

The Efficiency Cost of Child Tax Benefits

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Abstract

Child tax benefits in the U.S. federal income tax are estimated at \$160 billion for 2008. This paper examines the efficiency implications of child tax benefits in a model with endogenous fertility. In the model, the efficiency of a chosen tax policy depends on three key parameters that account for the responsiveness of labor supply to the income tax rate, the responsiveness of fertility to the child subsidy level, and the cross-price substitution effect between leisure and children. This paper uses data from the National Longitudinal Survey of Youth (NLSY) to estimate the cross-price substitution effect because no estimates of this parameter exist in the applied literature. The resulting estimate implies that a tax on children rather than a child subsidy would be optimal. This implies that the annual cost of child tax benefits, when one accounts for efficiency costs, is in the \$190 to \$200 billion range.

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1 Introduction

Families with children receive preferential treatment in the U.S. federal income tax. The budgetary cost of these child tax benefits was about \$140 billion in 2006, or about \$1,900 per child.¹ This is larger than the tax expenditure from the deductibility of mortgage interest for owner-occupied homes, larger than the tax expenditure from the deductibility of state and local taxes (including property taxes), and even larger than the tax expenditure from the exclusion of employer contributions to medical insurance premiums. The \$300 per child subsidy in the 2008 tax rebate stimulus package is estimated to increase the budgetary cost of child tax benefits by about \$20 billion. As shown in Table 1, the real value of child tax benefits approximately doubled over the past decade and a half due to the expansion of existing tax provisions and the creation of new provisions.

Table 1: Estimated Budgetary Cost of Child Tax Benefits
(billions of dollars)

	1992	1996	1999	2004	2006
Dependent Exemption	24.1	30.7	35.8	36.4	35.9
Earned Income Credit	13.0	28.2	31.3	38.0	40.2
Child Tax Credit	–	–	19.9	31.2*	56.2
Child Care Expenses	3.4	3.4	3.1	3.6	3.9
Head of Household Status	3.0	3.5	3.7	3.9	4.1
TOTAL	43.5	65.8	93.8	113.1	140.3
Number of Children (millions)	66.5	70.2	71.9	73.3	73.7
Expenditure per Child	\$654	\$937	\$1,305	\$1,543	\$1,904
Real Expenditure per Child	\$940	\$1,204	\$1,579	\$1,647	\$1,904

* does not include the early child tax credit payments made in 2003

Sources: OMB analytical perspectives tables 5-1 and 19-1 various years, IRS statistics of income publications 1304, U.S. Census Bureau Table CH-1 (2007) Living Arrangements of Children Under 18 Years Old, and author's calculations.

The 2008 estimated cost of \$160 billion is a direct measure of the value of tax revenue

¹The budgetary cost of child tax benefits is the government expenditure on refundable child tax benefits combined with the tax expenditure of child tax benefits. The tax expenditure for a tax policy is a measure of the loss of government revenue due to the policy.

not received and payment made to families with children due to child tax benefit provisions. However, a measure of the true cost of providing child tax benefits should also incorporate economic efficiency considerations. Understanding the efficiency implications of child tax benefits is the focus of this paper. The optimal tax results are derived using a representative agent model where the agent decides how much time to spend working and how many children to have. The main result is then shown to hold in a model with heterogeneous agents and a nonlinear income tax. An extension to the model allowing for heterogeneity in the cost of raising children (through time costs) is also considered.

The three key parameters in the model are the responsiveness of labor supply to the income tax rate, the responsiveness of fertility to the child subsidy level, and the cross-price substitution effect between leisure and children. There are many estimates of the labor supply elasticity, and a few estimates of the fertility elasticity, but to my knowledge there are no estimates of the cross-price elasticity for leisure and children. I use data from the National Longitudinal Survey of Youth (NLSY) to estimate this cross-price elasticity. The resulting estimate implies that a tax on children rather than a child subsidy would be optimal.

This result should not be interpreted to mean that it is optimal to tax parents for the number of children that they have. The result is derived in a model that abstracts away from the benefits of providing child subsidies. I only consider the efficiency implications of providing child tax benefits and shows that when one considers the distortions associated with child subsidies, the cost is even larger than the reported budgetary cost.²

²The term budgetary cost is more appropriate than tax expenditure here for two reasons. The first is that a large fraction of child tax benefits come through refundable credits (EITC and Child Tax Credit), thus for recipients who have no tax liability, child tax benefits are cash payments. Second, the personal exemption is not usually included as a tax expenditure, but is instead thought of as part of the normal tax.

2 Model

Models with children can be very complicated. I make a strong attempt to keep this model simple. A more complicated extension is considered in the appendix, however, one assumption that is maintained throughout the paper is that parents, to a large extent, determine the number of children in their family. Therefore, government subsidization of children may distort fertility choices. In this paper, agents choose how many children to have, they gain utility from their children, and they bear the cost of raising those children.

Another important assumption is that the preferences of families can be represented by a single utility function. The number of children enters that utility function, but the model is static, so children do not grow up and become adults with their own utility functions. This assumption allows us to avoid the debate on whether social welfare should be normalized to population size, but it also means that in the model the welfare of children is only considered through its impact on family utility.³ This assumption seems most reasonable if we consider families with young children where each child's utility is fully incorporated into the parent's utility due to parental altruism.

2.1 Representative Agent Model

Consider a simple representative agent model in a Ramsey setting where the agent chooses how much time to spend working and the number of children to have. Social welfare in this model is represented by the following utility function

$$U(C, L, N) \tag{1}$$

where L is leisure (time endowment less market work time), N is the number of children, and C is the consumption of other goods. In this specification of the model, raising children

³Kaplow (2008b) contains an excellent discussion of how to compare social welfare between populations of different sizes in chapter 14.

does not require time, only money. Or alternatively, time spent raising children is considered leisure time by the parents. By assumption, each argument of the utility function is a good, meaning that children are a net source of enjoyment to their parents. The government has the ability to impose a linear income tax and can either subsidize or tax children, but must raise revenue R . There are no lump-sum taxes or subsidies and consumption is untaxed.⁴

In this model it is possible to derive conditions under which is it optimal to subsidize, rather than tax, the presence of children in a family. The optimal tax policy is the policy that is most efficient at raising the required government revenue. As will be shown, the optimal tax treatment of children in this simple model primarily depends on the cross-price substitution effect between leisure and children.

