively, 0.648, 0.570, 0.610, and 0.518. Table A2 reports the means and standard deviations of these ratios for our soft and hard-skill occupations (classified using our benchmark leadership principal component). Row (1) ~(4) are, respectively, specifications (1) ~(4). The means are higher for the soft-skill occupations than for the hard-skill occupations for all of the 4 specifications.

We now further explore whether these ratios are good candidates for occupation classification, by using the wage-AFQT regression. For each specification, we pick the cutoff point to have the same percentile as that of the value of 3 in the distribution of our benchmark leadership principal component. We do so in order to have a common and comparable classification across specifications. We then estimate the wage-AFQT regression using these alternative classifications, and report the results in Table A1, where columns (3) ~(6) correspond to, respectively, specifications (1) ~(4). The key variable, the interaction between the soft-skill dummy and AFQT score, is statistically insignificant in all 4 columns, in contrast to column (1).

We have also experimented with measuring soft skills using the O*NET characteristics of investigative skills, originality, and social skills. Originality is about coming up with "unusual or clever ideas about a given topic or situation", or developing "creative ways to solve a problem". 1.A.1.b.2. Social skills involve "working with, communicating with, and teaching people". 1.B.1.d. Investigative skills involve "working with ideas" and "searching for facts and figuring out problems mentally", and require "an extensive amount of thinking"; 1.B.1.b. For each of these occupation characteristic, we take its importance metric, and use the value of 3 as our cut-off. As reported in Table A3, the AFQT coefficient of the soft skill sub-sample is larger than the hard-skill sub-sample, which is counter-intuitive.

In summary, we are *not* proposing leadership as *the* measure for soft skills; instead, we have shown that it is *a* useful measure that enables us to compare countries' productivities for soft-skill human capital in the absence of a direct measure along the soft-skill dimension. It is beyond the scope of our paper to explore all the occupation characteristics used in the literature, or those in O*NET.

7.2 Notes for Other Data and Parameter Values

7.2.1 ILO Employment-by-Occupation Data

We map the O*NET occupation codes into the ISCO-88 codes using the crosswalk at the National Crosswalk center ftp://ftp.xwalkcenter.org/DOWNLOAD/xwalks/. We drop the following observations from the ILO raw data because of data quality issues. 1. All data from Cyprus, because the data source is official estimate (source code "E"). 2. Year 2000 for Switzerland, because over 1 million individuals, a large fraction of the Swiss labor force, are "not classified". 3. Uganda, Gabon, Egypt, Mongolia, Thailand, Poland in 1994 and Romania in 1992, because the aggregate employment of the sub-occupation categories does not equal the number under "Total". 4. Estonia in 1998, S. Korea in 1995, and Romania in 2000, because the data is in 1-digit or 2-digit occupation codes.

Table A4 shows the list of soft- and hard-skill occupations and the summary statistics of their employment shares. The top (bottom) panel is for hard(soft)-skill occupations. The occupation codes follow ISCO-88, and their descriptions are available from standard sources, such as https://www.ilo.org/public/english/bureau/stat/isco/isco88/major.htm.

The summary statistics are for the distribution of employment shares across our sample of high-income countries minus the U.S., which we have left out because U.S. occupation codes are O*NET and not ISCO-88. Note that some occupation codes show up for a single country, in which cases the median is equal to the 25th and 75th percentiles and the standard deviation is not shown.

Most countries have a single year of data around 2000. In Figure A2 we plot the soft-skill employment share for all the countries that have multiple years of data. Within countries, the soft-skill employment share shows limited variation over time. As a result, for this set of countries we keep the single year of data closest to 2000; e.g. 1990 for Switzerland, 2000 for the U.S., etc. By construction, the soft and hard-skill employment shares sum to 1 by country.

7.2.2 Test Score Data

PISA samples students in a nationally representative way, covers many countries, and controls qualities of the final data (e.g. the 2000 UK scores and 2006 US reading scores are dropped because of quality issues). However, when PISA first started in 2000, only the reading test was administered, and only a small set of countries participated. In order to obtain PISA scores in all three subjects for every country in our sample, we calculate simple averages over time by country by subject, using all years of available data; e.g. Germany's PISA math score is the simple average of 03, 06, 09 and 2012, U.K.'s reading score the average of 06, 09 and 2012, etc. We have tabulated over-time changes of PISA scores within countries and found limited variation. For example, for the U.S. reading score the mean is 499.26 and the standard deviation is 3.93. The summary statistics by country is available upon request.

There have been several international tests on adults: IALS (International Adult Literacy Survey), administered in 1994-1998, ALLS (Adult Literacy and Life Skills Survey), conducted in 2002-2006, and PIAAC (Program for the International Assessment of Adult Competencies), conducted in 2013. The response rate of IALS, 63%, is substantially lower than the initial wave of PISA in 2000, 89% (Brown et al. 2007). ALLS was designed as a follow-up to IALS, but only 5 countries participated. Of the 26 countries in our high-income-country sample, only 16 participated in IALS, and only 19 in PIAAC. This would represent a 38% and 27% reduction in the number of observations, respectively.

We regress the 2012 PISA scores on 2013 PIAAC scores, for reading and math, for all the countries that participated in both tests, including those that are not in our sample. We obtain, respectively, the coefficient estimate of 0.938 and 1.067, and R-square of 0.508 and 0.527. These results are reported in Table A5.

7.2.3 Employment Data by Occupation by Education

EuroStat covers Switzerland for 1990, plus the following countries for 2000 or 2001: Austria, Bulgaria, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Netherlands, Poland, Portugal, Romania, Slovakia, Spain, and the U.K. The EuroStat data is by 1-digit ISCO88 occupation, and so we use the more disaggregated ILO data (discussed in sub-section 7.2.1 above) to compute soft and hard-skill employment shares by 1-digit occupation. We then construct, for each 1-digit occupation, the number of soft and hard-skill workers with primary, secondary and tertiary education. Finally, we aggregate across 1-digit occupations to obtainsoft and hard-skill employment shares by three educational groups, primary, secondary, and tertiary, by country.

To obtain data on U.S. immigrants' occupations and education, we use the 5% public sample of the 2000 U.S. Census. We look at the adult immigrants (age 18-65 in year 2000) who came to the U.S. at least 6 years after their expected graduation dates (i.e. year of entry $\geq$ 2000 $-$ age + 6 + schooling-years + 6). We drop those in school, in the military, or unemployed.

In preparation for extrapolation, we run the following two regressions across occupation-by-country. We include Bulgaria, Latvia, Luthiana and Romania, which are not in our high-income-country sample, to obtain a better fit. In the first regression, the dependent variable is the fraction of occupation-i-country-k workers with tertiary education. The explanatory variables are the fraction of i-k immigrants in the U.S. with tertiary education, dummy for the soft-skill occupation, and fraction of k's workers with tertiary education (obtained from World Development Indicators, or WDI). With 42 observations, our adjusted R^2 is 0.95. The second regression is similar to the first one, except that it is for primary education. Here, our adjusted R^2 is 0.97 (the full results of both regressions are available upon request). We do not use the regression for secondary education because of lower goodness of fit.

