7 Theory Appendix

7.1 Equation (8)

To simplify notation, we drop the superscript \( k \). In addition, let \( \omega_c = \omega_c h_c \), \( \omega_n = \omega_n h_n \), \( F_c = \frac{\partial F}{\partial \omega_c} \), and \( F_{nc} = \frac{\partial^2 F}{\partial \omega_c \partial \omega_n} \). Using the definition of \( p_n \), we have

\[
p_n = \Pr(\omega_n \varepsilon_n \geq \omega_c \varepsilon_c) = \int_0^\infty \int_0^{\omega_n \varepsilon_n} F_{nc} d\varepsilon_n d\varepsilon_c
\]

\[
= \int_0^\infty \left| F_c(\varepsilon_c, \varepsilon_n \to \infty) - F_c(\varepsilon_c, \varepsilon_n = \frac{\omega_c}{\omega_n} \varepsilon_c) \right| d\varepsilon_c
\]

\[
= \int_0^\infty F_c(\varepsilon_c, \varepsilon_n \to \infty) d\varepsilon_c - \int_0^\infty F_c(\varepsilon_c, \varepsilon_n = \frac{\omega_c}{\omega_n} \varepsilon_c) d\varepsilon_c
\]

Using the Frechet distribution (1), we have

\[
F_c(\varepsilon_c, \varepsilon_n) = AFT_c e^{-1} A = (1 - \rho) \theta(T_n \varepsilon_n^{-\theta} + T_c \varepsilon_c^{-\theta})^{-\rho}
\]

(1) When \( \varepsilon_n \to \infty \), \( A = (1 - \rho) \theta(T_c \varepsilon_c^{-\theta})^{-\rho} \) and \( F = \exp[-(T_c \varepsilon_c^{-\theta})^{1-\rho}] \). Therefore,

\[
F_c(\varepsilon_c, \varepsilon_n) \to \infty = (1 - \rho) \theta(T_c \varepsilon_c^{-\theta})^{-\rho} \exp[-(T_c \varepsilon_c^{-\theta})^{1-\rho}] \exp[-(T_c \varepsilon_c^{-\theta})^{1-\rho}]
\]

\[= \theta(1 - \rho)(T_c)^{1-\rho} \varepsilon_c^{-\theta(1-\rho)-1} \exp[-(T_c)^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}]
\]

and

\[
\int_0^\infty F_c(\varepsilon_c, \varepsilon_n) \to \infty d\varepsilon_c = \int_0^\infty \theta(1 - \rho)(T_c)^{1-\rho} \varepsilon_c^{-\theta(1-\rho)-1} \exp[-(T_c)^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}] d\varepsilon_c
\]

\[= \int_0^\infty \frac{d \exp[-(T_c)^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}]}{d\varepsilon_c} = \left( \exp[-(T_c)^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}] \right)_0^\infty = 1
\]

(2) When \( \varepsilon_n = \frac{\omega_c}{\omega_n} \varepsilon_c \),

\[A = (1 - \rho) \theta[T_n \varepsilon_c^{-\theta} (\frac{\omega_c}{\omega_n})^{-\theta} + T_c \varepsilon_c^{-\theta})^{-\rho} = (1 - \rho) \theta(\varepsilon_c^{-\theta})^{-\rho} B^{-\rho}, B = T_n (\frac{\omega_c}{\omega_n})^{-\theta} + T_c
\]

and,

\[F(\varepsilon_c, \varepsilon_n = \frac{\omega_c}{\omega_n} \varepsilon_c) = \exp\left[-T_n \varepsilon_c^{-\theta} (\frac{\omega_c}{\omega_n})^{-\theta} + T_c \varepsilon_c^{-\theta}\right]^{1-\rho} = \exp[-B^{1-\rho}(\varepsilon_c^{-\theta})^{1-\rho}]\]

Therefore,

\[
F_c(\varepsilon_c, \varepsilon_n) = \frac{\omega_c}{\omega_n} \varepsilon_c) = (1 - \rho) \theta(\varepsilon_c^{-\theta})^{-\rho} B^{-\rho} \exp[-B^{1-\rho}(\varepsilon_c^{-\theta})^{1-\rho}]\exp[-(T_c \varepsilon_c^{-\theta})^{1-\rho}]
\]

\[= (1 - \rho) \theta T_c \varepsilon_c^{-\theta(1-\rho)-1} B^{-\rho} \exp[-B^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}] \]

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and

\[
\int_0^\infty F_c(\varepsilon_c, \varepsilon_n) \, d\varepsilon_c = \int_0^\infty \frac{\omega_c \varepsilon_c}{\omega_n} \, d\varepsilon_c = \int_0^\infty (1 - \rho) \theta T_c \varepsilon_c^{-\theta(1-\rho)-1} B^{-\rho} \exp[-B^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}] \, d\varepsilon_c
\]

\[
= \frac{T_c B^{-1}}{T_c} \int_0^\infty \frac{d \exp[-B^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}]}{d\varepsilon_c}
\]

\[
= \left( T_c B^{-1} \exp[-B^{1-\rho} \varepsilon_c^{-\theta(1-\rho)}] \right)_{0}^{\infty} = T_c B^{-1}
\]

(3) Using (1) and (2) above we have

\[
p_n = 1 - T_c B^{-1} = \frac{T_n(\omega_c)^{-\theta} (\omega_n)^{\theta}}{T_c + T_n(\omega_c)^{-\theta} (\omega_n)^{\theta}} = \frac{T_n(\omega_n)^{\theta}}{T_c(\omega_c)^{\theta} + T_n(\omega_n)^{\theta}}
\]

This is equation (8).

### 7.2 Equations (9) and (12)

To simplify notation, we drop the superscript \( k \). We note that the Frechet distribution is max stable; i.e. the max of Frechet variables is still Frechet. To be specific, consider the random variable \( \varepsilon^* = \max \{ w_c h_c \varepsilon_c, w_n h_n \varepsilon_n \} \). By our discussions in section 2, \( \varepsilon^* = w_n h_n \varepsilon_n \) if and only if the individual chooses occupation \( n \).

We now obtain the cdf of the distribution of \( \varepsilon^* \)

\[
\Pr(\varepsilon^* \leq y) = \Pr(w_c h_c \varepsilon_c \leq y \text{ and } w_n h_n \varepsilon_n \leq y)
\]

\[
= F\left( \frac{y}{w_c h_c}, \frac{y}{w_n h_n} \right)
\]

\[
= \exp[-B_1 y^{-\theta(1-\rho)}], B_1 = \left( T_c \left( \frac{w_c h_c}{P} \right)^{\theta} + T_n \left( \frac{w_n h_n}{P} \right)^{\theta} \right)^{1-\rho}
\]

where we have used the Frechet distribution (1) in the second equality.

Consider the mean of non-cognitive workers’ net income, \( I_n \), conditional on choosing the non-cognitive occupation, \( n \). By the expression of \( I_n \), (7), we know that it is proportional to the mean of \( (w_n h_n \varepsilon_n) \frac{1}{1-\eta} \), conditional on choosing occupation \( n \). This conditional mean is, by Bayesian rule, the mean of \( (w_n h_n \varepsilon_n) \frac{1}{1-\eta} \) for those choosing occupation \( n \), divided by the employment share \( p_n \). The mean of \( (w_n h_n \varepsilon_n) \frac{1}{1-\eta} \) for those choosing occupation \( n \), in turn, is the mean of \( (\varepsilon^*) \frac{1}{1-\eta} \) for all workers times the employment share \( p_n \). As a result, the conditional mean of \( I_n \) is proportional to the mean of \( (\varepsilon^*) \frac{1}{1-\eta} \), which equals

\[
\int_0^\infty y^{\frac{1}{1-\eta}} \frac{d \exp[-B_1 y^{-\theta(1-\rho)}]}{dy} = \int_0^\infty y^{\frac{1}{1-\eta}} \exp[-B_1 y^{-\theta(1-\rho)}] B_1 \theta (1 - \rho) y^{-\theta(1-\rho)-1} \, dy
\]
We then use change-of-variables to calculate the value of this expression, because the Gamma function is defined as

$$\Gamma(a + 1) = \int_0^\infty t^ae^{-t}dt,$$

where $a$ is a constant. Let $x = B_1y^{-\theta(1 - \rho)}$. Then $y = \left(\frac{x}{B_1}\right)^{-\frac{1}{\theta(1 - \rho)}}$, and $dy = -\frac{1}{\theta(1 - \rho)}B_1^{-\frac{1}{\theta(1 - \rho)}}x^{-\frac{1}{\theta(1 - \rho)}}\frac{1}{\theta(1 - \rho)}dx$. In addition, as $y \to 0$, $x \to \infty$; as $y \to \infty$, $x \to 0$. Therefore,

\[
\begin{align*}
\int_0^\infty y^{\frac{1}{1-\eta}} \frac{d\exp[-B_1y^{-\theta(1-\rho)}]}{dy} \\
= \int_0^\infty y^{\frac{1}{1-\eta}} \exp[-B_1y^{-\theta(1-\rho)}]B_1\theta(1-\rho)y^{-\theta(1-\rho)-1}dy \\
= \int_\infty^0 \left(\frac{x}{B_1}\right)^{-\frac{1}{\theta(1-\rho)}} \frac{1}{\theta(1-\rho)}B_1^{-\frac{1}{\theta(1-\rho)}}x^{-\frac{1}{\theta(1-\rho)}} \frac{1}{\theta(1-\rho)}e^{-x}dx \\
= B_1^{-\frac{1}{\theta(1-\rho)(1-\eta)}} \int_0^\infty x^{-\frac{1}{\theta(1-\rho)(1-\eta)}} e^{-x}dx = B_1^{-\frac{1}{\theta(1-\rho)(1-\eta)}} \Gamma(1 - \frac{1}{\theta(1-\rho)(1-\eta)}) \\
= \gamma \left[ T_c \left( \frac{w_c}{P} h_c \right)^\theta + T_n \left( \frac{w_n}{P} h_n \right)^\theta \right]^{\frac{1}{1-\eta}}, \gamma = \Gamma(1 - \frac{1}{\theta(1-\rho)(1-\eta)})
\end{align*}
\]

Therefore, the average net income of non-cognitive workers, $I_n$, equals $(1-\eta)\eta^{\frac{\eta}{1-\eta}} \gamma \left[ T_c \left( \frac{w_c}{P} h_c \right)^\theta + T_n \left( \frac{w_n}{P} h_n \right)^\theta \right]^{\frac{1}{1-\eta}}$. This is equation (9).