The efficiency cost of a tax policy is measured by its excess burden, that is, the loss of utility greater than would have occurred had the tax revenue been collected as a lump sum (Rosen, 1978).⁵ In other words, the excess burden of a tax policy is the loss in social welfare due to the distortion in relative prices only and not that which is due to the tax-induced loss of income. Exact measures of excess burden are defined by Diamond and McFadden (1974) and Auerbach and Rosen (1980). However it is necessary to have an explicitly-specified utility function in order to calculate the exact excess burden. Rather than assume a particular utility function, we will use the well-known approximation developed by Hotelling (1938), Hicks (1939), and Harberger (1964):

$$EB = -\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N t_i t_j S_{ij} \quad (2)$$

where t_i is the tax rate on good i and S_{ij} is the substitution effect for good i given an increase

⁴The price of consumption is the numeraire. This model with a linear income tax is equivalent to a model with a consumption tax and no income tax. One can either think of an income tax that decreases the price of leisure relative to the price of consumption, or a consumption tax that increases the price of consumption relative to the price of leisure. The tax or subsidy of children is simply disproportionate taxation of children relative to consumption goods.

⁵Auerbach (1985) provides a comprehensive mathematical and graphical descriptions of the excess burden and its relation to optimal tax theory.

in the price of good j .

Corlett and Hague (1954) were the first to examine the excess burden of a tax policy. Following their framework, the optimal tax policy is the one that minimizes the Hotelling-Hicks-Harberger approximation of excess burden while raising the required government revenue R .⁶ Assuming that the prices of all goods are constant in the relevant range and that there are no distortions in the economy (other than those caused by taxation), the excess burden of a tax policy is approximated by the following expression:

$$EB = -\frac{1}{2} (\Delta P_C \Delta C^c + \Delta P_L \Delta L^c + \Delta P_N \Delta N^c) \quad (3)$$

where P_C , P_L , and P_N are the prices and C^c , L^c , N^c are the compensated demands. The excess burden is simply the sum of the three compensated deadweight loss triangles for consumption, leisure, and children. By assumption the only price distortions in the economy are those caused by the tax policy.⁷ Consumption is untaxed (see footnote 4), thus the change in the price of consumption is always zero. This enables us to drop the first of the three compensated deadweight loss triangles in the expression. The remaining compensated demands for leisure and children are both potentially affected by changes in either price:

$$EB = -\frac{1}{2} \left[\Delta P_L \left(\frac{\partial L^c}{\partial P_L} \Delta P_L + \frac{\partial L^c}{\partial P_N} \Delta P_N \right) + \Delta P_N \left(\frac{\partial N^c}{\partial P_N} \Delta P_N + \frac{\partial N^c}{\partial P_L} \Delta P_L \right) \right]. \quad (4)$$

If the compensated demand curves are highly linear, equation (4) will be a close approximation of the true excess burden of the tax policy.⁸ Once again using the assumption that the only distortions in the economy are those caused by the tax policy, the price change for

⁶One particularly important application using a similar 3-good representative agent model is Boskin and Sheshinski (1983). Their application was the optimal tax treatment of the labor supply for married couples where the three goods are consumption, the leisure of the husband, and the leisure of the wife. In this paper, I follow the Boskin and Sheshinski terminology.

⁷All goods are produced under constant returns by competitive firms employing labor as the only input, so there are no profits.

⁸Green and Sheshinski (1979) show when this method provides an accurate approximation (even for large tax changes) and derive an approximation built from a higher order Taylor series expansion.

leisure is given by $\Delta P_L = (1 - \tau)P_L - P_L = -\tau P_L$, where τ is the income tax rate. Similarly for children, $\Delta P_N = (1 + \theta)P_N - P_N = \theta P_N$, where θ defines the tax treatment of children. A positive value of θ is a tax on children whereas a negative value of θ is a child subsidy.

While it is natural to think of the price of leisure as the after-tax wage, it is less clear what is meant by the price of children. We will take P_N to represent the level of expenditure necessary to raise a child. By assumption, the agent cannot chose to spend less than P_N for each child and any child-related expenditure above the necessary level will be considered consumption.⁹

The agent has an endowment of time, T , that can be taken as leisure L , or market work, H . The assumption that $T - H = L$ is less restrictive than it first appears because T can be defined to exclude time required for household and personal maintenance including sleep. However, we are assuming that T is given exogenously and does not depend on the number of children. This must be taken to mean that the agent considers time spent raising children as leisure.¹⁰

Returning to the approximation of excess burden, the symmetry of the Slutsky matrix allows (4) to be rewritten as:

$$EB = -\frac{1}{2} \left((-\tau P_L)^2 S_{LL} + (\theta P_N)^2 S_{NN} + 2 S_{LN} (-\tau P_L) (\theta P_N) \right) \quad (5)$$

where S represents the substitution effect. S_{LN} is the change in the compensated demand for good L due to a one unit change in the price of good N . For analytical convenience, we will scale the units of each good so that prices are unity. This means that consumption,

⁹This assumption can be relaxed by adding a fourth good, child quality, to the model. The main result of this paper (Result 2.1) still holds if the number of children and the quality of children (additional expenditure on children) are substitutes and leisure time and expenditure on children are also substitutes.