To check the usefulness of these two regressions for prediction, we drop Finland, Greece and Switzerland, whose output per worker is close to the median of the EuroStat sample, and obtain very similar coefficient estimates. We then generate predicted values for Finland, Greece and Switzerland, and plot them against the actual values. Figure A3 is for tertiary education, and Figure A4 for primary education. These figures show that the actual and predicted values are highly correlated.

Finally, we use our tertiary-education and primary-education regressions (with Finland, Greece and Switzerland) to generate the predicted values for Belgium, Hong Kong, Iceland, S. Korea, Luxembourg, Norway, Slovenia, and Sweden (we obtain data for the U.S. from 2000 U.S. Census). The predicted values for secondary education are one minus the values for primary education minus those for tertiary education. All predicted values are between 0 and 1.

7.2.4 Estimating the Value of θ

Let I^G denote gross income. It is proportional to net income, $I_i(\varepsilon_i)$, given in equation (7)

$$x_i(\varepsilon_i) = \frac{1}{1-\eta} I_i(\varepsilon_i) = \eta^{\frac{\eta}{1-\eta}} \left(\frac{w_i^k}{P^k} h_i^k \varepsilon_i^k \right)^{\frac{1}{1-\eta}}.$$

This expression says that $I_i^G(\varepsilon_i)$ depends on $(\varepsilon_i)^{\frac{1}{1-\eta}}$. So we first derive the joint distribution of $z_i = (\varepsilon_i)^{\frac{1}{1-\eta}}$, and then the distribution of I_i^G. We leave out country superscripts, to keep the exposition compact.

First,

$$
\begin{aligned}
\Pr((\varepsilon_c)^{\frac{1}{1-\eta}} &\leq z_c \text{ and } (\varepsilon_n)^{\frac{1}{1-\eta}} \leq z_n) \\
&= \Pr((\varepsilon_c) \leq (z_c)^{1-\eta} \text{ and } (\varepsilon_n) \leq (z_n)^{1-\eta}) = F((z_c)^{1-\eta}, (z_c)^{1-\eta}) \\
&= \exp\left(-\left(T_c z_c^{-\theta(1-\eta)} + T_n z_n^{-\theta(1-\eta)}\right)^{1-\upsilon}\right)
\end{aligned} \tag{71}
$$

where we have used the Frechet distribution of (1). Next, the derivation of the distribution of I^G follows similar steps as in our Theory Appendix 6.2. Let LIP denote probabilities, such as 25%. Then we have

$$
\begin{aligned}
LIP(I^G) &= \Pr(GrossIncome \leq I^G) = \Pr(\max_i\{\eta^{\frac{\eta}{1-\eta}}\left(\frac{w_i^k}{P^k}h_i^k\varepsilon_i^k\right)^{\frac{1}{1-\eta}}\} \leq I^G) \\
&= \Pr(\eta^{\frac{\eta}{1-\eta}}\left(\frac{w_c^k}{P^k}h_c^k\varepsilon_c^k\right)^{\frac{1}{1-\eta}} \leq I^G \text{ and } \eta^{\frac{\eta}{1-\eta}}\left(\frac{w_n^k}{P^k}h_n^k\varepsilon_n^k\right)^{\frac{1}{1-\eta}} \leq I^G) \\
&= \Pr(z_c = \left(\varepsilon_c^k\right)^{\frac{1}{1-\eta}} \leq I^G\eta^{-\frac{\eta}{1-\eta}}\left(\frac{w_c^k}{P^k}h_c^k\right)^{-\frac{1}{1-\eta}} \text{ and } z_n \leq I^G\eta^{-\frac{\eta}{1-\eta}}\left(\frac{w_n^k}{P^k}h_n^k\right)^{-\frac{1}{1-\eta}}) \\
&= \exp[-B_2(I^G)^{-\theta(1-\eta)(1-\upsilon)}], B_2 = \left(T_c\left(\eta^\eta\frac{w_c}{P}h_c\right)^\theta + T_n\left(\eta^\eta\frac{w_n}{P}h_n\right)^\theta\right)^{1-\upsilon}
\end{aligned}
$$

where we have used the distribution (71). This expression implies that

$$
\frac{\ln(LIP(I^G))}{\ln(LIP(I_0^G))} = \frac{(I^G)^{-\theta(1-\eta)(1-\upsilon)}}{(I_0^G)^{-\theta(1-\eta)(1-\upsilon)}}
$$

where I_0^G is a benchmark, such as the median level of gross income. Taking logs, we get

$$
\ln[-\ln(LIP(I^G))] = b_1 - \theta(1-\eta)(1-\upsilon)\ln I^G \tag{72}
$$

Equation (72) is our model's prediction for the distribution of life-time labor earnings, and comes directly from the property of the Frechet distribution. On the left-hand side, the $LIP(.)$'s are probabilities, such as 1%, 2%, ..., 99%, and on the right-hand side, I^G represents the corresponding percentiles of these probabilities. Equation (72) says that the double logs of the probabilities are negatively related to the logs of their corresponding percentiles. Note that equation (72) holds in both closed- and open-economy settings of our model.

To implement equation (72), we use the panel data of the U.S. PSID, which covers 1968-2017. For each individual, we deflate her income using U.S. CPI, and assume the discount rate of 0.96, as in the macro literature. We compute the total life-time labor earnings in the sample period, and then convert them into year 2000 US dollars. Because PSID over-samples low-income individuals and top codes labor earnings in some years, we drop the individuals whose life-time labor earnings are below the median earning in a single year (about $27,200 in year-2000 dollars) or above $1 million.

We report our results in Table A6. Column (1) is our main specification. Here, we regress the raw life-time labor earnings on the demographic controls of gender, race and year-1968-age cohort, and use the residuals (plus the constant) to compute the earnings percentiles. Although we have a single explanatory variable in the regression, the R^2 is 0.722, indicating that our model's prediction of (72) accounts for over 70% of the observed distribution of life-time labor earnings in the data. The coefficient estimate implies that $\theta = 2.2$, given our value of $\eta = 0.27$ and the additional assumption that $\upsilon = 0$. We can also reject the null hypothesis that $\theta = 1$ (which implies a coefficient estimate of -0.73, given that $\eta = 0.27$ and $\upsilon = 0$).

In column (2), we increase the number of age cohorts from 10 to 20. In column (3), we use the raw life-time labor earnings, without controlling for demographics. The results are similar to column (1). Note, in particular, that we obtain a higher R^2 of 0.815 in column (3).