Meanwhile, the average real income of a worker in occupation $i$ is $I$, so the total real income of workers in occupation $i$ is $L_p I$. The real wage of a unit of effective labor of type $i$ is $w_i/P$ and the number of effective units is $L_i$ but we must net out expenditure on education. Hence, we must have

$$\frac{w_i}{P} L_i (1-\eta) = L_p I.$$

Substituting using (7), we obtain

\[
\begin{align*}
L_i &= \frac{L_p}{w_i} P \gamma \eta^{\frac{\eta}{1-\eta}} \left[ T_c \left( \frac{w_c}{P} h_c \right)^\theta + T_n \left( \frac{w_n}{P} h_n \right)^\theta \right]^{\frac{1}{1-\eta}} \\
&= \frac{L_p}{w_i} \gamma^{\frac{\eta}{1-\eta}} \left[ P^{1-\eta} \left( T_c \left( \frac{w_c}{P} h_c \right)^\theta + T_n \left( \frac{w_n}{P} h_n \right)^\theta \right)^\frac{\theta}{P} \right]^{\frac{1}{1-\eta}}.
\end{align*}
\]
7.3 Equations (19) and (20)

We normalize $P^k = 1$. By equation (4),

$$Y^k = \Theta^k \left( A_c \left( L^k_c \right)^{\frac{\alpha-1}{\alpha}} + A_n \left( L^k_n \right)^{\frac{\alpha-1}{\alpha}} \right)^{\frac{1}{\alpha-1}}$$

$$= \Theta^k L^k_c \left( A_c + A_n \left( \frac{L^k_n}{L^k_c} \right)^{\frac{\alpha-1}{\alpha}} \right)^{\frac{1}{\alpha-1}}$$

By equation (16),

$$\frac{L^k_c}{L^k_n} = \left( \frac{p^k_c A_n}{p^k_n A_c} \right)^{\frac{\alpha}{\alpha-1}}.$$

Substituting this expression into the output equation yields

$$Y^k = \Theta^k L^k_c \left( A_c \frac{p^k_c}{p^k_c} \right)^{\frac{\alpha}{\alpha-1}}$$

Substituting out $p^k_c$ using equation (13), we obtain

$$w^k_c = \Theta^k \left( p^k_c \right)^{-\frac{1}{1-\eta}} \left( A_c \right)^{\frac{\alpha}{\alpha-1}} \tag{34}$$

Rearranging equation (10),

$$E^k_c = \left( \eta w^k_c h^k_c \right)^{\frac{1}{1-\eta}} \left( \frac{T_c}{p^k_c} \right)^{\frac{1}{\eta(1-\eta)}} \gamma,$$

Given that $E^k = \eta Y^k / L^k$, we can substitute $w^k_c$ in equation (34) to obtain, after rearranging

$$\frac{Y^k}{L^k} = \left( \Theta^k h^k_c \left( \frac{p^k_c}{p^k_c} \right)^{-\frac{\phi}{\eta^{(\alpha-1)}}} \left( A_c \right)^{\frac{\alpha}{\alpha-1}} \left( T_c \right)^{\frac{1}{\eta}} \right)^{\frac{1}{1-\eta}} \frac{\gamma}{\eta},$$

where we have defined $\phi \equiv \alpha + \theta - 1$. Substituting out $p^k_c$ using its definition, we obtain

$$\frac{Y^k}{L^k} = \left( \Theta^k h^k_c \left( 1 + \frac{T_n (h^k_n)^{\theta}}{T_c (h^k_c)^{\theta}} \left( \frac{w^k_n}{w^k_c} \right)^{\theta} \left( A_n \right)^{\frac{\alpha}{\phi}} \left( A_c \right)^{\frac{\phi}{\eta^{(\alpha-1)}}} \left( T_c \right)^{\frac{1}{\eta}} \right)^{\frac{1}{\eta}} \frac{\gamma}{\eta} \right).$$

We then substitute equation (18) into this expression, to obtain an expression with no endogenous variables

$$\frac{Y^k}{L^k} = \left( \Theta^k h^k_c \left( 1 + \left( \frac{T_n (h^k_n)^{\theta}}{T_c (h^k_c)^{\theta}} \left( A_n \right)^{\frac{\alpha}{\phi}} \left( A_c \right)^{\frac{\phi}{\eta^{(\alpha-1)}}} \left( T_c \right)^{\frac{1}{\eta}} \right)^{\frac{1}{\eta}} \frac{\gamma}{\eta} \right).$$
Therefore,
\[
\frac{Y^k/L^k}{Y^0/L^0} = \left( \frac{\Theta^k}{\Theta^0} \left( h^0_c \left( 1 + \left( \frac{T_n(h^0_c)^\theta}{T_c(h^0_c)^\theta} \right) \frac{A_n}{A_c} \alpha \frac{\phi}{\tau(\alpha-1)} \right)^{1/(1-\eta)} \right) \right) \ .
\]

Combining equations (8) and (18) for the base country, we have
\[
\left( \frac{A_n}{A_c} \right)^{\frac{\theta-1}{\phi}} \left( \frac{T_n}{T_c} \right)^{\frac{\alpha-1}{\phi}} = \left( \frac{h^0_c}{(h^0_c)^\theta} \right)^{\frac{\theta-1}{\phi}} \left( \frac{p^0_c}{p^0_c} \right)^{\frac{\phi}{\tau(\alpha-1)}}.
\]

Substituting this expression into the expression for \( \frac{Y^k/L^k}{Y^0/L^0} \), and simplifying, we arrive at our decomposition:
\[
\frac{Y^k/L^k}{Y^0/L^0} = \left( \frac{\Theta^k}{\Theta^0} \left( p^0_c \left( \frac{h^0_c}{(h^0_c)^\theta} \right)^{\frac{\theta-1}{\phi}} + p^0_n \left( \frac{h^0_n}{(h^0_n)^\theta} \right)^{\frac{\theta-1}{\phi}} \right)^{\frac{\phi}{\tau(\alpha-1)}} \right) \ .
\]

### 7.4 Equation (25)

Using equations (27), (10) and (12), we can show that
\[
\frac{L^{kS}}{L^k} = \frac{p^k_c}{w^k_c} \left( (\eta(P^k)^{-1})^{\eta} \left( T_c \left( u^k_c h^k_c \right)^{\theta} + T_n \left( u^k_n h^k_n \right) \right)^{1/(1-\eta)} \right) \ \Rightarrow \ u^k_c = \frac{E^k}{L^{kS}/L^k \eta(P^k)^{-1}}
\]

We now use equation (8) to obtain that \( T_c \left( u^k_c h^k_c \right)^{\theta} + T_n \left( u^k_n h^k_n \right) = \frac{T_n(u^k_c h^k_c)^\theta}{p^0_c} \). This expression allows us to substitute out the term \( T_c \left( u^k_c h^k_c \right)^{\theta} + T_n \left( u^k_n h^k_n \right) \) in equation (12), giving us, together with equation (27), that
\[
\frac{L^{kS}}{L^k} = p^k_c \left( h^k_c (\eta(P^k)^{-1}) u^k_c \right)^{1/(1-\eta)} \gamma
\]
\[
= (p^k_c)^{1-\frac{1}{\tau(1-\eta)}} \gamma \eta^{\frac{n}{1-\eta}} (T_c)^{\frac{1}{\tau(1-\eta)}} (p^k_c)^{1-\frac{1}{\tau(1-\eta)}} (E^k)^{\frac{n}{1-\eta}} (h^k_c)^{1-\eta}
\]

We then substitute out \( u^k_c \) using \( b^k_p \frac{E^k}{\eta} \frac{L^{kS}/L^k \eta(P^k)^{-1}} \) to obtain
\[
\frac{L^{kS}}{L^k} = (p^k_c)^{1-\frac{1}{\tau(1-\eta)}} \gamma \eta^{\frac{n}{1-\eta}} (T_c)^{\frac{1}{\tau(1-\eta)}} (p^k_c)^{1-\frac{1}{\tau(1-\eta)}} (E^k)^{\frac{n}{1-\eta}} (h^k_c)^{1-\eta}
\]
\[
= \left( \frac{L^{kS}/L^k}{L^k} \right)^{\frac{n}{1-\eta}} \gamma \eta^{\frac{n}{1-\eta}} (T_c)^{\frac{1}{\tau(1-\eta)}} (p^k_c)^{1-\frac{1}{\tau(1-\eta)}} (E^k)^{\frac{n}{1-\eta}} (h^k_c)^{1-\eta}
\]
\[
\Leftrightarrow \frac{L^{kS}}{L^k} = \gamma^{1-\eta} \eta^{\frac{n}{1-\eta}} (T_c)^{\frac{1}{\tau(1-\eta)}} (p^k_c)^{1-\frac{1}{\tau(1-\eta)}} (E^k)^{\frac{n}{1-\eta}} (h^k_c)^{1-\eta}
\]
Taking the ratio of this expression with respect to country $0$, and substituting out $E^k$ using equation (11), we get equation (25).