¹⁰An extension of this model that allows for a time cost of raising children is considered in the appendix. The main result of this paper (Result 2.1) still holds in the representative agent setting. However, as is show in the numerical simulation in Section 4, adding time costs to a model with heterogeneous agents and a nonlinear income tax can produce very different results because time costs have a strong influence on the marginal utility of consumption.

leisure, and children are all expressed in dollar terms. For example, two children implies a value for N of $2P_N$. This normalization allows us to express the agent's "full income" budget constraint as:

$$T(1 - \tau) = C + L(1 - \tau) + N(1 + \theta). \quad (6)$$

The optimal tax policy is the one that minimizes the excess burden of the tax policy subject to raising government revenue R :

$$\min_{\tau, \theta} \left\{ -\frac{1}{2} [\tau^2 S_{LL} + \theta^2 S_{NN} - 2\tau\theta S_{LN}] - \lambda [\tau H + \theta N - R] \right\}. \quad (7)$$

The first order conditions with respect to τ and θ can be solved to yield:

$$\tau = \frac{-\lambda (S_{NN}H + S_{LN}N)}{S_{NN}S_{LL} - (S_{LN})^2} \quad (8)$$

$$\theta = \frac{-\lambda (S_{LL}N + S_{LN}H)}{S_{NN}S_{LL} - (S_{LN})^2} \quad (9)$$

where λ is the multiplier on the government budget constraint. The denominator for either expression is non-negative because it is the determinant of a second-order principal minor of the Slutsky matrix which is negative semidefinite. The multiplier λ is positive because the excess burden of a tax policy increases in the revenue requirement, R .¹¹ Determining whether it is optimal to subsidize or tax the presence of children in a family is then reduced to signing the following expression:

$$S_{LL}N + S_{LN}H. \quad (10)$$

¹¹The optimal tax policy as a function of the required government revenue, R , is given by the following:

$$\tau = \frac{R(S_{LN}N + S_{NN}H)}{S_{NN}H^2 + 2S_{LN}HN + S_{LL}N^2} \quad \theta = \frac{R(S_{LL}N + S_{LN}H)}{S_{NN}H^2 + 2S_{LN}HN + S_{LL}N^2}.$$

These expressions are used later in the chapter as part of a back of the envelope calculation of the optimal tax treatment of children.

A child subsidy is optimal if and only if $S_{LL}N + S_{LN}H > 0$. Both N and H are constrained to be non-negative and S_{LL} is non-positive by definition, so it is the value of S_{LN} that is key in determining if a child subsidy is optimal.¹² A necessary condition for the optimal tax policy to include a child subsidy is that children and leisure be substitutes.

Result 2.1. *If leisure and children are complements ($S_{LN} < 0$) then it is not optimal to subsidize children.*

An intuitive explanation for this result comes from considering how the compensated demand for each good is affected by the tax policy. By totally differentiating the first order conditions from the optimal tax problem, we can derive how the agent's demand for leisure and children are affected by τ and θ :

$$\begin{aligned} \frac{\partial L}{\partial \tau} &= -S_{LL} - Hi_L & \frac{\partial L}{\partial \theta} &= S_{LN} - Ni_L \\ \frac{\partial N}{\partial \tau} &= -S_{LN} - Hi_N & \frac{\partial N}{\partial \theta} &= S_{NN} - Ni_N. \end{aligned}$$

The income effects, i_L and i_N , are not relevant in the excess burden measure because the same level of tax revenue, R , is raised under any policy considered. If leisure and children are complements ($S_{LN} < 0$) then an increase in τ increases the compensated demand for both leisure and children whereas an increase in θ decreases the compensated demand for both leisure and children. The income tax distortions are reduced by imposing a tax on children ($\theta > 0$) because it pushes the compensated demands back in the opposite direction. A tax on children also raises revenue, enabling the government to raise R with a lower income tax rate.

If leisure and children are substitutes ($S_{LN} > 0$), an increase in τ increases the compensated demand for leisure but decreases the compensated demand for children. The income

¹²However, S_{LN} , the cross-price substitution effect for leisure and children, is the only parameter of this model for which empirical estimates do not exist. Section 3 will turn to the task of estimating this parameter.

tax distortions are reduced by giving a child subsidy ($\theta < 0$). Providing child tax benefits is costly in that τ must be increased in order to finance the benefits, so only when leisure and children are strong substitutes as defined by Result 2.2 is it optimal to provide child tax benefits.

Result 2.2. *If $S_{LN} > -S_{LL} \left(\frac{N}{H}\right)$ then it is optimal to subsidize children.*

From this we see that subsidizing children is more likely to be optimal if there is a low demand for children and a high supply of labor. In fact, if work effort, H , is zero then regardless of the value of S_{LN} it is never optimal to subsidize children.¹³ The intuition for Result 2.2 is that the income tax distortion can be somewhat offset by a child tax subsidy if leisure and children are substitutes. Figure 1 shows this graphically by depicting how the optimal child tax treatment varies with the cross-price substitution effect for leisure and children.

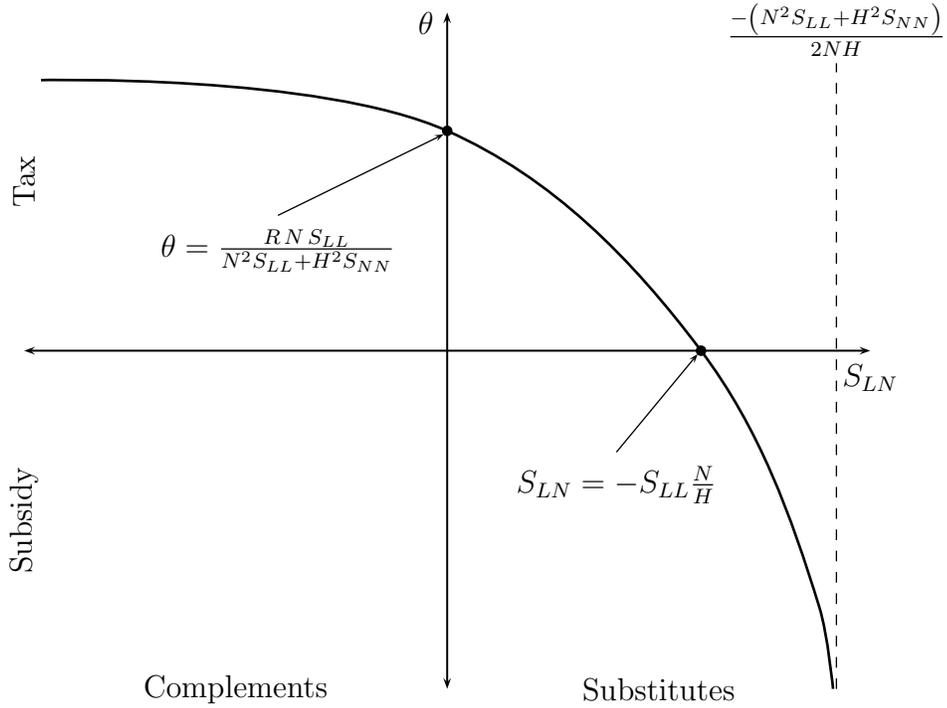
It appears from Result 2.2 that if labor is very inelastically supplied (S_{LL} close to zero), this would make the optimal tax policy more likely to include a child subsidy instead of a tax. Note however that the properties of the Slutsky matrix put bounds on the relationship between S_{LN} and S_{LL} . We know that $S_{NN}S_{LL} > (S_{LN})^2$ which implies that inelastic labor supply is associated with a smaller absolute value of S_{LN} . A similar reasoning about the demand for children is expressed as Result 2.3.