7.2.5 Estimating the Value of α

We first derive the equation that we use to estimate the value of α, by using our open-economy setting as laid out in sub-section 2.6. We can then obtain the results for the closed-economy setting as a special case. Our derivation draws on our work in Theory Appendices 6.5 and 6.7.2. By equation (13), $w_c^k L_c^{kS} = b p_c^k P^k Y^k$, and so

$$\begin{aligned} \frac{Y^k}{L_c^{kSk}(1-x_c^k)} &= \frac{1}{bp_c^k(1-x_c^k)} \frac{w_c^k}{P^k} = \frac{\Theta^k}{bp_c^k(1-x_c^k)} \frac{w_c^k}{\left((A_c)^\alpha \left(w_c^k\right)^{1-\alpha} + (A_n)^\alpha \left(w_n^k\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha}}} \\ &= \frac{\Theta^k}{(A_c)^{\frac{\alpha}{1-\alpha}} bp_c^k(1-x_c^k)} \left(\frac{(A_c)^\alpha \left(w_c^k\right)^{1-\alpha}}{(A_c)^\alpha \left(w_c^k\right)^{1-\alpha} + (A_n)^\alpha \left(w_n^k\right)^{1-\alpha}}\right)^{\frac{1}{1-\alpha}} \\ &= \frac{\Theta^k}{(A_c)^{\frac{\alpha}{1-\alpha}} bp_c^k(1-x_c^k)} \left(s_c^k\right)^{\frac{1}{1-\alpha}} \end{aligned}$$

Since

$$s_c^k = p_c^k(1-x_c^k)$$

we have

$$\frac{Y^k}{L_c^{kS}(1-x_c^k)} = \frac{\Theta^k}{(A_c)^{\frac{\alpha}{1-\alpha}} b} \left(p_c^k(1-x_c^k)\right)^{\frac{\alpha}{1-\alpha}}$$

and finally

$$1 + \frac{p_n^k(1-x_n^k)}{p_c^k(1-x_c^k)} = \frac{p_c^k(1-x_c^k) + p_n^k(1-x_n^k)}{p_c^k(1-x_c^k)} = \frac{1 - p_c^k x_c^k - p_n^k x_n^k}{p_c^k(1-x_c^k)} = \frac{1}{p_c^k(1-x_c^k)} = \frac{1}{s_c^k}$$

where the last expression follows from trade balance. hence

$$\frac{Y^k}{L_c^{kS}(1-x_c^k)} = \frac{\Theta^k}{(A_c)^{\frac{\alpha}{1-\alpha}} b} \left(p_c^k(1-x_c^k)\right)^{\frac{\alpha}{1-\alpha}} = \frac{\Theta^k}{(A_c)^{\frac{\alpha}{1-\alpha}} b} \left(1 + \frac{p_n^k(1-x_n^k)}{p_c^k(1-x_c^k)}\right)^{\frac{\alpha}{\alpha-1}}$$

This implies that

$$\log\left(\frac{Y^k}{L_c^{kS}}\frac{1}{1-x_c^k}\right)-\log\left(\frac{Y^0}{L_c^{0S}}\frac{1}{1-x_c^0}\right)=\frac{\alpha}{\alpha-1}[\log\left(1+\frac{p_n^k\left(1-x_n^k\right)}{p_c^k\left(1-x_c^k\right)}\right)-\log\left(1+\frac{p_n^0\left(1-x_n^0\right)}{p_c^0\left(1-x_c^0\right)}\right)]+\log\frac{\Theta^k}{\Theta^0}$$

or that

$$\log\left(\frac{Y^k}{L_c^{kS}}\frac{1}{1-x_c^k}\right)=F+\frac{\alpha}{\alpha-1}\log\left(1+\frac{p_n^k\left(1-x_n^k\right)}{p_c^k\left(1-x_c^k\right)}\right)+\log\Theta^k \tag{73}$$

where the constant F has absorbed all the variables pertaining to the benchmark country of 0.

Equation (73) is the prediction under the open-economy setting. The prediction under the closed-economy setting can be obtained by setting $x_c^k = x_n^k = 0$ in (73).

Intuitively, (73) is an input-output relationship. The output is Y^k, and there are two inputs. The first is hard-skill human capital, L_c^{kS}. We adjust it by $\left(1-x_c^k\right)$ to take into account net export of the services of hard-skill human capital. The second input is the relative quantity of soft-skill human capital, and it is a monotonic function of $\frac{p_n^k\left(1-x_n^k\right)}{p_c^k\left(1-x_c^k\right)}$. Therefore, equation (73) shows how aggregate output, normalized by the quantity of hard-skill human capital, varies with the relative quantity of soft-skill human capital.

The estimation of (73) using our macro data, then, is similar to the estimation of the aggregate production function. Table A7 shows the results of fitting our data using (73), implemented as a regression with aggregate output as weight. Note that both Y^k and L_c^{kS} are unit-free in our regression, since the variation we use comes from their values relative to those of the benchmark country of 0, as shown in the expression above (73).

Column (1) is our main specification. Here, we use the closed-economy setting (i.e. having $x_c^k = x_n^k = 0$ in (73)) and measure L_c^{kS} using metric 1. The coefficient estimate is positive and statistically significant. This allows us to reject the hypothesis that $\alpha < 1$, which implies a negative coefficient estimate. We can also reject the hypothesis that $\alpha = 1$, in which case the coefficient estimate approaches infinity. The magnitude of our coefficient estimate, 2.36, implies that $\alpha = 1.73$, which is not statistically different from Burnstein et al. (2019)'s estimate of $\alpha = 1.78$.

In column (2), we maintain the closed-economy setting and measure L_c^{kS} using metric 2. In columns (3) and (4), we use the open-economy setting, and bring in the additional data of x_c^k and x_n^k. Our results are very similar to column (1).

7.3 Notes for External Validation

First, we provide more details for the validation in Table 4. Column (1) of Table A8 reports the wage-AFQT regression, where we have re-standardized the AFQT score to PISA scale. The coefficient estimate of log AFQT is 0.912. In column (2) of Table A8, we re-estimate the regression in column (1) using raw AFQT score. The coefficient estimate of log AFQT is now 0.150. Note that the re-standardization of AFQT scores does not affect our occupation classification. e.g. with AFQT scores in PISA scale, the coefficient estimate for log AFQT is 0.706 for the soft-skill sub-sample and 0.927 for the hard-skill sub-sample.

Second, we discussed how we obtained data on U.S. immigrants' occupations in sub-section 7.2.3 above.

Third, the EuroStat data on average annual gross earnings are by 1-digit ISCO88 occupations, and cover the following countries for 2002: Austria, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden and the U.K. (we drop Belgium because of missing data for several occupations). We use the more dis-aggregated ILO data to compute soft and hard-skill employment shares by 1-digit occupation, as in sub-section 7.2.3. We then construct the aggregate employment and wage bill for soft and hard-skill occupations by country, and use them to calculate w_n^k/w_c^k.

EuroStat also provides a historical series on average annual earnings by 1-digit ISCO88 occupation. As compared with the previously-discussed data, the historical series distinguishes between full-time and part-time workers, but covers fewer countries: Austria, Denmark, Finland, Germany, Greece, Iceland, Portugal, Slovakia, Sweden and the U.K. When we calculate w_n^k/w_c^k using the historical series data for 2002 and full-time workers, we find that its correlation with soft-skill employment share has a similar magnitude, -0.474, but the p-value is larger (0.14) because of fewer countries in the data.

The data on relative supplies of skill and skill premia are from Table A4 of Caselli and Coleman (2006), and we use the definition of college for high skill.