### 7.5 Iso-PISA Score Curve: Theoretical Approach

First, we examine how average cognitive human capital, $L^{ks}_c / L^k$, changes in response to changes in cognitive productivity, $h^k_c$, and non-cognitive productivity, $h^k_n$, and perform comparative statics. We keep the labor-force size, $L^k$, and output TFP, $\Theta^k$, fixed. Equations (14) and (17) imply that

$$ \frac{L^{ks}_c}{L^k} = \left( \frac{A_c p^k_n A_n}{p^k_c} \right)^{\frac{\alpha}{\theta - 1}}. $$

We then substitute out the term $L^{kD}_n$ in (4) using this expression and $L^{ks}_c$, to obtain

$$ Y^k = \Theta^k \{ A_c (L^{ks}_c)^{\frac{\alpha}{\theta}} + A_n [L^{ks}_c \left( \frac{p^k_n A_c}{p^k_c A_n} \right)^{\frac{\alpha}{\theta - 1}}]^{\frac{\alpha}{\theta - 1}} \}^{\frac{1}{\alpha - 1}} = \Theta^k L^{ks}_c (A_c)^{\frac{\alpha}{\theta - 1}} (1 + \frac{p^k_n}{p^k_c})^{\frac{\alpha}{\theta - 1}}$$

This expression, the identity $p^k_n + p^k_c = 1$, and equations (24) and (25) imply that

$$ d \ln p^k_n = -(d \ln p^k_c) \frac{p^k_c}{p^k_n}$$

$$(d \ln p^k_c) - d \ln p^k_n = \frac{\theta (\alpha - 1)}{\theta + \alpha - 1} (d \ln h^k_c - d \ln h^k_n)$$

$$d \ln L^{ks}_c - \eta d \ln Y^k = (1 - \frac{1}{\theta}) d \ln p^k_c + d \ln h^k_c$$

$$d \ln Y^k - d \ln L^{ks}_c = -\frac{\alpha}{\alpha - 1} d \ln p^k_c$$

These four equations allow us to solve $d \ln L^{ks}_c$ in terms of $d \ln h^k_c$ and $d \ln h^k_n$.

Next, we map $d \ln L^{ks}_c$ to changes in test score by using equation (27) and metric 2 for cognitive human capital, which has $g(.) = (t^k)^{0.75}$. Since we focus on test score, we assume that the number of schooling years remains unchanged. We thus obtain $0.75 d \ln t^k = d \ln L^{ks}_c$. As a result, we have

$$ B_1 d \ln t^k = (1 + B_2 p^k_c) d \ln h^k_c - (B_2 p^k_n) d \ln h^k_n, B_1 = 0.75(1 - \eta), B_2 = \frac{(\theta - 1)(\alpha - 1) - \alpha \eta}{\theta + \alpha - 1}, \quad (35)$$

where $B_1 = 0.55$ and $B_2 = 0.11$ according to our parameter values. The iso-PISA score curve that equation (35) implies is similar to the one in Figure 5.

### 7.6 Cognitive and Non-cognitive Intensive Occupations

In this sub-subsection, we formalize the intuition that non-cognitive and cognitive human capital are packages of skills. Assume that the workers accumulate human capital for occu-
occupation $i$, $i = 1, 2$, according to
\begin{align}
  l_i^k &= h_c^i(e)^{\beta_i} h_n^i(e)^{1-\beta_i} \varepsilon_i, \\
  h_c^i(e) &= h_c^i e^n, h_n(e) = h_n^i e^n.
\end{align}

In this expression, $\beta_i$ captures the cognitive intensity of occupation $i$. Let $\beta_1 > \beta_2$; i.e. occupation 1 is cognitive-intensive. $h_c^k$ and $h_n^k$ are the non-cognitive and cognitive productivities. We adapt the other aspects of our setting in section 2 for occupations 1 and 2; e.g. workers now draw their birth talents for occupations 1 and 2, $\varepsilon_1$ and $\varepsilon_2$, from the Frechet distribution, (1). We show, below, that

**Proposition 3** Let $\tilde h_i^k = (h_c^k)_{\beta_i} (h_n^k)^{1-\beta_i}$, $i = 1, 2$. Our results in sections 2-6 hold for $\tilde h_i^k$. e.g. equation (19) continues to hold, and the HCAP index is

\[(\Omega^k)^{\frac{1}{1-\eta}} = \left( p_1^0 \left( \frac{\tilde h_1^k}{h_1^0} \right)^\frac{\theta}{(\alpha-1)} + p_2^0 \left( \frac{\tilde h_2^k}{h_2^0} \right)^\frac{\theta}{(\alpha-1)} \right)^{\frac{\phi}{(\alpha-1)}} \frac{1}{1-\eta}.
\]

The intuition of our results is as follows. Both occupations now use non-cognitive and cognitive human capital, and so both occupations are packages of skills. What matters for human-capital productivity of country $k$, then, is the productivities of these packages, $\tilde h_i^k$ and $\tilde h_2^k$. On the other hand, the structure and mechanism of our model remain unchanged, and so all the results there hold for $\tilde h_1^k$ and $\tilde h_2^k$.

We now show Proposition 3. The worker who chooses occupation $i$ solves the following optimization problem

\[ w_i \tilde h_i^k e^n \varepsilon_i - P^k e \]

which is similar to section 2. The solution is similar to equation (6):

\[ e(\varepsilon_i) = \left( \frac{w_i^k \tilde h_i^k \varepsilon_i}{P^k} \right)^{\frac{1}{1-n}}. \]

Net income for occupation $i$ is similar to equation (7)

\[ I_i(\varepsilon_i) = (1-\eta) \eta^{\frac{n}{1-n}} \left( \frac{w_i^k \tilde h_i^k \varepsilon_i}{P^k} \right)^{\frac{1}{1-n}}. \]

So we chose occupation 1 if $w_2^k \tilde h_2^k \varepsilon_2^k \leq w_1^k \tilde h_1^k \varepsilon_1^k$. From this, the share of workers who choose occupation $i$ is, similar to equation (8),

\[ p_i^k = \frac{T_i \left( \tilde h_i^k w_i^k \right) \theta}{\Delta^k}, \Delta^k = T_1 \left( \tilde h_1^k w_1^k \right) \theta + T_2 \left( \tilde h_2^k w_2^k \right) \theta, \]

(37)
The aggregate supply of human capital in occupation \(i\) is similar to equation (12)

\[
L_i^{kS} = \frac{L_k p_k^i}{w_i^k} \left( \eta^n (P^k)^{1-\eta} \left( T_c \left( \frac{w_k^i}{P_k h_1^k} \right) + T_n \left( \frac{w_n^i}{P_k h_2^k} \right) \right) \right)^{\frac{1}{1-\eta}}
\]

and average spending is similar to equation (10).

\[
E = \gamma \eta^{\frac{1}{1-\eta}} (\Delta^k)^{\frac{1}{\eta(1-\eta)}} \left( \frac{1}{P_k} \right)^{\frac{1}{1-\eta}}.
\]  

(38)

The relative supply of occupation 1 is similar to equation (14)

\[
\frac{L_1^{kS}}{L_2^{kS}} = \frac{T_1}{T_2} \left( \frac{h_1^k}{h_2^k} \right)^{\theta} \left( \frac{w_1^k}{w_2^k} \right)^{\theta-1}
\]

and relative demand is the same as equation (16). So the relative return is similar to equation (18),

\[
\frac{w_1^k}{w_2^k} = \left( \frac{T_2}{T_1} \left( \frac{h_2^k}{h_1^k} \right)^{\theta} \left( \frac{A_1}{A_2} \right)^{\alpha} \right)^{\frac{1}{\eta+\alpha-1}}
\]

and comparative advantage is similar to equation (24).

\[
\frac{h_1^k/h_2^k}{h_1^0/h_2^0} = \left( \frac{p_1^k/p_2^k}{p_1^0/p_2^0} \right)^{\frac{\phi}{\eta+\alpha-1}}.
\]

The derivation of the HCAP index follows similar steps as Theory Appendix 3.

### 7.7 Additional Equations and Derivations: Open Economy

#### 7.7.1 Additional Equilibrium Conditions and Equilibrium Definition

International equilibrium requires that countries’ exports of cognitive human capital must be equal to other countries’ imports of cognitive human capital. Defining \(M_c\) as the set of countries that import cognitive labor (i.e. \(x_c^k < 0\)) and \(X_c\) as the set of countries that export cognitive labor (i.e. \(x_c^k > 0\)). International factor market clearing requires that

\[
\sum_{k \in X_c} x_c^k I_c^{kS} + \sum_{k \in M_c} x_c^k I_c^{kS} = 0.
\]  

(39)

where \(I_c^{kS}\) must satisfy (12). Let \(w_c\) and \(w_n\) denote the prices of non-cognitive and cognitive human capital on the international factor market clearinghouse. Then factor prices in country \(k\) are given by

\[
w_c^k = \begin{cases} w_c, & w_n^k = w_n, k \in X_c, \\ w_c r^k, & w_n^k = w_n, k \in X_n. \end{cases}
\]  

(40)
We are now in a position to define the equilibrium of our model.

**Definition 1** An equilibrium to our model is a set of international factor prices \( w_c \) and \( w_n \) that imply local factor prices via (40) and that imply quantities of factors supplied locally, given by (14), and factors demanded, given by (16). These quantities clear domestic factor markets, given by (29), and the associated factor trades clear the international market for cognitive human capital, given by (39) in conjunction with (12).