Result 2.3. *If $|S_{NN}| < \left|S_{LL} \left(\frac{N}{H}\right)^2\right|$ then it is not optimal to subsidize children.*

While there are only a few estimates of the own-price substitution effect for children, it seem generally accepted that S_{NN} is close to zero. Common experience leads us to believe that the demand for children is not very price sensitive and empirical work by Baughman and Dickert-Conlin (2003) confirms this. Result 2.3 points out that if this is correct, the

¹³This does not imply that a particular tax unit with no earned income should receive no child subsidy. This result only applies in the representative agent model and so implies that if no one in the economy is working, it would not be optimal to subsidize children. Of course, with no earned income, the income tax would produce no revenue and children would have to be taxed in order to meet the revenue requirement.

Figure 1: Optimal Child Tax Treatment.



theoretically implied bound on S_{LN} may make it impossible for leisure and children to be strong substitutes (as defined in Result 2.2). In fact, if the compensated demand for children is completely inelastic ($S_{NN} = 0$), then the optimal tax policy is to tax children quite heavily. In this case, a child tax would have the ability to raise a large amount of revenue while causing no distortions.

While this simple model of labor supply and fertility choice abstracts from various characteristics of children, externalities associated with children, pre-existing taxes and subsidies, and differences across families, it provides a useful starting point to examine the efficiency cost of child tax benefits. In this model, the case for subsidizing children primarily depends on the cross-price substitution effect for leisure and children, a parameter that has received little attention in the empirical literature. I present evidence in Section 3 that this cross-price substitution effect for leisure and children is negative (complements) implying that a child tax is optimal.

2.2 Heterogeneous Agents with a Nonlinear Income Tax

The main result from the representative agent model, that it is optimal to tax children, is confirmed in a model with heterogeneous agents that face a nonlinear income tax. As in the representative agent model, utility depends only on leisure time, l , the number of children, n , and consumption of other goods, c . However, there are many agents that differ in their wage rates, w , but have identical utility functions. The distribution of agent wage rates is given by the density function $f(w)$. Hours of work is given by h , the time endowment minus leisure time, l . The aggregate level of consumption, C is given by $\int_0^\infty c(w)f(w)dw$, where $c(w)$ is the level of consumption chosen by an agent of wage type w . Similarly, the total number of children, N , is given by $\int_0^\infty n(w)f(w)dw$, where $n(w)$ is the number of children chosen by an agent of wage type w .

The government imposes a nonlinear income tax, $t(wh)$, and a tax on children of θ . Note that as before, a positive value for θ is a tax on children while a negative value for θ is a child subsidy. The government's objective is to maximize social welfare,

$$\int_0^\infty W(U(w)) f(w)d(w), \quad (11)$$

where W is the social welfare function and $U(w)$ is the utility of an agent of wage type w . The government must raise revenue R which implies the following budget constraint:

$$\int_0^\infty t(wh(w)) f(w)d(w) + \int_0^\infty \theta n(w)f(w)d(w) = R. \quad (12)$$

Under standard assumptions on the utility and social welfare function, a progressive nonlinear income tax may cause some agents to reduce their labor supply h to imitate the wage types that are just below them. Following Mirrlees (1971), an agent's utility must rise with the wage type at a sufficient rate to prevent this mimicking behavior. This incentive

constraint is given by

$$\frac{dU(w)}{dw} = \frac{\partial U(c, l, n)}{\partial l} \frac{h}{w} = \frac{U_l h}{w}. \quad (13)$$

The government chooses the function $t(wh)$ and θ to maximize (11) subject to (12) and (13). Following Atkinson and Stiglitz (1976), the Hamiltonian is expressed as:

$$H(U, w, n, h, \lambda, \mu) = W(U(w)) f(w) + \lambda(wh(w) - n(w) - R) f(w) - \mu(w) \frac{U_l h}{w}. \quad (14)$$

Maximizing this expression with respect to θ and substituting the agent's first order conditions yields the optimal tax formula:¹⁴

$$\theta = \frac{\mu(w)}{\lambda U_c} \frac{h(w)}{w f(w)} (U_{ln} - U_{lc}) \quad (15)$$

The Atkinson-Stiglitz result, that no differential commodity taxation is optimal (i.e. no subsidization or taxation of children in this model), depends on the assumption of labor being weakly separable. Weak separability implies that U_{lc} and U_{ln} are both zero. Thus, the optimal tax treatment of children, θ , is zero if we assume weak separability.¹⁵ However, if U_{ln} and U_{lc} are not zero, then some special tax treatment of children is optimal.

If leisure and children are complements, this implies that as the agent increases the amount of leisure time, the relative value of children to other consumption rises ($\partial U_n / \partial l > \partial U_c / \partial l$). The other terms in (15) including μ and λ are all positive. Thus, the optimal value of θ is positive if leisure and children are complements. This result shows that the intuition from the representative agent model - that it is optimal to tax families with children - extends to a model with wage heterogeneity and a nonlinear income tax.