Finally, for the validation in Table 5, we collect aggregate import and export for the 31 NAICS manufacturing industries in the 2000 U.S. census, and the 9 1-digit service industries in the UN service-trade database, for each country in our sample. We obtain the 6-digit HS (Harmonized System) import and export data for merchandise trade from COMTRADE, and convert the HS6 codes to 1997 NAICS codes using the mapping of Pierce and Schott (2009). We obtain the data for service trade from the United Nations Service Trade database. To convert the service-industry codes of NAICS 1997 into the 1-digit service-trade codes, we start from the mapping of Liu and Trefler (2011) and augment it with our own mapping.

7.4 Open-Economy HCAP Index

Our computation of factor content of trade follows similar steps as Costinot and Rodriguez-Clare (2014). We first use US 2000 Census to get data for wage bill by industry for soft and hard-skill type workers, where our industries are the same as in the previous sub-section. We then use the NBER Productivity Database to get data for output for manufacturing industries, and the United Nations UNIDO Database to get those for service industries. For each industry, we compute the value of hard-skilled (soft-skilled) type service embodied in trade as net export multiplied by the ratio of hard-skilled (soft-skilled) wage bill to output. We then sum across industries and divide the total by country k's aggregate output. These numbers do not correspond to the variable x_i^k in our model; rather, they correspond to $nx_i^k = w_i^k \left(L_i^{kS} - L_i^{kD}\right)/(P^k y^k)$, which is the value of net exports of type i human capital normalized by output. It is easy to show that $x_i^k = nx_i^k / p_i^k$, and this expression allows us to compute x_i^k using nx_i^k. In our computation, we have implicitly assumed that hard and soft-skill types have the same cost shares across countries, because we only have cost-share data for the U.S. This assumption is also used in Costinot and Rodriguez-Clare (2014). The absolute value of net exports of hard skills, $|x_c^k|$, has a mean of 0.016 in our sample, and that of net exports of soft skills, $|x_n^k|$, has a mean of 0.030.

We then compute the HCAP index with factor content of trade using equation (23), and report the dispersion of this variable in Table A9. The log of the open-economy version of the HCAP index has a variance of 0.071 with metric 1, the standard single-type measure of human capital, and its variance is 0.15 with metric 2, based on Schoellman (2012). The 90-10 ratio of the open-economy version of the HCAP index is 1.99 with metric 1 and 2.65 with metric 2. These results are very similar to our main results in Table 6.

Finally, an alternative approach to calculate factor content of trade is to use industry employment as raw data, rather than wage bill (e.g. Davis and Weinstein 2001). Yet another approach is to use total factor requirements. We have experimented with both approaches, and obtained similar results (which are available upon request).

7.5 Iso-PISA Score Curve

We regress PISA math score on h_c^k and h_n^k, treating all three variables as data. We weigh the observations by aggregate output, because the countries in our sample vary a lot in size (e.g. Switzerland, Germany, and the U.S.). We obtain

$$\begin{aligned} Math &= 421.16 + 82.61 h_c^k - 11.93 h_n^k, \\ & \quad (30.00) \quad (32.74) \qquad (9.13) \end{aligned}$$

where standard errors are in the brackets, $R^2 = 0.22$, and $N = 26$. We obtain the predicted PISA math score for the U.S. by having $h_c^k = h_n^k = 1$ in this expression. In this exercise, PISA math score is in levels, while h_c^k and h_n^k are relative to the U.S. Rescaling PISA math score to be relative to the U.S. also rescales the coefficient estimates by the same factor (these results are available upon request), and does not affect the iso-PISA score curve.

7.6 Notes for Section 4

7.6.1 Additional Data for Lower-Income Countries

We obtain additional data for lower-income countries from a variety of sources, mainly from IPUMS. First, we follow the same procedure as outlined in sub-section 7.2 above, to obtain the data for Bulgaria, Estonia, Latvia, Lithuania and Romania from ILO and EuroStat. These data are for 2000 or 2001. Second, the data for Thailand 2000 is available from ILO. Finally, IPUMS data with 3-digit ISCO-88 occupations are available for the following countries, mainly around 2000: Bolivia 2001, Botswana 2001, Cambodia 1998, Costa Rica 2000, Cuba 2002, Ecuador 2001, France 1999, Greece 2001, Honduras 2001, Jordan 2004, Malaysia 2000, Mongolia 2000, Panama 2000, Papua New Guinea 2000, Paraguay 2002, Philipines 2000, Romania 2002, Rwanda 2002, Senegal 2002, South Africa 2001, Switzerland 2000, Thailand 2000, Trinidad and Tobago 2000, Uganda 2002, Uruguay 1996, and Venezuela 2001. We have dropped Guinea because of missing Barro-Lee data.

Among the countries covered in IPUMS, some show up in the ILO and EuroStat data as well, albeit with slightly different years. For these countries, their data from different sources are consistent. For example, France, Greece, Portugal, Romania, Switzerland and Thailand are also covered in the ILO data. For these six countries, the correlation in ILO and IPUMS labor-force sizes is 0.9992. As another example, France, Greece, Portugal, Romania and Switzerland also have data from EuroStat. For these five countries, the correlation in the EuroStat and IPUMS employment shares of tertiary education is 0.9755. We use ILO or EuroStat data when there is overlapping coverage.

In the IPUMS data, most of the workers in subsistence farming have limited education. Pooling across all the countries, we find that 98.4% of those in subsistence farming have primary education or less.

7.6.2 Data on School Duration and Skill Premium

We use a later version of the Barro-Lee dataset (version 2.2) than Jones (2014) (CC 2019 use the same data as Jones 2014). This allows us to include the following countries that are not in their data: Bulgaria, Cambodia, Cuba, Czech Republic, Estonia, Germany, Hungary, Latvia, Lithuania, Mongolia, Poland, Slovakia, Slovania, and Vietnam. This also means that, for these countries, we do not have Jones (2014)'s data for primary- and secondary-school duration, $durprim^k$ and $durscd^k$. The mean values of $durprim^k$ and $durscd^k$ are 6.15

and 5.95 years for our sample countries that are also in Jones (2014)'s data, and so we set $durprim^k$ and $durscd^k$ to 6 years for our sample countries that are not in Jones (2014)'s data.

Meanwhile, we use CC (2019)'s codes to compute the earnings of skilled and unskilled workers, and then skill premium, SP^k. The correlation between SP^k and output per worker is -0.454, and -0.633 for our sample countries that are also in the data of Jones (2014) and CC (2019). These patterns are consistent with the literature.

7.6.3 Data on Private plus Public Education Spending

As compared with other data sources, such as WDI, the UNESCO data includes private and public spending on education. A number of countries in our sample have missing values for this variable: Bolivia, Botswana, Cambodia, Costa Rica, Ecuador, Estonia, Greece, Honduras, Hong Kong, Iceland, Ireland, Jordan, Malaysia, Mongolia, Norway, Panama, Papua New Guinea, Philippines, Romania, Rwanda, Senegal, South Africa, Switzerland, Trinidad and Tobago, Uganda, the U.K., Venezuela, and Vietnam. We extrapolate the missing data points using public educational spending. To be specific, we merge the UNESCO data with Penn World Tables, and regress the share of private plus public education in GDP on GDP and the share of public education spending in GDP, with GDP as weights. We use all the countries with available data in this regression, including the ones not in our sample, to obtain a better fit. The R square of our regression is 0.90 (additional details are available upon request). We then use this regression to obtain the fitted values of the share of private plus public education in GDP.