### 7.7.2 Equations (30) and (31)

We start by substituting equation (12) into equation (13), to obtain

\[
\frac{Y^k}{L^k} = \frac{1}{P^k} \left( \frac{\eta}{P^k} \right)^{\frac{n}{\eta}} \left[ T_c \left( \frac{w^k_c}{h_c} \right)^\theta + T_n \left( \frac{w^k_n}{h_n} \right)^\theta \right]^{\frac{1}{\sigma(1-\eta)}}
\]

and so relative to the base country we have

\[
\frac{Y^k / L^k}{Y^0 / L^0} = \left( \frac{P^0}{P^k} \right)^{\frac{n}{\eta}} \left[ \frac{w^0_c}{w^k_c} \left( \frac{h_c}{h_0^c} \right)^\theta \frac{w^0_n}{w^k_n} \left( \frac{h_n}{h_0^n} \right)^\theta \right]^{\frac{1}{\sigma(1-\eta)}}
\]

rearranging

\[
\frac{Y^k / L^k}{Y^0 / L^0} = \left( \frac{w^k_c}{w^0_c} \right)^{\frac{n}{\eta}} \left[ \frac{w^0_n}{w^k_n} \left( \frac{h_n}{h_0^n} \right)^\theta + \frac{w^0_c}{w^k_c} \left( \frac{h_c}{h_0^c} \right)^\theta \right]^{\frac{1}{\sigma(1-\eta)}}
\]

(41)

Let \( n x^k_i = p^k_i x^k_i, i = n, c. \) We now use equations (14), (16) and (17) to show that

\[
\frac{w^k_c}{w^k_n} = \left( \frac{A_c}{A_n} \right)^{\frac{1}{\alpha-1}} \left( \frac{p^k_c - n x^k_n}{p^k_c - n x^k_c} \right)^{\frac{1}{\alpha-1}}
\]

(42)

From the price index we have

\[
P^k = \frac{1}{\Theta^k} \left( (A_c)^\alpha (w^k_c)^{1-\alpha} + (A_n)^\alpha (w^k_n)^{1-\alpha} \right)^{\frac{1}{1-\alpha}}
\]

(43)

\[
P^k = \frac{w^k_c}{\Theta^k} \left( (A_c)^\alpha + (A_n)^\alpha \left( \frac{w^k_c}{w^k_n} \right)^{\alpha-1} \right)^{\frac{1}{1-\alpha}}
\]

Combine this expression with equation (42), we have

\[
\frac{w^k_c}{P^k} = \Theta^k \left( (A_c)^\alpha + (A_n)^\alpha \left( \frac{A_c}{A_n} \right)^{\frac{1}{\alpha-1}} \left( \frac{p^k_c - n x^k_n}{p^k_c - n x^k_c} \right)^{\frac{1}{\alpha-1}} \right)^{\alpha-1} \left( \frac{p^k_c - n x^k_n}{p^k_c - n x^k_c} \right)^{\frac{1}{\alpha-1}}
\]

\[
= \Theta^k (A_c)^{\frac{\alpha}{\alpha-1}} \left( \frac{p^k_c - n x^k_n}{p^k_c - n x^k_c} \right)^{\frac{1}{\alpha-1}}
\]

\[
= \Theta^k (A_c)^{\frac{\alpha}{\alpha-1}} (p^k_c - n x^k_c)^{-\frac{1}{\alpha-1}}
\]

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where the last equality uses \( p_n^k - n x_n^k + p_c^k - n x_c^k = 1 \), which is implied by equation (8) and balance of trade. So relative real wage is

\[
\frac{w_c^k}{w_c^0} = \frac{\Theta^k}{\Theta^0} \left( \frac{p_c^k - n x_c^k}{p_c^0 - n x_c^0} \right)^{-\frac{1}{\alpha - 1}} (44)
\]

We substitute equations (42) and (44) into (41):

\[
\frac{Y^k/L^k}{Y^0/L^0} = \left( \frac{\Theta^k}{\Theta^0} \left( \frac{p_c^k - n x_c^k}{p_c^0 - n x_c^0} \right)^{-\frac{1}{\alpha - 1}} \right)^{\frac{1}{1-\eta}} + \left( \frac{p_c^0 \left( h_c^k / h_c^0 \right)^{\theta} + p_n^0 \left( \frac{p_c^0 - n x_c^0}{p_c^0 - n x_c^k} \right)^{\frac{1}{\alpha - 1}} \left( h_n^k / h_n^0 \right)^{\theta}}{p_c^0 \left( h_c^0 / h_c^0 \right)^{\theta} + p_n^0 \left( \frac{p_c^0 - n x_c^0}{p_c^0 - n x_c^k} \right)^{\frac{1}{\alpha - 1}} \left( h_n^0 / h_n^0 \right)^{\theta}} \right)^{\frac{1}{1-\eta}}
\]

This is equation (30). Meanwhile, under free trade, \( P^k = (\Theta^k)^{-1} \), and so equation (12) implies

\[
L_i^k = \frac{L_k p_i^k}{w_i} \left( (\eta \Theta^k)^{\eta} \left( T_c \left( w_ch_c^k \right)^{\theta} + T_n \left( w_n h_n^k \right)^{\theta} \right) \right)^{1/(1-\eta)}
\]

Combine this expression with equation (13), we can write real output per capita in country \( k \) relative to a base country 0 as

\[
\frac{Y^k/L^k}{Y^0/L^0} = \left( \frac{\Theta^k}{\Theta^0} \left( \frac{T_c \left( w_ch_c^0 \right)^{\theta} + T_n \left( w_n h_n^0 \right)^{\theta} \right)}{T_c \left( w_ch_c^0 \right)^{\theta} + T_n \left( w_n h_n^0 \right)^{\theta}} \right)^{\frac{1}{1-\eta}}
\]

Rearranging, we obtain

\[
\frac{Y^k/L^k}{Y^0/L^0} = \left( \frac{\Theta^k}{\Theta^0} \left( \frac{T_c \left( w_ch_c^0 \right)^{\theta} + T_n \left( w_n h_n^0 \right)^{\theta} \right)}{T_c \left( w_ch_c^0 \right)^{\theta} + T_n \left( w_n h_n^0 \right)^{\theta}} \right)^{\frac{1}{1-\eta}} \left( \frac{T_c \left( w_ch_c^k \right)^{\theta} + T_n \left( w_n h_n^k \right)^{\theta} \right)^{\frac{1}{\theta}} \right)^{\frac{1}{1-\eta}}
\]

(45)
now replacing the expressions with occupation shares from the base country, we obtain
\[
\frac{Y^k/L^k}{Y^0/L^0} = \left( \Theta^k \left( \frac{P_c^0 (h_n^k \overline{h}_n^0)}{h_0^c} + \frac{P_n^0 (h_n^k \overline{h}_n^0)}{h_0^n} \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{1-\gamma}}
\]

7.7.3 Computation of the Free-Trade Equilibrium: Outline

As we know from equations (20) and (31), a country’s HCAP index depends on the extent of openness to factor service trade. We show, below, that the international equilibrium condition (39) can be rewritten as
\[
\sum_k H \left( \frac{h_n^k \overline{h}_n^0}{h_0^c} \right)^{\theta} \left( \Theta^k \right)^{\frac{\alpha-1}{1-\gamma}} L^k \left( H \left( \frac{h_n^k \overline{h}_n^0}{h_0^c} \right)^{\theta} + \left( \frac{h_n^k \overline{h}_n^0}{h_0^n} \right)^{\theta} \right)^{\frac{1}{1-\gamma}} = \frac{\left( \overline{\omega} \right)^{1-\alpha}}{\left( \overline{\omega} \right)^{1-\alpha} + 1}, \quad (46)
\]

where
\[
H = \left( \frac{P_c^0}{P_n^0} \right)^{\frac{\theta+\alpha-1}{\alpha-1}} \overline{\omega} = \left( \frac{A_c}{A_n} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{w_c}{w_n} \right).
\]

In equation (46), the superscript "0" denotes the same base country as used in equation (31) (which is the U.S. in our computation). The only unknown variable in (46) is \( \overline{\omega} \); all the other variables are known, either data or parameters. This means we can solve equation (46) for \( \overline{\omega} \), recover relative demand from the expression (where the superscript "\( T \)" denotes the free-trade equilibrium)
\[
s_c^{kT} = \frac{\left( \overline{\omega} \right)^{1-\alpha}}{\left( \overline{\omega} \right)^{1-\alpha} + 1}
\]

and equation (16), and then recover relative supply using
\[
p_c^{kT} = \frac{H \left( \frac{h_n^k \overline{h}_n^0}{h_0^c} \right)^{\theta}}{H \left( \frac{h_n^k \overline{h}_n^0}{h_0^c} \right)^{\theta} + \left( \frac{h_n^k \overline{h}_n^0}{h_0^n} \right)^{\theta}}
\]

and equation (14). Finally, factor service trade can be recovered from the expression \( s_c^{kT} = p_c^{kT} (1 - x_c^k) \).