The assumption that the government does not allow θ to depend on the wage type w may be important. While I suspect that allowing the government to select a function $\theta(w)$ would

¹⁴See Salanié (2003) or Kaplow (2008a) for an explanation of how to arrive at this optimal tax formula

¹⁵Kaplow (2006) shows that under the assumption of weak separability, even if the nonlinear income tax is not optimal, it is not optimal to have differential commodity taxation.

not change the overall result, I have not shown this. It is possible that the optimal $\theta(w)$ would take on a negative value (a subsidy) for some wage types. Even with identical utility functions, a good can be a leisure complement for some wage types and a leisure substitute for others.

Another important assumption is that there is no time cost of raising children. With time costs, an agent's choice of children is correlated with the wage, and therefore the government can use the observability of the number of children to better infer an agent's wage. Because a higher wage rate would mean higher time costs, observing the number of children would be a signal to the government about the agent's wage type. Thus a child subsidy may be optimal. As shown in the appendix, the inclusion of a time cost in the representative agent model does not change the result that it is optimal to tax children. However, the simple numerical simulations in Section 4 suggest that adding time costs to this model with heterogeneous agents can lead to the optimality of child subsidies. If the marginal utility of consumption increases strongly in the number of children, this implies that transfers to agents with more children are optimal.

3 Leisure and Children Cross-Price Effect Estimation

3.1 Data

The objective in this section is to estimate the cross-price substitution effect for leisure and children. The data for this exercise is a sample of women from the 1979 National Longitudinal Survey of Youth (NLSY). The NLSY contains detailed labor supply and fertility information for each respondent from 1979 to 2004. The sample is restricted to women who were 16 to 20 when first interviewed in 1979. This restriction enables labor supply and earnings histories to be constructed from age 19 until age 43. The number of children born to each woman by

age 43 is also obtained.¹⁶

The women in the sample were interviewed annually from 1979 to 1994 and then biennially from 1996 to 2004. Missed interviews do not necessarily prevent the construction of a complete labor supply history because interviewers attempt to ask questions from the previous interviews if missed. However, multiple missed interviews, especially if consecutive, do prevent the creation of the labor supply history. Women for which it is not possible to construct a complete labor supply or fertility history are dropped from the sample.

The decision to use a sample of women rather than a sample of married couples is motivated by the fact that approximately one-third of all births in the United States are to unmarried women. This is not simply due to teenage mothers. While about 90 percent of teenage mothers are unmarried at the time they give birth, teenage mothers make up less than a quarter of the total number of births to unmarried women each year. Births to unmarried women age 20 or higher accounted for 26 percent of total births in 2003 (Martin et al., 2005).

Three relationship categories are defined: married, partnership, single. Nearly all of the women in the sample are single at age 19 and about 88 percent are married for some period of time between age 19 and 43. Only 8 percent never report being married or in a partnership. Some women move between relationship categories several times and the relationship history is taken as exogenous. Like the relationship status, the labor supply and earnings of husbands and partners is also taken as exogenous.¹⁷ Male and other income is

¹⁶For the youngest cohort, birth at age 42 is assumed if pregnancy is reported in 2004. Some women in the sample may have children after age 43 and we would ideally have this in the data; however, this would be only a small number of births. The U.S. National Center for Health Statistics reports that less than 1 percent of women have a child after age 40 and that only 0.05 percent of women have a child after age 45 (Martin et al., 2005).

¹⁷This requires the addition of nonwage income to the model of Section 2. The agent's budget constraint becomes:

$$T(1 - \tau) + M(1 - \tau) = C + L(1 - \tau) + N(1 + \theta)$$

where M is male earnings and other nonwage income (including transfer payments). The government budget constraint is also adjusted to reflect this addition:

$$R = \tau H + \tau M + \theta N.$$

subject to the linear income tax, but does not respond to changes in tax treatment. This common assumption is often justified by the finding that the labor supply elasticity for men is very low. For example, MaCurdy, Green, and Paarsch (1990) estimate that both the substitution and income effects for male labor supply are close to zero.¹⁸ The choice variables in this application of the representative agent model are the average labor supply of the woman (age 19 to 43) and the total number of children that she has. The average hours of market work over this period is created from a series of more than a thousand questions asking how many hours she worked week by week over the previous period.¹⁹

After removing observations that do not have complete birth, work, and earnings histories, the sample used in this analysis consists of 4,169 of the 6,283 women in the NLSY. All dollar amounts are inflation-adjusted to year 2000 dollars using the CPI-U before averaging. An implied wage is calculated as the real average annual earnings divided by the average annual hours. The summary statistics for this sample are given in Table 2.

The married, partner, and single variables measure the fraction of time from age 19 until 43 that the individual is either married, living with a partner, or single. The income of the husband or partner as well as any nonwage income, including welfare benefits, are combined into a single nonwage income variable. The “moved” variable indicates whether the individual’s family moved to a different town while she was growing up. The summary statistics for variables indicating whether the individual lived with both biological parents until age 14 and whether either parent is an immigrant are also listed.

For those women with no reported hours of market work over the full time period, no wage calculation can be made. This is unfortunate because an observed wage rate is essential in estimating the cross-price substitution effect for leisure and children. Therefore, the

¹⁸There is considerable evidence that investment and other non-wage income is quite responsive to the tax treatment, thus a worthwhile extension of the model that I have not attempted would be to explicitly model investment behavior.

¹⁹This measure of the average hours of work does not distinguish between a part-time worker who is employed continuously from age 19 to 43 and a full-time worker who is employed for only half that time period.