References

[1] Liu, Runjuan and Daniel Trefler, 2011. A Sorted Tale of Globalization: White Collar Jobs and the Rise of Service Offshoring, NBER working paper 17559.

[2] Pierce, Justin and Peter Schott, 2009, "A Concordance Between Ten-Digit U.S. Harmonized System Codes and SIC/NAICS Product Classes and Industries". NBER working paper 15548.

Figure A1 Histogram of Leadership Principal Component

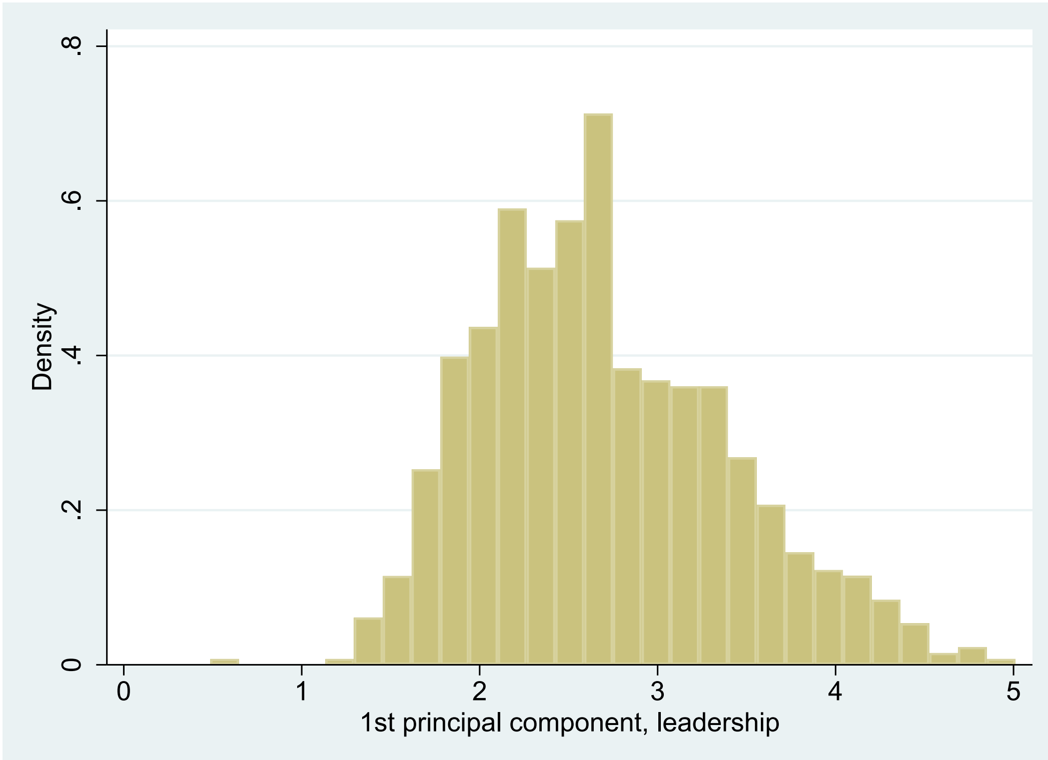

Figure A2 Soft-Skill Employment Share over Time for the Countries with Available Data

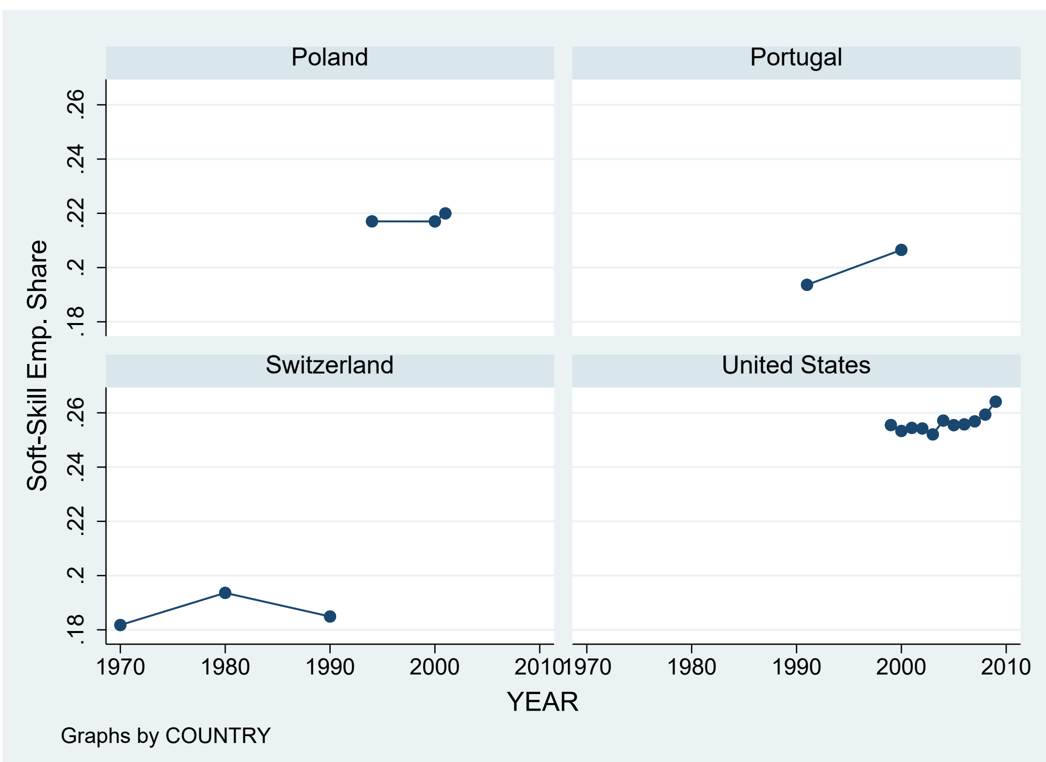

Figure A3 Predicted vs. Actual Shares of Tertiary Education, by Occupation-Country