We now derive equation (46). With free trade, (39) simplifies to \( \sum_k L_c^{kS} = \sum_k L_c^{kD} \). Using equation (13), the first-order condition for cost minimization, and factor price equalization, this equilibrium condition becomes
\[
\sum_k p_c^k \frac{P_c^0 Y^k}{\sum_{k'} P_{k'}^k Y_{k'}} = s_c
\]
where a country’s output weight can be written

\[
P^k Y^k \sum_{k'} P^{k'} Y^{k'} = \frac{(\Theta^k)^{\frac{n}{1-n}} L^k \left( T_c \left( \frac{w_c}{w_n} h^k_c \right)^\theta + T_n \left( h^k_n \right)^\theta \right)^{\frac{1}{\beta}}}{\sum_{k'} (\Theta^{k'})^{\frac{n}{1-n}} L^{k'} \left( T_c \left( \frac{w_c}{w_n} h^{k'}_c \right)^\theta + T_n \left( h^{k'}_n \right)^\theta \right)^{\frac{1}{\beta}} - \frac{1}{\beta}}.
\]

Substituting for factor supplies, factor demands, and for income weights, the international equilibrium condition becomes

\[
\sum_k \frac{T_c}{T_n} \left( \frac{w_c}{w_n} \right)^\theta \left( \frac{A_n}{A_c} \right)^{\frac{1}{1-\alpha}} \left( \frac{w_c}{w_n} \right)^{1-\alpha} + (h^k_n)^\theta \sum_{k'} (\Theta^{k'})^{\frac{n}{1-n}} L^{k'} \left( \frac{T_c}{T_n} \left( \frac{w_c}{w_n} \right)^\theta + (h^{k'}_n)^\theta \right)^{\frac{1}{\beta}}
\]

Defining \( \tilde{\omega} = \left( \frac{A_n}{A_c} \right)^{\frac{1}{1-\alpha}} \left( \frac{w_c}{w_n} \right) \), we can substitute out \( \frac{w_c}{w_n} \) using \( \frac{w_c}{w_n} = \tilde{\omega} \left( \frac{A_n}{A_c} \right)^{\frac{1}{1-\alpha}} \), to obtain

\[
\sum_k \frac{T_c}{T_n} \left( \frac{A_n}{A_c} \right)^{\frac{1}{1-\alpha}} \tilde{\omega}^{1-\alpha} h^k_c \left( \frac{A_n}{A_c} \right)^{\frac{1}{1-\alpha}} \left( \frac{w_c}{w_n} \right)^{1-\alpha} + (h^k_n)^\theta \sum_{k'} (\Theta^{k'})^{\frac{n}{1-n}} L^{k'} \left( \frac{T_c}{T_n} \left( \frac{A_n}{A_c} \right)^{\frac{1}{1-\alpha}} \tilde{\omega} h^{k'}_c \right)^\theta + (h^{k'}_n)^\theta \right)^{\frac{1}{\beta}}
\]

Defining \( \Psi \equiv \frac{T_c}{T_n} \left( \frac{A_n}{A_c} \right)^{\frac{1}{\beta}} \), this expression becomes

\[
\sum_k \Psi \left( \tilde{\omega} \right)^\theta \left( \frac{A_n}{A_c} \right)^{\frac{1}{1-\alpha}} \left( \frac{w_c}{w_n} \right)^{1-\alpha} + (h^k_n)^\theta \sum_{k'} (\Theta^{k'})^{\frac{n}{1-n}} L^{k'} \left( \Psi \left( \tilde{\omega} \right)^\theta + (h^{k'}_n)^\theta \right)^{\frac{1}{\beta}} = \frac{(\tilde{\omega})^{1-\alpha}}{(\tilde{\omega})^{1-\alpha} + 1}
\]

Rearranging this expression so that all human capital productivities appear as ratios, we obtain

\[
\frac{(\tilde{\omega})^{1-\alpha}}{(\tilde{\omega})^{1-\alpha} + 1} = \sum_k \Psi \left( \frac{w_c}{w_n} \right)^\theta \left( \frac{h^k_c}{h^{k'}_c} \right)^\theta \left( \tilde{\omega} \right)^\theta \left( \frac{h^k_n}{h^{k'}_n} \right)^\theta \sum_{k'} (\Theta^{k'})^{\frac{n}{1-n}} L^{k'} \left( \Psi \left( \frac{w_c}{w_n} \right)^\theta \left( \frac{h^{k'}_c}{h^{k'}_c} \tilde{\omega} \right)^\theta + (h^{k'}_n)^\theta \right)^{\frac{1}{\beta}}
\]

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Now, let us assume that the data we observe can be well approximated by the closed economy equilibrium. We have, by equation (8),
\[
\frac{p^0_c}{p^0_n} = \frac{T_c}{T_n} \left( \frac{h^0_c w^0_c}{h^0_n w^0_n} \right) \theta.
\]
Substituting for the autarky equilibrium wages in the base country using equation (18), we obtain
\[
\Psi \left( \frac{h^0_c}{h^0_n} \right)^\theta = \left( \frac{p^0_c}{p^0_n} \right)^{\frac{\theta + \alpha - 1}{\alpha - 1}}.
\]
Substituting this expression back into the labor market clearing condition, we obtain equation (46).

### 7.7.4 Adding China and Japan

Since we lack occupation employment data for Japan, we assume that Japan has the same occupation shares and the same non-cognitive and cognitive productivities as S. Korea. We then use Japan’s output-per-worker data and equations (19) and (20) to calculate Japan’s output TFP. Likewise, we assume that China is the same as Hong Kong except for labor-force size and output TFP.

### 7.7.5 Gains From Trade

Assume that there is no change in \( L^k, \Theta^k, h^k_c \) or \( h^k_n \). Re-interpret the country 0 in equation (41) as the initial equilibrium of country \( k \), under free trade,
\[
1/\hat{Y}^k = \left[ \left( p^k_c \left( \frac{w^k_c}{P^k} \right)^\theta + p^k_n \left( \frac{w^k_n}{P^k} \right)^\theta \right)^{\frac{1}{\eta}} \right]^{\frac{\eta}{1-\eta}},
\]
where
\[
\hat{P}^k = \left( s^k_c \left( \frac{w^k_c}{P^k} \right)^\theta + s^k_n \left( \frac{w^k_n}{P^k} \right)^\theta \right)^{\frac{1}{1-\eta}}.
\]
Equation (47) says that the change in output depends on the weighted power mean of the changes in cognitive and non-cognitive workers’ real earnings. This weighted mean is amplified by the power \( 1/(1 - \eta) \) because changes in real earnings affect workers’ human capital investment, by equation (6).

Now let the subsequent equilibrium be autarky, indicated by primes’. Re-write equation (47) as
\[
1/\hat{Y}^k = \left( \frac{w^k_c}{P^k} \right)^{\frac{1}{\eta}} \left[ p^k_c + p^k_n \left( \frac{w^k_n}{w^k_c} \right)^\theta \right]^{\frac{\eta}{1-\eta}}.
\]
By equation (43), we have

$$\frac{\hat{w}_c^k}{\hat{p}_c^k} = \frac{(A_c)^\alpha + (A_n)^\alpha \left(\frac{w_n}{w_c}\right)^{1-\alpha}}{(A_c)^\alpha + (A_n)^\alpha \left(\frac{w_n}{w_c}\right)^{1-\alpha}} \frac{1}{1-\alpha}$$

$$= \left(\frac{A_c}{A_n}\right)^\alpha \left(\frac{w_n}{w_c}\right)^{1-\alpha} \frac{1}{1-\alpha}$$

$$= \left(p_c^k(1 - x_c^k) + p_n^k(1 - x_n^k) \left(\frac{\hat{w}_n^k}{\hat{w}_c^k}\right)^{1-\alpha}\right) \frac{1}{1-\alpha}$$

where the last equality uses the result \(s_i^k = p_i^k(1 - x_i^k), i = n, c\), which is implied by equations (14), (16), and (29). Substituting this expression into equation (48), we obtain

$$1/\hat{Y}_c^k = \left(p_c^k(1 - x_c^k) + p_n^k(1 - x_n^k) \left(\frac{\hat{w}_n^k}{\hat{w}_c^k}\right)^{1-\alpha} \left(p_c^k + p_n^k \left(\frac{\hat{w}_n^k}{\hat{w}_c^k}\right)^\theta\right)\right)^\frac{1}{1-\alpha}. \quad (49)$$

By equations (8) and (42), we have

$$\frac{w_n}{w_c} = \left(\frac{1 - x_c^k}{1 - x_n^k}\right)^\theta \left(\frac{A_n}{A_c}\right)^\alpha \left(\frac{h_c}{h_n}\right)^\theta \frac{1}{\theta + \alpha - 1}.$$

This means that

$$\frac{\hat{w}_n^k}{\hat{w}_c^k} = \left(\frac{1 - x_n^k}{1 - x_c^k}\right)^\frac{1}{\theta + \alpha - 1},$$

where we have used \(x_c^{k'} = x_n^{k'} = 0\) at the subsequent equilibrium of autarky. Substituting this back into (49), we obtain

$$1/\hat{Y}_c^k = \left(\frac{p_c^k(1 - x_c^k) + p_n^k(1 - x_n^k) \left(\frac{1 - x_n^k}{1 - x_c^k}\right)^{1-\alpha} \left(\frac{1}{\theta + \alpha - 1}\right)^\frac{1}{\alpha + \theta - 1}}{\left(p_c^k + p_n^k \left(\frac{1 - x_n^k}{1 - x_c^k}\right)^\theta\right)^\frac{1}{\alpha + \theta - 1}}\right)^\frac{1}{1-\alpha}.$$

To simplify the steps let \(p_c^k(1 - x_c^k)^\frac{\theta}{\theta + \alpha - 1} + p_n^k (1 - x_n^k)^\frac{\theta}{\theta + \alpha - 1} = C\). Then we have

$$1/\hat{Y}_c^k = \left((1 - x_c^k)^\frac{\theta}{\theta + \alpha - 1}\right)^\frac{1}{\alpha + \theta - 1} \left(p_c^k(1 - x_c^k)^{1+\frac{\theta}{\theta + \alpha - 1}} + p_n^k (1 - x_n^k)^{1+\frac{\theta}{\theta + \alpha - 1}}\right)^\frac{1}{\alpha + \theta - 1} \frac{C^\frac{1}{\alpha + \theta - 1} + \frac{1}{\alpha + \theta - 1}}{(1 - x_c^k)^\frac{\theta}{\theta + \alpha - 1}} \frac{1}{\alpha + \theta - 1}$$

$$= \left(p_c^k(1 - x_c^k)^\frac{\theta}{\theta + \alpha - 1} + p_n^k (1 - x_n^k)^\frac{\theta}{\theta + \alpha - 1}\right)^\frac{1}{\alpha + \theta - 1} \frac{1}{\alpha + \theta - 1},$$