Table 2: Summary Statistics

	Obs	Mean	Sd.Dev.	Min	Max
Weekly Hours	4169	26.43	11.71	0	65.67
Annual Earnings	4169	18,084.51	13,435.37	0	104,252.40
Nonwage Income	4169	27,867.33	19,347.70	131.47	123,649.40
Hourly Wage	4059	12.45	6.48	4.81	65.14
Children	4169	1.958	1.340	0	11
Married	4169	0.566	0.327	0	1
Partner	4169	0.071	0.137	0	1
Single	4169	0.363	0.306	0	1
White	4169	0.776	0.417	0	1
Hispanic	4169	0.062	0.242	0	1
Black	4169	0.144	0.351	0	1
Other Race	4169	0.017	0.131	0	1
Rural	4169	0.219	0.414	0	1
Moved	4169	0.570	0.495	0	1
Mother Education	4169	11.54	2.734	0	20
Lived w/ 2 parents	4169	0.618	0.486	0	1
Immigrant Parents	4169	0.086	0.281	0	1

The reported summary statistics are computed using sample weights.

110 women with no observed hours of market work are dropped from the sample. The characteristics of the full sample, those with positive hours of work, and those with no market work are shown in Table 3.

It is apparent from Table 3 that the women for whom it is not possible to calculate an average wage rate are quite different from the remaining sample. This is particularly true with respect to their family background and fertility choices. No effort is made to correct for the selected nature of the remaining sample. Therefore, estimates are interpreted as only applying to the 99 percent of women in the full sample with an observable wage.²⁰

²⁰Sample selection correction would require the specification of a participation equation, a wage equation, and an exclusion restriction. Because nearly 99 percent of the sample has an observable wage, the sample selection correction would not likely yield much additional insight, but would certainly add to the complexity.

Table 3: Sample Average by Hours of Work

	All	Hours > 0	Hours = 0
Observations	4,169	4,059	110
Sample Weight	1	0.985	0.015
Weekly Hours	26.43	26.83	0
Annual Earnings	18,084.51	18,362.11	0
Nonwage Income	27,867.33	28,001.11	19,152.40
Hourly Wage	12.45	12.45	-
Children	1.958	1.944	2.856
Married	0.566	0.569	0.366
Partner	0.071	0.070	0.123
Single	0.363	0.361	0.511
White	0.776	0.782	0.394
Hispanic	0.062	0.060	0.191
Black	0.144	0.140	0.382
Other Race	0.017	0.017	0.033
Rural	0.219	0.220	0.132
Moved	0.570	0.571	0.517
Mother Education	11.54	11.57	9.20
Both Parents (14)	0.616	0.618	0.439
Immigrant Parents	0.086	0.086	0.111

The reported summary statistics are computed using sample weights.

3.2 Estimation

A common approach to estimating a substitution effect in a static model is to estimate a linear demand equation that includes a wage variable and a nonwage income variable. This is particularly common in the labor supply literature where an econometrician estimates a labor supply function of this form:

$$\text{Hours}_i = \alpha_0 + \alpha_1 \text{Wage}_i + \alpha_2 \text{Nonwage Income}_i + \alpha_3 X_i + \epsilon_i. \quad (16)$$

The vector X_i represents the predetermined characteristics of the individual that are observed by the econometrician and ϵ_i represents those unobserved characteristics that affect labor supply. A true labor supply function would depend not only on the wage, but also on the

prices of all other goods. In practice, other prices are not frequently included in the regression equation, particularly when using cross-sectional data. The usual assumption used to justify this is that the prices for all other goods are not individual-specific. Among other things, this assumption implies that there are no geographical differences in other prices.

In the labor supply equation, α_1 is the change in labor supply due to a one dollar increase in the hourly wage. This total effect is comprised of an income effect and a substitution effect as given by the Slutsky equation:

$$\frac{\partial \text{Hours}}{\partial \text{Wage}} = \left(\frac{\partial \text{Hours}}{\partial \text{Wage}} \right)^c + \text{Hours} \frac{\partial \text{Hours}}{\partial \text{Income}}. \quad (17)$$

This same approach to estimating an own-price substitution effect for labor supply (or leisure demand) can be used to estimate the cross-price substitution effect for leisure and children. We specify a linear child demand function that depends on the wage, nonwage income, a vector of predetermined and observed characteristics, and an error term that represents those unobserved characteristics that affect child demand:

$$\text{Children}_i = \beta_0 + \beta_1 \text{Wage}_i + \beta_2 \text{Nonwage Income}_i + \beta_3 X_i + \eta_i. \quad (18)$$

Ideally, the “price” of children (cost of raising children) should also be included in this specification. The absence of this variable is cause for some concern because the demand of a good clearly depends on its own price. The high degree of uncertainty about the level of expenditure required to raise a child—and how this level changes with family size, family income, and other factors—severely complicates determining an individual-specific cost of raising a child. Similar to the argument for the exclusion of other prices in a labor supply equation, one could argue that there is little individual specific variation in the direct cost of raising a child. However, differences in child tax benefits by income level, family economies of scale, and geographical differences in the cost of food, housing, and health care suggest

that this may not be the case. The direction of any bias in the estimates of β_1 and β_2 from this heterogeneity in the cost of raising children is not clear.

Thus, for this exercise, we make the assumption that there are no individual differences in the monetary cost of children. This does not rule out differences in the opportunity cost of raising children. Rather, the assumption is that each woman faces the same out-of-pocket expenditure necessary to raise a child. If it were possible to determine an individual specific P_{Ni} , this variable could be used in an alternative method for identifying the cross-price substitution effect for leisure and children: a regression of labor supply on P_{Ni} , the wage rate, and nonwage income.

For the identification of β_1 , there is a serious concern that the decision to have a child has a direct influence on the wage. For example, Miller (2006) shows that the timing of the first birth has a strong effect on a woman's future wage growth. The child demand equation in this paper is in regards to the number of children, not the timing of children, thus the Miller (2006) result does not apply directly. A valid instrument for wage in the child demand equation is a predetermined characteristic that affects the individual's wage but not the demand for children or other factors that affect the demand for children. Several observed characteristics like the month and year of birth and measures of the reading habits of the individual's parents satisfy the definition in this sample. However, instrumental variables estimation gives very similar estimates of β_1 as OLS and using a Hausman test of endogeneity, we fail to reject that the wage is exogenous.²¹ This is not conclusive evidence, however, the assumption that the wage is exogenous is maintained in the discussion that follows.