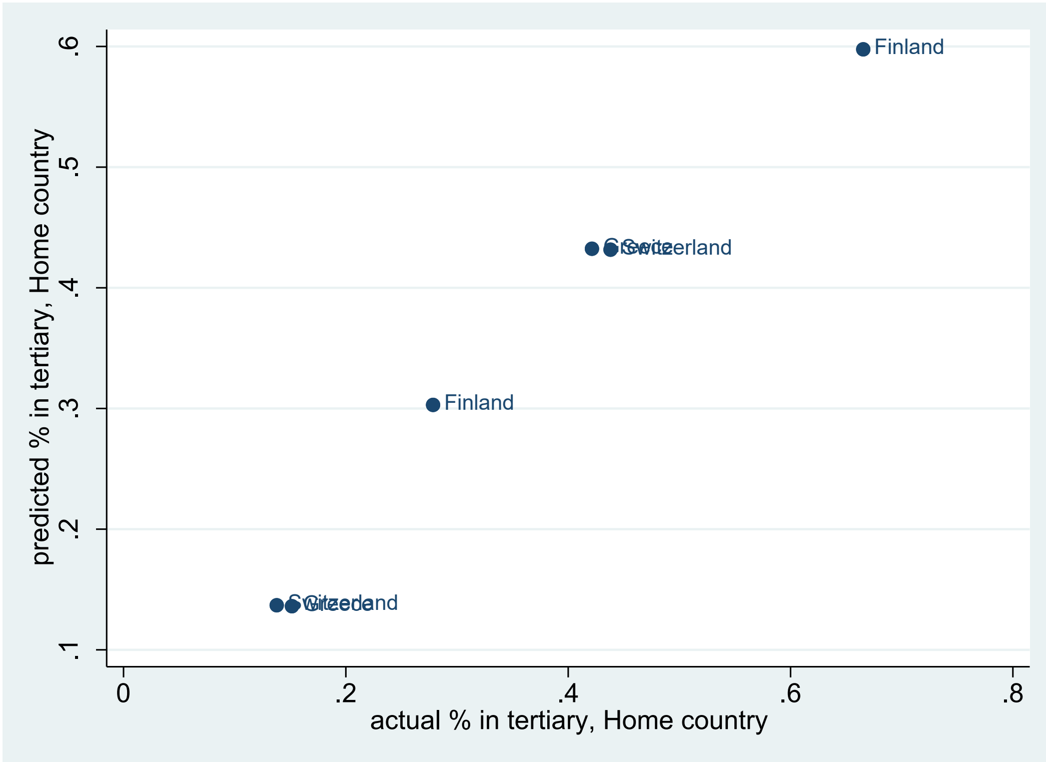

Figure A4 Predicted vs. Actual Shares of Primary Education, by Occupation-Country

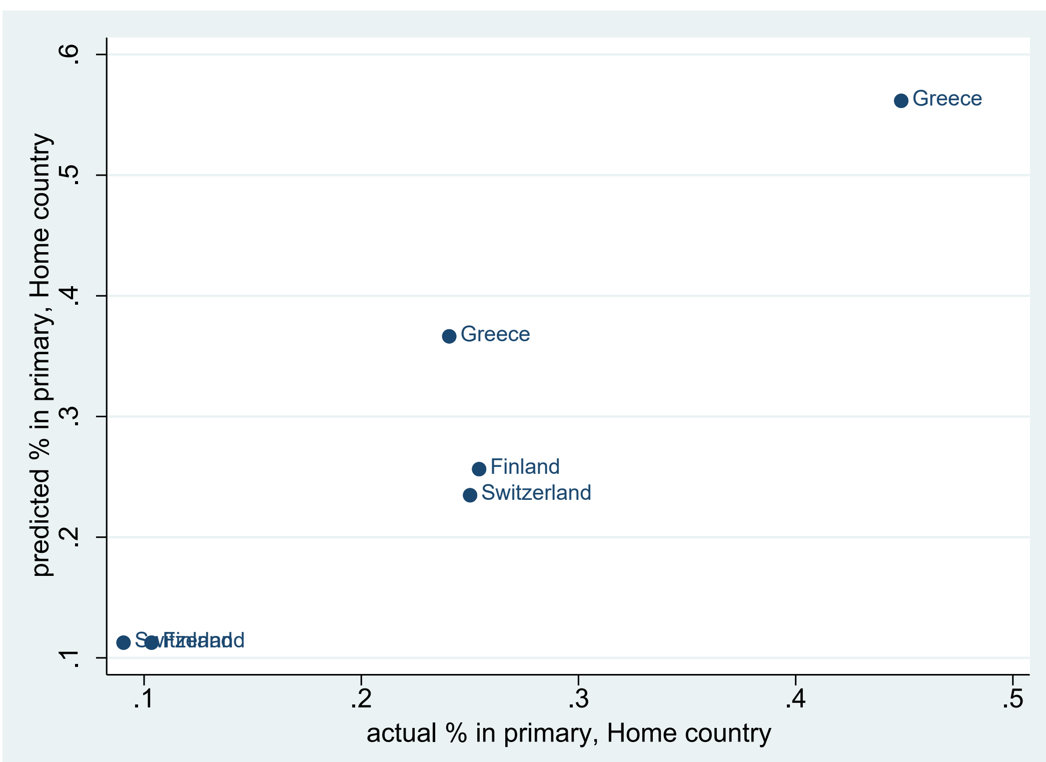

Table A1 Wages and AFQT Score for Ratios of Principal Components

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
Black	-0.0656	-0.0661	-0.067	-0.067	-0.068	-0.066
	(3.43)	(3.46)	(-3.48)	(-3.44)	(-3.50)	(-3.42)
Hispanics	0.0424	0.0413	0.044	0.044	0.044	0.044
	(2.07)	(2.00)	(2.12)	(2.13)	(2.11)	(2.12)
Age	0.0320	0.0323	0.033	0.033	0.033	0.033
	(4.66)	(4.69)	(4.69)	(4.77)	(4.70)	(4.79)
Soft-Skill Occp.	0.131	0.121	-0.021	-0.032	-0.013	-0.044
	(8.04)	(7.42)	(-1.43)	(-2.10)	(-0.83)	(-2.83)
College	0.179	0.187	0.215	0.216	0.214	0.216
	(6.75)	(7.08)	(8.12)	(8.16)	(8.07)	(8.19)
AFQT	0.140	0.137	0.135	0.138	0.139	0.135
	(12.17)	(11.91)	(10.90)	(11.25)	(11.69)	(11.11)
$AFQT^2$	-0.0356	-0.0369	-0.039	-0.040	-0.039	-0.039
	(3.76)	(3.88)	(-4.07)	(-4.12)	(-4.11)	(-4.10)
AFQT x Soft-Skill	-0.0541	-0.0345	-0.010	-0.017	-0.024	-0.012
	(3.38)	(2.17)	(-0.69)	(-1.08)	(-1.48)	(-0.77)
AFQT x College	0.0582	0.0525	0.043	0.041	0.043	0.041
	(2.38)	(2.14)	(1.75)	(1.66)	(1.74)	(1.67)
constant	6.218	6.218	6.253	6.248	6.249	6.249
	(57.05)	(57.05)	(56.80)	(56.83)	(56.81)	(56.87)
Obs. No.	3,210	3,210	3,210	3,210	3,210	3,210
R^2	0.217	0.214	0.2	0.201	0.2	0.202

Notes: t-statistics in parentheses. Column (1) recaps column (5) of Table 1. In column (2), we use the single O*NET characteristic of guiding and directing sub-ordinates to classify occupations. The specifications in columns (3)~(6) are discussed in Data Appendix 7.1.