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or

$$1/Y_k = (p^k_c(1 - x^k_c) \varpi^{\alpha-1} + p^k_n(1 - x^k_n) \varpi^{\alpha-1}) \frac{\alpha}{\alpha-1} \frac{1}{1-\eta}. \quad (50)$$

We now prove that $Y_k > 1$ if $x_i^k < 1$ for at least one $i$ in equation (50). This equation can be written as

$$(Y_k)^{-(1-\eta)(\alpha-1)} = (p^k_c(1 - x^c_c) \varpi^{\alpha-1} + p^k_n(1 - x^c_n) \varpi^{\alpha-1})^{\theta + \alpha - 1}$$

Substituting using equation (28), we have

$$(Y_k)^{-(1-\eta)(\alpha-1)} = \left( p^k_c\left( \frac{L^D_c}{L^B_c} \right) \varpi^{\alpha-1} + p^k_n\left( \frac{L^D_n}{L^B_n} \right) \varpi^{\alpha-1} \right)^{\theta + \alpha - 1}$$

From this expression it is clear that to gain from trade (i.e. $Y_k > 1$) it must be that $L^D_c < 1$ for at least one $i$. Trade balance requires that if $L^D_c > 1$, then $L^D_n < 1$, and vice versa.

We now prove by contradiction. Suppose that $Y_k < 1$. Then, we must have

$$p^k_c\left( \frac{L^D_c}{L^B_c} \right) \varpi^{\alpha-1} + p^k_n\left( \frac{L^D_n}{L^B_n} \right) \varpi^{\alpha-1} > 1 \quad (51)$$

Without loss of generality, let $X \equiv \frac{L^D_c}{L^B_c} > 1$ and $Y \equiv \frac{L^D_n}{L^B_n} < 1$. Then trade balance requires

$$p^k_cX + p^k_nY = 1.$$  

We can rewrite condition (51) as

$$p^k_c f(X) + p^k_n f(Y) > 1$$

$$p^k_c f(X) + (1 - p^k_c) f(Y) > 1$$

$$p^k_c (f(X) - f(Y)) > 1 - f(Y)$$

where $f$ is continuous, increasing, and concave, i.e. $\alpha > 1$ and

$$f(z) = z^{\varpi^{\alpha-1}}.$$  

The trade balance condition can be written

$$p^k_c (X - Y) = 1 - Y$$

Now dividing the rearranged condition by the trade balance, we have

$$\frac{f(X) - f(Y)}{X - Y} > \frac{1 - f(Y)}{1 - Y}$$

But $X > 1 > Y > 0$ and $f(.)$ is increasing and concave, hence (note that $f(1) = 1$)

$$\frac{f(X) - f(Y)}{X - Y} < \frac{1 - f(Y)}{1 - Y}.$$  

This contradicts the assertion, so it must be the case that $Y_k > 1$.  

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8 Data Appendix

8.1 Notes for Occupation Classification

8.1.1 O*NET Data for Leadership

We first list the O*NET ID's of the leadership characteristics we discuss in the text. Guiding and directing subordinates is 4.A.4.b.4; leadership in work style = 1.C.2.b; coordinating the work and activities of others = 4.A.4.b.1; developing and building teams = 4.A.4.b.2; coaching and developing others = 4.A.4.b.5; recruiting and promoting employees = 4.A.4.c.2; monitoring and controlling resources and spending = 4.A.4.c.3; and coordinate or lead others in work = 4.C.1.b.1.g.

For each occupation characteristic, O*NET provides two metrics, importance and level. The importance metric ranges from 1 to 5, with 1 = not important, 3 = important, and 5 = extremely important, etc. These interpretations are common to all occupation characteristics. The level metric ranges from 0 to 7, and its interpretation is specific to each characteristic. For example, for guiding and directing subordinates, 2 = "work occasionally as a backup supervisor", 4 = "supervise a small number of subordinates in a well-paid industry", and 7 = "manage a severely downsized unit"; for developing and building teams, 1 = "encourage two coworkers to stick with a tough assignment", 4 = "lead an assembly team in an automobile plant", and 6 = "lead a large team to design and build a new aircraft".

We use the importance metric, not the level metric, because the former has the same intuitive interpretation for all occupation characteristics (e.g. a value of 3 means this characteristic is important). We also recognize that the importance metric is ordinal, not cardinal, and so we use its ordinal properties to classify occupations. To be specific, we rescale the numerical value of the leadership principal component to also range from 1 to 5. The non-cognitive occupations have importance metric of 3 or above, and the cognitive occupations have importance metric below 3. Figure A1 shows the histogram of this metric.

8.1.2 Sample Cuts for NLSY-79 Data

Following Neal and Johnson (1996) we: (1) use the 1989 version of AFQT and drop the observations with missing AFQT scores; (2) drop the individuals whose wage is above $75 or below $1 in 1991; and (3) drop those who are older than 17 when they take the AFQT.

8.1.3 Alternative Occupation Classifications

We first experiment with ratios of occupation characteristics. We specify two types of cognitive characteristics, math and information processing. For information processing,
we use the following characteristics to construct the principal component: processing information (4.A.2.a.2) and evaluating information to determine compliance with standards (4.A.2.a.3). For math, we use the following characteristics to construct the principal component: mathematical reasoning (1.A.1.c.1), number facility (1.A.1.c.2), advanced math (2.D.3.g), mathematics (2.A.1.e and 2.C.4.a), basic math (2.D.3.f), mathematics - entry requirement (3.B.1.e). We then calculate the ratio of the leadership principal component, which we have been using, to the sum of the principal components of leadership, math and information processing.

For the occupation characteristics we examine, the level and importance metrics are highly correlated. So we examine the following 4 specifications. (1) We use the level metrics to compute principal components, and obtain the ratio of the leadership principal component to the sum of the principal components of leadership, math, and information processing. (2) We use the level metric, but drop information processing from the denominator. (3) We use the importance metric, and have all the 3 principal components in the denominator. (4) We use the importance metric, and again drop information processing.

These ratios are highly correlated with the numerical values of our benchmark leadership principal component. The correlation coefficients are, respectively, 0.648, 0.570, 0.610, and 0.518. Table A1 reports the means and standard deviations of these ratios for our non-cognitive and cognitive occupations (classified using our benchmark leadership principal component). Row (1) – (4) are, respectively, specifications (1) – (4). The means are higher for the non-cognitive occupations than for the cognitive occupations for all of the 4 specifications.

We now further explore whether these ratios are good candidates for occupation classification, by using the wage-AFQT regression. For each specification, we pick the cutoff point to have the same percentile as that of the value of 3 in the distribution of our benchmark leadership principal component. We do so in order to have a common and comparable classification across specifications. We then estimate the wage-AFQT regression using these alternative classifications, and report the results in Table A2. Table A2 has the same rows as column (5) of Table 1, and columns (1) – (4) correspond to, respectively, specifications (1) – (4). The key variable, the interaction between the non-cognitive dummy and AFQT score, is statistically insignificant in all 4 columns.

We have also experimented with measuring non-cognitive skills using the O*NET characteristics of investigative skills, originality, and social skills. Originality is about coming up with “unusual or clever ideas about a given topic or situation”, or developing “creative ways to solve a problem”. 1.A.1.b.2. Social skills involve “working with, communicating with, and teaching people”. 1.B.1.d. Investigative skills involve “working with ideas” and “searching for facts and figuring out problems mentally”, and require “an extensive amount of thinking”; 1.B.1.b. For each of these occupation characteristic, we take its importance metric,
and use the value of 3 as our cut-off. As reported in Table A3, the AFQT coefficient of the non-cognitive sub-sample is larger than the cognitive sub-sample, which is counter-intuitive.

In summary, we are not proposing leadership as the measure for non-cognitive skills; instead, we have shown that it is a useful measure that enables us to compare countries’ productivities for non-cognitive human capital in the absence of a direct measure along the non-cognitive dimension. It is beyond the scope of our paper to explore all the occupation characteristics used in the literature, or those in O*NET.

8.2 Notes for Other Data and Parameter Values

8.2.1 ILO Employment-by-Occupation Data

We map the O*NET occupation codes into the ISCO-88 codes using the crosswalk at the National Crosswalk center ftp://ftp.xwalkcenter.org/DOWNLOAD/xwalks/. We drop the following observations from the ILO raw data because of data quality issues. 1. All data from Cyprus, because the data source is official estimate (source code “E”). 2. Year 2000 for Switzerland, because over 1 million individuals, a large fraction of the Swiss labor force, are “not classified”. 3. Uganda, Gabon, Egypt, Mongolia, Thailand, Poland in 1994 and Romania in 1992, because the aggregate employment of the sub-occupation categories does not equal the number under “Total”. 4. Estonia in 1998, S. Korea in 1995, and Romania in 2000, because the data is in 1-digit or 2-digit occupation codes.

Most countries have a single year of data around 2000. In Figure A2 we plot the non-cognitive employment share for all the countries that have multiple years of data. Within countries, the non-cognitive employment share shows limited variation over time. As a result, for this set of countries we keep the single year of data closest to 2000; e.g. 1990 for Switzerland, 2000 for the U.S., etc. By construction, the non-cognitive and cognitive employment shares sum to 1 by country.