Using the estimates for β_1 and β_2 , the cross-price substitution effect for leisure and

²¹The two-stage least squares results are available from the author by request. In each regression, the estimate of β_1 was larger in magnitude (more negative, although not statistically different) using instrumental variables estimation than the corresponding OLS regression. If having an additional child (not the timing of the birth) had a direct effect on the wage, one would expect the instrumental variables estimation to produce estimates of β_1 that were smaller in magnitude.

children are given by $(\beta_1 - \text{Hours} \beta_2)$ as indicated by the Slutsky decomposition:

$$\frac{\partial \text{Children}}{\partial \text{Wage}} = \left(\frac{\partial \text{Children}}{\partial \text{Wage}} \right)^c + \text{Hours} \frac{\partial \text{Children}}{\partial \text{Income}}. \quad (19)$$

In the representative agent model of Section 2, we normalize the units of all goods so that prices are unity. This same normalization is easily made to the total, income, and substitution effects by multiplying through by the appropriate prices:

$$S_{LN} = P_N \left(P_L \hat{\beta}_1 - H \hat{\beta}_2 \right). \quad (20)$$

Given this normalization, S_{LN} is the change in the compensated demand for children for a doubling of the wage. At the average wage in the sample, a 100 percent wage increase would place it at about the 95th wage percentile.

The estimation of equation (18) is given in Table 4. The region controls include indicators for the region of the country the individual was raised in, either the Northeast, South, Central, or West. The family controls provide information about the family during the woman's growing-up years, whether it was a rural or urban area, if the family had access to a local library, and if the family ever moved. Also included is the number of siblings and indicators for oldest or youngest child, immigrant parents, and if the woman grew up in a home with both biological parents.²² Religion controls are for the religion that the individual was raised in and also a measure of how often the family went to religious services.²³

There are several issues to consider in this type of estimation. Concern is often expressed about the measurement of nonwage income. The NLSY income variables are top-coded

²²Youngest child is an indicator that the individual has at least one sibling and was the youngest child in her family; oldest child indicates that the individual has at least one sibling and was the oldest child in her family.

²³Included in these variables are indicator for being raised in a family that went to religious services twice a month or more and an indicator of having converted to a religion other than the religion raised in. The religion categories (in order of size in the sample) are: Catholic, Baptist, Other Christian Religion, Methodist, Lutheran, Presbyterian, None, Pentecostal, Episcopalian, Jewish, and Non-Christian Religion.

Table 4: Linear Child Demand Estimation

	(1)	(2)	(3)	(4)	(5)
Wage	-0.0308 (0.0031)**	-0.0232 (0.0032)**	-0.0160 (0.0031)**	-0.0157 (0.0031)**	-0.0229 (0.0032)**
Nonwage Income (thousands)	0.0191 (0.0011)**	0.0206 (0.0011)**	0.0080 (0.0013)**	0.0078 (0.0013)**	0.0203 (0.0011)**
Hispanic	0.603 (0.086)**	0.316 (0.092)**	0.383 (0.089)**	0.374 (0.093)**	0.309 (0.096)**
Black	0.499 (0.061)**	0.331 (0.062)**	0.615 (0.062)**	0.575 (0.065)**	0.281 (0.065)**
Other race	-0.029 (0.154)	-0.229 (0.152)	-0.059 (0.147)	-0.020 (0.156)	-0.182 (0.161)
Married			1.372 (0.080)**	1.372 (0.080)**	
Partner			0.571 (0.153)**	0.593 (0.153)**	
Constant	1.701 (0.072)**	1.996 (0.133)**	1.156 (0.084)**	1.208 (0.165)**	1.918 (0.162)**
Region controls	yes	yes	yes	yes	yes
Family controls	no	yes	yes	yes	yes
Religion controls	no	no	no	yes	yes
Observations	4059	4059	4059	4059	4059
R-squared	0.1005	0.1422	0.2020	0.2063	0.1473
Total Effect	-0.383	-0.289	-0.199	-0.195	-0.285
Income Effect	0.351	0.378	0.147	0.143	0.373
Substitution Effect	-0.734	-0.667	-0.346	-0.339	-0.658

* significant at 5% ** significant at 1%. The reported values are computed using sample weights. Standard errors in parentheses.

Region controls include: northeast, central, and south. Family controls include: number of siblings, youngest child indicator, oldest child indicator, biological parents (14), immigrant parents, mother's education level, rural, moved, and a library indicator. Religion controls include: Catholic, Baptist, Methodist, Lutheran, Presbyterian, Pentecostal, Episcopalian, Jewish, Other Christian Religion, Non-Christian Religion, and a measure of frequency of attendance.

which puts a downward bias on both the average wage and the average nonwage income for an individual. The cutoff at which top-coding occurs and the procedure have changed over the 25 years of the survey. However, the number of individuals affected by top-coding is small. A second concern is that the procedure for calculating nonwage income is to subtract female earnings from total family income. This means that transfer payments that may not be independent of female earnings are included in the measure of nonwage. However, transfer payments are generally non-taxable and thus treating them as nonwage income is more appropriate. A third issue is the problem of nonresponse to income questions. Nearly all respondents in the NLSY report their own income and their spouse's income, however, approximately 30 percent of respondents living with a partner in a given year do not report their partner's income.²⁴ Even if the individual keeps her finances completely separate from her partner's, living together suggests that they probably share some common expenses like rent or house payments. Quite often, a woman who refuses to answer the partner's income question will answer the question in the next year and since the average woman in the sample spends only about 7 percent of her time in a partnership, the nonresponse bias is likely small. A regression of the percentage of the time living with a partner on other factors including wage, race, region, and religion reveals very little correlation between time living with a partner and any other characteristics.²⁵

The assumption of a linear child demand function implies a specific form of the utility function.²⁶ An alternate functional form assumption for the direct utility could yield different estimates for the substitution effects. Using OLS to estimate the child demand function may