Table A2 Summary Statistics: Ratios of Principal Components

	Soft-Skill Occupations		Hard-Skill Occupations	
	Mean	Std. Dev.	Mean	Std. Dev.
(1)	0.429	0.049	0.353	0.070
(2)	0.592	0.059	0.509	0.094
(3)	0.455	0.055	0.384	0.071
(4)	0.623	0.066	0.550	0.094

Notes: The specifications (1)~(4) are discussed in Data Appendix 7.1.

Table A3 Wages & AFQT Scores for Alternative Measures of Soft-Skill Occupations

VARIABLES	Originality	Not Originality	Social-skill	Not Social-skill	Investigative	Not Investigative
Black	-0.0735*	-0.0463**	0.0238	-0.0515**	0.010	-0.060***
	(0.0395)	(0.0216)	(0.0683)	(0.0202)	(0.091)	(0.02)
Hispanics	0.0380	0.0398*	0.119	0.0364*	0.036	0.039*
	(0.0402)	(0.0240)	(0.0788)	(0.0215)	(0.092)	(0.022)
Age	0.0569***	0.0220***	0.0557**	0.0325***	0.027	0.036***
	(0.0136)	(0.00798)	(0.0254)	(0.00722)	(0.030)	(0.007)
AFQT	0.182***	0.154***	0.204***	0.185***	0.188***	0.171***
	(0.0210)	(0.0109)	(0.0370)	(0.00979)	(0.060)	(0.010)
$AFQT^2$	0.00428	-0.0382***	-0.00483	-0.0172**	-0.043	-0.019**
	(0.0149)	(0.00996)	(0.0341)	(0.00807)	(0.032)	(0.008)
Constant	5.942***	6.414***	5.732***	6.292***	6.642***	6.212***
	(0.216)	(0.126)	(0.403)	(0.114)	(0.481)	(0.114)
Obs. No.	1,096	2,114	382	2,828	158	3052
R^2	0.164	0.126	0.127	0.181	0.106	0.148

Notes: The dependent variable is log wage, and the sample is NLSY 79. Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. This table is discussed in Data Appendix 7.1.

Table A4 Hard- and Soft-Skill Occupations and Summary Statistics of Employment Shares

ISCO-88 Code	Median	25th Pct.	75th Pct.	Std. Dev.
Hard-Skill Occupations				
2110	0.14%	0.08%	0.22%	0.09%
2112	0.18%	0.18%	0.18%	
2120	0.03%	0.01%	0.06%	0.03%
2130	0.59%	0.18%	1.30%	0.64%
2140	1.35%	0.75%	1.83%	0.78%
2350	0.23%	0.15%	0.36%	0.29%
2400	0.37%	0.20%	0.68%	0.34%
2420	0.42%	0.33%	0.51%	0.22%
2444	0.08%	0.08%	0.08%	
2450	0.56%	0.40%	0.68%	0.31%
2451	0.19%	0.19%	0.19%	
3100	0.15%	0.04%	0.26%	0.16%
3110	2.02%	0.88%	2.92%	1.31%
3118	0.14%	0.14%	0.14%	
3120	0.55%	0.21%	0.78%	0.39%
3130	0.22%	0.17%	0.26%	0.16%
3150	0.24%	0.13%	0.50%	0.24%
3200	0.11%	0.02%	0.20%	0.13%
3210	0.17%	0.06%	0.34%	0.29%
3220	0.70%	0.46%	0.85%	0.43%
3300	0.07%	0.02%	0.13%	0.38%
3310	0.43%	0.14%	0.83%	0.90%
3320	0.32%	0.16%	0.76%	0.64%
3330	0.12%	0.04%	0.41%	0.20%
3400	0.23%	0.05%	0.77%	0.39%
3410	2.71%	1.52%	3.60%	1.25%
3430	1.79%	1.23%	3.99%	1.85%
3432	0.14%	0.14%	0.14%	
3434	0.08%	0.08%	0.08%	
3440	0.37%	0.15%	0.79%	0.54%
3470	0.44%	0.21%	0.62%	0.25%
3471	0.09%	0.09%	0.09%	
4100	0.92%	0.29%	2.82%	1.76%
4110	2.27%	1.54%	2.83%	1.68%
4120	1.65%	1.06%	2.53%	1.55%
4130	1.57%	1.08%	1.77%	0.49%
4132	1.67%	1.67%	1.67%	
4140	0.61%	0.45%	0.91%	0.52%
4190	2.07%	0.80%	4.01%	2.36%
4200	0.39%	0.01%	1.07%	0.54%

4210	1.27%	0.95%	1.58%	0.58%
4220	0.66%	0.52%	0.87%	0.36%
5100	0.06%	0.01%	0.49%	0.41%
5110	0.18%	0.15%	0.29%	0.10%
5120	3.31%	2.45%	3.61%	0.91%
5130	2.02%	0.96%	3.06%	2.80%
5140	0.77%	0.68%	1.05%	0.26%
5141	0.88%	0.82%	0.95%	0.10%
5143	0.02%	0.02%	0.02%	
5200	1.78%	0.04%	2.09%	3.27%
5210	0.01%	0.00%	0.03%	1.54%
5220	5.17%	4.55%	6.13%	1.83%
6100	0.04%	0.01%	0.47%	0.42%
6110	0.98%	0.54%	2.41%	3.00%
6120	0.52%	0.29%	0.99%	1.10%
6130	0.97%	0.23%	2.95%	3.54%
6140	0.12%	0.07%	0.23%	0.13%
6150	0.05%	0.03%	0.19%	0.58%
7100	0.10%	0.01%	0.19%	0.13%
7110	0.13%	0.06%	0.26%	0.17%
7130	1.96%	1.67%	2.35%	0.63%
7140	0.66%	0.58%	0.81%	0.20%
7200	0.10%	0.04%	0.16%	0.08%
7210	0.97%	0.67%	1.33%	0.49%
7220	0.85%	0.44%	1.30%	1.03%
7230	2.05%	1.61%	2.52%	0.71%
7240	1.32%	0.85%	1.78%	0.48%
7300	0.04%	0.01%	0.08%	0.05%
7310	0.19%	0.11%	0.29%	0.16%
7320	0.13%	0.05%	0.19%	0.12%
7330	0.05%	0.02%	0.11%	0.12%
7340	0.37%	0.26%	0.42%	0.17%
7400	0.11%	0.07%	0.15%	0.06%
7410	0.89%	0.49%	1.19%	0.66%
7420	0.54%	0.24%	0.94%	0.42%
7430	0.59%	0.33%	1.52%	0.90%
7440	0.08%	0.04%	0.18%	0.26%
8000	10.71%	10.71%	10.71%	
8100	0.05%	0.04%	0.06%	0.02%
8110	0.03%	0.02%	0.08%	0.07%
8120	0.22%	0.13%	0.31%	0.20%
8130	0.07%	0.04%	0.12%	0.06%
8140	0.12%	0.08%	0.44%	0.24%
8150	0.18%	0.13%	0.24%	0.17%