8.2.2 Test Score Data

PISA samples students in a nationally representative way, covers many countries, and controls qualities of the final data (e.g. the 2000 UK scores and 2006 US reading scores are dropped because of quality issues). However, when PISA first started in 2000, only the reading test was administered, and only a small set of countries participated. In order to obtain PISA scores in all three subjects for every country in our sample, we calculate simple averages over time by country by subject, using all years of available data; e.g. Germany’s PISA math score is the simple average of 03, 06, 09 and 2012, U.K.’s reading score the average of 06, 09 and 2012, etc. We have tabulated over-time changes of PISA scores within countries and found limited variation. For example, for the U.S. reading score the mean is
499.26 and the standard deviation is 3.93. The summary statistics by country is available upon request.

There have been several international tests on adults: IALS (International Adult Literacy Survey), administered in 1994-1998, ALLS (Adult Literacy and Life Skills Survey), conducted in 2002-2006, and PIAAC (Program for the International Assessment of Adult Competencies), conducted in 2013. The response rate of IALS, 63%, is substantially lower than the initial wave of PISA in 2000, 89% (Brown et al. 2007). ALLS was designed as a follow-up to IALS, but only 5 countries participated. Of the 26 countries in our narrow sample, only 16 participated in IALS, and only 19 in PIAAC. This would represent a 38% and 27% reduction in the number of observations, respectively.

We regress the 2012 PISA scores on 2013 PIAAC scores, for reading and math, for all the countries that participated in both tests, including those that are not in our sample. We obtain, respectively, the coefficient estimate of 0.938 and 1.067, and R-square of 0.508 and 0.527. These results are reported in Table A4.

8.2.3 Employment Data by Occupation by Education

EuroStat covers Switzerland for 1990, plus the following countries for 2000 or 2001: Austria, Bulgaria, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Netherlands, Poland, Portugal, Romania, Slovakia, Spain, and the U.K. IPUMS covers Thailand (2000). The EuroStat data is by 1-digit ISCO88 occupation, and so we use the more dis-aggregated ILO data (discussed in sub-section 8.2.1) to compute non-cognitive and cognitive employment shares by 1-digit occupation. We then construct, for each 1-digit occupation, the number of non-cognitive and cognitive workers with primary, secondary and tertiary education. Finally, we aggregate across 1-digit occupations to obtain non-cognitive and cognitive employment shares by three educational groups, primary, secondary, and tertiary, by country.

To obtain data on U.S. immigrants’ occupations and education, we use the 5% public sample of the 2000 U.S. Census. We look at the adult immigrants (age 18-65 in year 2000) who came to the U.S. at least 6 years after their expected graduation dates (i.e. year of entry \( \geq 2000 - \text{age} + 6 + \text{schooling-years} + 6 \)). We drop those in school, in the military, or unemployed.

In preparation for extrapolation, we run the following two regressions across occupation-by-country. We use the countries in EuroStat to obtain a better fit. In the first regression, the dependent variable is the fraction of occupation-\(i\)-country-\(k\) workers with tertiary education. The explanatory variables are the fraction of \(i-k\) immigrants in the U.S. with tertiary education, dummy for the non-cognitive occupation, and fraction of \(k\)'s workers with tertiary education (obtained from World Development Indicators, or WDI). With 42 observations,
our adjusted $R^2$ is 0.95. The second regression is similar to the first one, except that it is for primary education. Here, our adjusted $R^2$ is 0.97 (the full results of both regressions are available upon request). We do not use the regression for secondary education because of lower goodness of fit.

To check the usefulness of these two regressions for prediction, we drop Finland, Greece and Switzerland, whose output per worker is close to the median of the EuroStat sample, and obtain very similar coefficient estimates. We then generate predicted values for Finland, Greece and Switzerland, and plot them against the actual values. Figure A3 is for tertiary education, and Figure A4 for primary education. These figures show that the actual and predicted values are highly correlated.

Finally, we use our tertiary-education and primary-education regressions (with Finland, Greece and Switzerland) to generate the predicted values for Belgium, Estonia, Hong Kong, Iceland, S. Korea, Luxembourg, Norway, Slovenia, and Sweden (we obtain data for the U.S. from 2000 U.S. Census). The predicted values for secondary education are one minus the values for primary education minus those for tertiary education. All predicted values are between 0 and 1.

8.2.4 Mincer Regressions for Metrics 2 and 3 of Human Capital

We estimate the Mincer wage regressions using our NLSY sample, and report the results in Table A5. In column (1), we include years of schooling, and obtain a coefficient estimate of 0.10. In column (2), we include the log of AFQT, scaled to the PISA scale of mean 500 and standard deviation 100. Its coefficient estimate is 0.75. We do not include schooling years in column (2). In column (3), we include both schooling years and log AFQT, and obtain coefficient estimates of 0.068 and 0.57. The coefficient estimates in columns (1) and (2) are used for metric 2, and those in column (3) for metric 3.

8.2.5 Iso-PISA Score Curve

We regress PISA math score on $h^k_c$ and $h^k_n$ for our narrow-sample countries, treating all three variables as data. We weigh the observations by aggregate output, because the countries in our sample vary a lot in size (e.g. Switzerland, Germany, and the U.S.). We obtain

\[
\text{Math} = 421.16 + 82.61h^k_c - 11.93h^k_n; \\
(30.00) \quad (32.74) \quad (9.13)
\]

where standard errors are in the brackets, $R^2 = 0.22$, and $N = 26$. We obtain the predicted PISA math score for the U.S. by having $h^k_c = h^k_n = 1$ in this expression. In this exercise, PISA math score is in levels, while $h^k_c$ and $h^k_n$ are relative to the U.S. Rescaling PISA math
score to be relative to the U.S. also rescales the coefficient estimates by the same factor (these results are available upon request), and does not affect the iso-PISA score curve.

8.3 Notes for External Validation: I

First, we discussed how we obtained data on U.S. immigrants’ occupations in sub-section 8.2.3.

Second, Table A6 reports the full correlation table among our output TFP estimates, $\Theta^k$, and those reported in the literature. Ours = our estimates for $\Theta^k$; HJ98 = Hall and Jones (1998) TFP (A); KRC97 = Klenow and Rodriguez-Clare (1997); HRG95 = Harrigan (1995); PWT_90 = Penn World Tables 8.0, current PPP, year 1990; PWT_00 = PWT 8.0, current PPP, 2000. The correlation coefficients between our $\Theta^k$ and the literature’s estimates, reported in the first column of Table A6 and in boldface, are comparable to those among the literature’s estimates, reported in the rest of Table A6.

Finally, the EuroStat data on average annual gross earnings are by 1-digit ISCO88 occupations, and cover the following countries for 2002: Austria, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden and the U.K. (we drop Belgium because of missing data for several occupations). We use the more dis-aggregated ILO data to compute non-cognitive and cognitive employment shares by 1-digit occupation, as in sub-section 8.2.3. We then construct the aggregate employment and wage bill for non-cognitive and cognitive occupations by country, and use them to calculate $w^k_n/w^k_c$.

EuroStat also provides a historical series on average annual earnings by 1-digit ISCO88 occupation. As compared with the previously-discussed data, the historical series distinguishes between full-time and part-time workers, but covers fewer countries: Austria, Denmark, Finland, Germany, Greece, Iceland, Portugal, Slovakia, Sweden and the U.K. When we calculate $w^k_n/w^k_c$ using the historical series data for 2002 and full-time workers, we find that its correlation with non-cognitive employment share has a similar magnitude, -0.474, but the p-value is larger (0.14) because of fewer countries in the data.

The data on relative supplies of skill and skill premia are from Table A4 of Caselli and Coleman (2006), and we use the definition of college for high skill.

8.4 Notes for Open Economy

8.4.1 External Validation: II

For each country in our sample, we collect aggregate import and export for the 31 NAICS manufacturing industries in the 2000 U.S. census, and the 9 1-digit service industries in the UN service-trade database. We obtain the 6-digit HS (Harmonized System) import and
export data for merchandise trade from COMTRADE, and convert the HS6 codes to 1997
NAICS codes using the mapping of Pierce and Schott (2009). We obtain the data for service
trade from the United Nations Service Trade database. To convert the service-industry codes
of NAICS 1997 into the 1-digit service-trade codes, we start from the mapping of Liu and
Trefler (2011) and augment it with our own mapping.

8.4.2 Factor Content of Trade

Our computation of factor content of trade follows similar steps as Costinot and Rodriguez-
Clare (2014). We first use US 2000 Census to get data for wage bill by industry for cogni-
tive and non-cognitive type workers, where our industries are the same as in the previous
sub-section. We then use the NBER Productivity Database to get data for output for man-
ufacturing industries, and the United Nations UNIDO Database to get those for service
industries. For each industry, we compute the value of cognitive (non-cognitive) type service
embodied in trade as net export multiplied by the ratio of cognitive (non-cognitive) wage
bill to output. We then sum across industries and divide the total by country \( k \)’s aggregate
output. These numbers do not correspond to the variable \( x_i^k \) in our model; rather, they cor-
respond to \( n x_i^k = w_i^k \left( L_i^{kS} - L_i^{kD} \right) / (P_i y_i^k) \), which is the value of net exports of type \( i \) human
capital normalized by output. It is easy to show that \( x_i^k = n x_i^k / p_i^k \), and this expression allows
us to compute \( x_i^k \) using \( n x_i^k \). In our computation, we have implicitly assumed that cognitive
and non-cognitive types have the same cost shares across countries, because we only have
cost-share data for the U.S. This assumption is also used in Costinot and Rodriguez-Clare
(2014).

An alternative approach to calculate factor content of trade is to use industry employment
as raw data, rather than wage bill (e.g. Davis and Weinstein 2001). Yet another approach is
to use total factor requirements. We have experimented with both approaches, and obtained
similar results (which are available upon request).

References

Jobs and the Rise of Service Offshoring, NBER working paper 17559.

nized System Codes and SIC/NAICS Product Classes and Industries”. NBER working
paper 15548.
Figure A1 Histogram of Leadership Principal Component

Figure A2 Non-Cognitive Employment Share Over Time for the Countries with Available Data
Figure A3 Predicted vs. Actual Shares of Tertiary Education, by Occupation-Country

Figure A4 Predicted vs. Actual Shares of Primary Education, by Occupation-Country
Table A1 Summary Statistics: Ratios of Principal Components

<table>
<thead>
<tr>
<th></th>
<th>Non-Cog Occupations</th>
<th>Cog Occupations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>(1)</td>
<td>0.429</td>
<td>0.049</td>
</tr>
<tr>
<td>(2)</td>
<td>0.592</td>
<td>0.059</td>
</tr>
<tr>
<td>(3)</td>
<td>0.455</td>
<td>0.055</td>
</tr>
<tr>
<td>(4)</td>
<td>0.623</td>
<td>0.066</td>
</tr>
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</table>

Notes: The specifications (1)–(4) are discussed in Data Appendix 1.

Table A2 Wages and AFQT Score for Ratios of Principal Components

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>-0.067</td>
<td>-0.067</td>
<td>-0.068</td>
<td>-0.066</td>
</tr>
<tr>
<td></td>
<td>(-3.48)</td>
<td>(-3.44)</td>
<td>(-3.50)</td>
<td>(-3.42)</td>
</tr>
<tr>
<td>Hispanics</td>
<td>0.044</td>
<td>0.044</td>
<td>0.044</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>(2.12)</td>
<td>(2.13)</td>
<td>(2.11)</td>
<td>(2.12)</td>
</tr>
<tr>
<td>Age</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>(4.69)</td>
<td>(4.77)</td>
<td>(4.70)</td>
<td>(4.79)</td>
</tr>
<tr>
<td>Non-cog. Occp.</td>
<td>-0.021</td>
<td>-0.032</td>
<td>-0.013</td>
<td>-0.044</td>
</tr>
<tr>
<td></td>
<td>(-1.43)</td>
<td>(-2.10)</td>
<td>(-0.83)</td>
<td>(-2.83)</td>
</tr>
<tr>
<td>College</td>
<td>0.215</td>
<td>0.216</td>
<td>0.214</td>
<td>0.216</td>
</tr>
<tr>
<td></td>
<td>(8.12)</td>
<td>(8.16)</td>
<td>(8.07)</td>
<td>(8.19)</td>
</tr>
<tr>
<td>AFQT</td>
<td>0.135</td>
<td>0.138</td>
<td>0.139</td>
<td>0.135</td>
</tr>
<tr>
<td></td>
<td>(10.90)</td>
<td>(11.25)</td>
<td>(11.69)</td>
<td>(11.11)</td>
</tr>
<tr>
<td>AFQT$^2$</td>
<td>-0.039</td>
<td>-0.040</td>
<td>-0.039</td>
<td>-0.039</td>
</tr>
<tr>
<td></td>
<td>(-4.07)</td>
<td>(-4.12)</td>
<td>(-4.11)</td>
<td>(-4.10)</td>
</tr>
<tr>
<td>AFQT x Non-Cog.</td>
<td>-0.010</td>
<td>-0.017</td>
<td>-0.024</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>(-0.69)</td>
<td>(-1.08)</td>
<td>(-1.48)</td>
<td>(-0.77)</td>
</tr>
<tr>
<td>AFQT x College</td>
<td>0.043</td>
<td>0.041</td>
<td>0.043</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>(1.75)</td>
<td>(1.66)</td>
<td>(1.74)</td>
<td>(1.67)</td>
</tr>
<tr>
<td>constant</td>
<td>6.253</td>
<td>6.248</td>
<td>6.249</td>
<td>6.249</td>
</tr>
<tr>
<td></td>
<td>(56.80)</td>
<td>(56.83)</td>
<td>(56.81)</td>
<td>(56.87)</td>
</tr>
<tr>
<td>Obs. No.</td>
<td>3,210</td>
<td>3,210</td>
<td>3,210</td>
<td>3,210</td>
</tr>
<tr>
<td>R$^2$</td>
<td>0.2</td>
<td>0.201</td>
<td>0.2</td>
<td>0.202</td>
</tr>
</tbody>
</table>

Notes: t-statistics in parentheses. The specifications (1)–(4) are discussed in Data Appendix 1.
## Table A3 Wages & AFQT Scores for Alternative Measures of Non-Cognitive Occupations

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>Originality</th>
<th>Not Originality</th>
<th>Social-skill</th>
<th>Not Social-skill</th>
<th>Investigative</th>
<th>Not Investigative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>-0.0735*</td>
<td>-0.0463**</td>
<td>0.0238</td>
<td>-0.0515**</td>
<td>0.010</td>
<td>-0.060***</td>
</tr>
<tr>
<td></td>
<td>(0.0395)</td>
<td>(0.0216)</td>
<td>(0.0683)</td>
<td>(0.0202)</td>
<td>(0.091)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Hispanics</td>
<td>0.0380</td>
<td>0.0398*</td>
<td>0.119</td>
<td>0.0364*</td>
<td>0.036</td>
<td>0.039*</td>
</tr>
<tr>
<td></td>
<td>(0.0402)</td>
<td>(0.0240)</td>
<td>(0.0788)</td>
<td>(0.0215)</td>
<td>(0.092)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Age</td>
<td>0.0569***</td>
<td>0.0220***</td>
<td>0.0557**</td>
<td>0.0325***</td>
<td>0.027</td>
<td>0.036***</td>
</tr>
<tr>
<td></td>
<td>(0.0136)</td>
<td>(0.00798)</td>
<td>(0.0254)</td>
<td>(0.00722)</td>
<td>(0.030)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>AFQT</td>
<td>0.182***</td>
<td>0.154***</td>
<td>0.204***</td>
<td>0.185***</td>
<td>0.188***</td>
<td>0.171***</td>
</tr>
<tr>
<td></td>
<td>(0.0210)</td>
<td>(0.0109)</td>
<td>(0.0370)</td>
<td>(0.00979)</td>
<td>(0.060)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>AFQT²</td>
<td>0.00428</td>
<td>-0.0382***</td>
<td>-0.00483</td>
<td>-0.0172**</td>
<td>-0.043</td>
<td>-0.019**</td>
</tr>
<tr>
<td></td>
<td>(0.0149)</td>
<td>(0.00996)</td>
<td>(0.0341)</td>
<td>(0.00807)</td>
<td>(0.032)</td>
<td>(0.008)</td>
</tr>
<tr>
<td></td>
<td>(0.216)</td>
<td>(0.126)</td>
<td>(0.403)</td>
<td>(0.114)</td>
<td>(0.481)</td>
<td>(0.114)</td>
</tr>
<tr>
<td>Obs. No.</td>
<td>1,096</td>
<td>2,114</td>
<td>382</td>
<td>2,828</td>
<td>158</td>
<td>3052</td>
</tr>
<tr>
<td>R²</td>
<td>0.164</td>
<td>0.126</td>
<td>0.127</td>
<td>0.181</td>
<td>0.106</td>
<td>0.148</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is log wage, and the sample is NLSY 79. Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. This table is discussed in Data Appendix 1.
Table A4 Correlation between 2012 PISA and 2013 PIAAC scores

<table>
<thead>
<tr>
<th>PISA Reading</th>
<th>PISA Math</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIAAC Literacy</td>
<td>0.938 (5.18)</td>
</tr>
<tr>
<td>PIAAC Numeracy</td>
<td>1.067 (5.38)</td>
</tr>
<tr>
<td>Constant</td>
<td>249.047 (5.13)</td>
</tr>
<tr>
<td>Obs. No.</td>
<td>28</td>
</tr>
<tr>
<td>R²</td>
<td>0.508</td>
</tr>
</tbody>
</table>

Notes: t-statistics in parentheses. This table is discussed in Data Appendix 2.

Table A5 Mincer Regressions for Metrics 2 and 3 of Human Capital

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>-0.195 (-11.15)</td>
<td>-0.068 (-3.50)</td>
<td>-0.095 (-4.85)</td>
</tr>
<tr>
<td>Hispanics</td>
<td>-0.023 (-1.14)</td>
<td>0.046 (2.18)</td>
<td>0.037 (1.79)</td>
</tr>
<tr>
<td>Experience</td>
<td>0.038 (4.55)</td>
<td>-0.025 (-4.56)</td>
<td>0.025 (3.04)</td>
</tr>
<tr>
<td>Experience Square</td>
<td>0.000 (0.54)</td>
<td>0.000 (0.03)</td>
<td>0.000 (0.02)</td>
</tr>
<tr>
<td>Schooling Years</td>
<td>0.103 (13.39)</td>
<td>0.068</td>
<td></td>
</tr>
<tr>
<td>log(AFQT PISA Scale)</td>
<td>0.746 (15.13)</td>
<td>0.572 (10.74)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>5.597 (69.43)</td>
<td>2.058 (6.77)</td>
<td>2.438 (8.00)</td>
</tr>
<tr>
<td>Obs. No.</td>
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<td>3210</td>
<td>3210</td>
</tr>
<tr>
<td>R²</td>
<td>0.17</td>
<td>0.182</td>
<td>0.199</td>
</tr>
</tbody>
</table>

Notes: t-statistics in parentheses. This table is discussed in Data Appendix 2.
Table A6 Correlation Coefficients for Output TFP Estimates

<table>
<thead>
<tr>
<th></th>
<th>Ours</th>
<th>HJ98</th>
<th>KRC97</th>
<th>HRG95</th>
<th>PWT_90</th>
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Notes: p-values under correlation coefficients. This table is discussed in Data Appendix 3.