8160	0.13%	0.07%	0.24%	0.21%
8170	0.08%	0.03%	0.11%	0.13%
8200	0.04%	0.03%	0.05%	0.01%
8210	0.35%	0.26%	0.65%	0.56%
8220	0.14%	0.09%	0.19%	0.10%
8230	0.31%	0.20%	0.43%	0.21%
8240	0.08%	0.02%	0.17%	0.18%
8250	0.19%	0.14%	0.31%	0.13%
8260	0.44%	0.27%	0.86%	0.73%
8270	0.36%	0.23%	0.60%	0.29%
8280	0.80%	0.40%	1.43%	0.65%
8290	0.23%	0.09%	0.49%	0.30%
8300	2.57%	0.01%	5.13%	3.62%
8310	0.23%	0.09%	0.37%	0.24%
8320	2.95%	2.65%	3.92%	0.79%
8330	0.94%	0.61%	1.03%	0.36%
8340	0.04%	0.02%	0.10%	0.06%
9100	0.08%	0.03%	0.10%	0.23%
9110	0.16%	0.08%	0.59%	0.27%
9130	3.19%	2.59%	4.03%	2.05%
9140	0.64%	0.22%	0.95%	0.60%
9150	0.65%	0.47%	1.01%	0.86%
9160	0.17%	0.07%	0.24%	0.13%
9200	0.01%	0.01%	0.01%	
9210	0.35%	0.17%	0.87%	0.49%
9300	0.01%	0.01%	0.01%	
9310	0.61%	0.31%	1.06%	0.65%
9320	0.95%	0.53%	1.70%	2.62%
Soft-Skill Occupations				
1100	0.51%	0.24%	0.61%	0.31%
1110	0.12%	0.06%	0.22%	0.30%
1120	0.01%	0.01%	0.04%	0.16%
1140	0.04%	0.02%	0.07%	0.05%
1200	0.52%	0.09%	0.82%	1.15%
1210	0.75%	0.16%	1.63%	0.90%
1220	1.31%	0.84%	1.91%	1.27%
1230	0.96%	0.59%	1.86%	1.27%
1300	0.11%	0.03%	0.47%	0.35%
1310	3.41%	2.01%	5.41%	2.10%
2200	0.01%	0.01%	0.08%	0.04%
2210	0.14%	0.08%	0.22%	0.12%
2220	0.95%	0.81%	1.27%	0.43%
2230	0.92%	0.32%	1.92%	1.00%
2300	0.09%	0.01%	0.13%	0.06%

2310	0.39%	0.27%	0.51%	0.19%
2320	1.64%	0.84%	2.09%	0.77%
2330	1.75%	1.37%	2.26%	0.74%
2340	0.14%	0.06%	0.20%	0.21%
2410	1.10%	0.55%	1.44%	0.84%
2430	0.14%	0.09%	0.21%	0.09%
2432	0.04%	0.04%	0.04%	
2440	0.54%	0.41%	0.94%	0.47%
2446	0.19%	0.19%	0.19%	
2460	0.11%	0.06%	0.15%	0.07%
3140	0.10%	0.08%	0.18%	0.24%
3230	1.61%	0.99%	2.16%	0.84%
3240	0.01%	0.01%	0.01%	
3340	0.28%	0.13%	0.53%	0.40%
3420	0.32%	0.16%	0.46%	0.28%
3450	0.12%	0.04%	0.19%	0.17%
3460	0.28%	0.16%	0.71%	0.34%
3480	0.05%	0.01%	0.09%	0.06%
5142	0.14%	0.14%	0.14%	
5149	0.41%	0.41%	0.41%	
5160	1.13%	0.69%	1.63%	0.57%
7120	2.62%	2.08%	3.13%	1.04%
9120	0.01%	0.01%	0.01%	0.86%
9330	0.74%	0.32%	1.06%	0.91%

Notes: This table is discussed in Data Appendix 7.2.

Table A5 Correlation between 2012 PISA and 2013 PIAAC scores

	PISA Reading	PISA Math
PIAAC Literacy	0.938	
	(5.18)	
PIAAC Numeracy		1.067
		(5.38)
Constant	249.047	215.948
	(5.13)	(4.13)
Obs. No.	28	28
R^2	0.508	0.527

Notes: t-statistics in parentheses. This table is discussed in Data Appendix 7.2.

Table A6 Double Log Probabilities and Log Life-Time-Income Percentiles

	(1)	(2)	(3)
VARIABLES	Baseline	More Cohorts	No Demographic Control
log(life time labor-income percentile)	-1.570***	-1.568***	-1.160***
	(0.0990)	(0.0992)	(0.0561)
Constant	19.69***	19.66***	13.85***
	(1.278)	(1.280)	(0.699)
Observations	99	99	99
R-squared	0.722	0.721	0.815

Notes: The dependent variable is the double log of the probabilities 1%, 2%, …, 99%, and the main explanatory variable the log of the corresponding percentiles of the distribution of life-time labor income, computed from PSID (see equation (72) in Data Appendix 7.2 for the specification). Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table A7 Normalized Output and Occupational Employment Shares

	Closed-Econ Model		Open-Econ Model	
	(1)	(2)	(3)	(4)
VARIABLES	Metric 1	Metric 2	Metric 1	Metric 2
$\ln(1+\frac{p_n^k}{p_c^k})$	2.363**	2.032**		
	(0.927)	(0.899)		
$\ln(1+\frac{p_n^k(1-x_n^k)}{p_c^k(1-x_c^k)})$			2.263**	1.932**
			(0.900)	(0.875)
Constant	8.111***	8.611***	8.135***	8.635***
	(0.265)	(0.257)	(0.257)	(0.250)
Observations	26	26	26	26
R-squared	0.213	0.175	0.209	0.169

Notes: The dependent variable is $\ln(\frac{Y^k}{L_c^{kS}})$ in columns (1) and (2), and $\ln(\frac{Y^k}{L_c^{kS}}\frac{1}{1-x_c^k})$ in columns (3) and (4) (see equation (73) in Data Appendix 7.2 for the specification). Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table A8 Scaled AFQT scores and Wages

	(1)	(2)
VARIABLES	PISA-Scaled AFQT	raw AFQT
Black	-0.0522***	-0.0804***
	(0.0195)	(0.0195)
Hispanics	0.0433**	0.0133
	(0.0211)	(0.0211)
Age	0.0202***	0.0240***
	(0.00710)	(0.00718)
log(AFQT score)	0.912***	0.150***
	(0.0437)	(0.00813)
Constant	0.798***	5.914***
	(0.283)	(0.113)
Obs. No.	3,210	3,210
R^2	0.169	0.147

Notes: The dependent variable is log wage, and the sample is NLSY 79. Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. This table is discussed in Data Appendix 7.3.

Table A9 Dispersions of Human Capital with Open-Economy HCAP

	Metric 1	Metric 2
Variance of log Human Cap		
HCAP Index Factor-Content Trade	0.0712	0.1503
90-10 Ratio of Human Cap		
HCAP Index Factor-Content Trade	1.9935	2.6545

Notes: The HCAP index with factor-content of trade is given in equation (23). This table is discussed in Data Appendix 7.4.