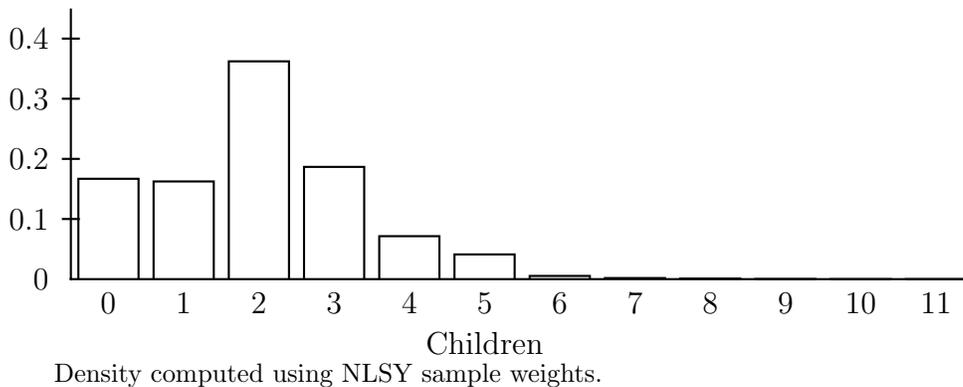
²⁴In the NLSY, partner income is not included in the constructed total family income variable. For this study, partner income was added to the constructed family income variable and then the respondent's earnings were subtracted to give nonwage income. Note that from 1979-1994 the partner income variables are separate from spouse income variables, but after 1994, these variables are combined.

²⁵This regression shows that women who live in the west and those who have lower wage rates spend more time living with a partner. None of the religion variables were significantly different than zero. The R-squared from the regression was 0.021, implying that the variables used in this analysis are not strongly correlated with time spent living with a partner.

²⁶See Pencavel (1986) for a discussion of the direct and indirect utility functions implied by linear demand equations.

also be inappropriate because of the nature of the dependent variable. For example, it is not possible for a woman to have a negative number of children. As Figure 2 shows, there is more bunching at zero than would be expected if children were distributed normally. While this suggests the possibility of censoring at zero (because individuals cannot have a negative number of children), the more serious issue is that the dependent variable that is not even approximately continuous and thus the normally distributed error term is probably not reasonable. The dependent variable in this exercise takes on only 12 different values, the natural numbers from 0 to 11. A discrete distribution, such as the Poisson is commonly used for this type of count data.

Figure 2: Number of Children Histogram



The Poisson distribution is determined by a single parameter, μ that indicates the intensity of the Poisson process. The value of μ is both the mean and the variance of the distribution. Thus, we only need to specify $\mu = E(\text{children}_i | \text{wage}_i, \text{nonwage income}_i, X_i)$, which we assume is given by the exponential function:

$$E(\text{children}_i | \text{wage}_i, \text{nonwage income}_i, X_i) = e^{(\beta_0 + \beta_1 \text{wage}_i + \beta_2 \text{nonwage income}_i + \beta_3 X_i)}. \quad (21)$$

This specification assures that μ will be positive for all values of wage, nonwage income, and X . In this model, the probability that the number of children equals the value j , conditional

on the independent variables, is

$$\Pr(\text{children} = j|X) = \frac{e^{-e^{(X\beta)}} (e^{(X\beta)})^j}{j!} \quad \text{for } j = 0, 1, 2, \dots \quad (22)$$

where X represents the full set of explanatory variables, including wage and nonwage income. Maximum likelihood estimation is used to obtain the parameter estimates. The estimated marginal effects evaluated at the mean are reported as the first two columns of Table 5.

Table 5: Poisson and Ordered Probit Child Demand Estimation

	Poisson	Poisson	Ordered Probit	Ordered Probit
Wage	-0.0170 (0.0051)**	-0.0252 (0.0056)**	-0.0148 (0.0040)**	-0.0208 (0.0042)**
Nonwage Income (thousands)	0.0074 (0.0013)**	0.0185 (0.0011)**	0.0074 (0.0013)**	0.0183 (0.0012)**
Hispanic	0.347 (0.081)**	0.281 (0.082)**	0.319 (0.068)**	0.244 (0.066)**
Black	0.646 (0.075)**	0.282 (0.066)**	0.545 (0.059)**	0.235 (0.054)**
Other race	0.006 (0.216)	-0.168 (0.194)	-0.005 (0.197)	-0.162 (0.183)
Married	1.450 (0.093)**		1.356 (0.086)**	
Partner	0.792 (0.204)**		0.635 (0.171)**	
Region controls	yes	yes	yes	yes
Family controls	yes	yes	yes	yes
Religion controls	yes	yes	yes	yes
Observations	4059	4059	4059	4059
Total Effect	-0.212	-0.314	-0.184	-0.259
Income Effect	0.136	0.340	0.136	0.336
Substitution Effect	-0.348	-0.653	-0.320	-0.595

* significant at 5% ** significant at 1%. The reported values are computed using sample weights. Standard errors in parentheses.

Region controls include: northeast, central, and south. Family controls include: number of siblings, youngest child indicator, oldest child indicator, biological parents (14), immigrant parents, mother's education level, rural, moved, and a library indicator. Religion controls include: Catholic, Baptist, Methodist, Lutheran, Presbyterian, Pentecostal, Episcopalian, Jewish, Other Christian Religion, Non-Christian Religion, and a measure of frequency of attendance.

As an alternative, an ordered probit model is also considered. The model is built around the concept of a latent demand for children that, conditioning on X , is normally distributed. This latent demand for children is continuously distributed with nothing preventing it from taking on negative values. However, this latent demand is not observed. What we do observe is the actual number of children (0, 1, 2, ...) selected by each individual. A set of cutoff values for the latent variable, for example $\{0.5, 1.5, 2.5, 3.5, \dots\}$, determine the observed number of children. If the latent variable were to have a value that falls between 0.5 and 1.5 (given the cutoff values in the example) then the individual would choose to have one child. The cutoff values and parameter estimates are obtained by maximum likelihood estimation. The estimated marginal effects evaluated at the mean are reported as the third and fourth columns of Table 5.

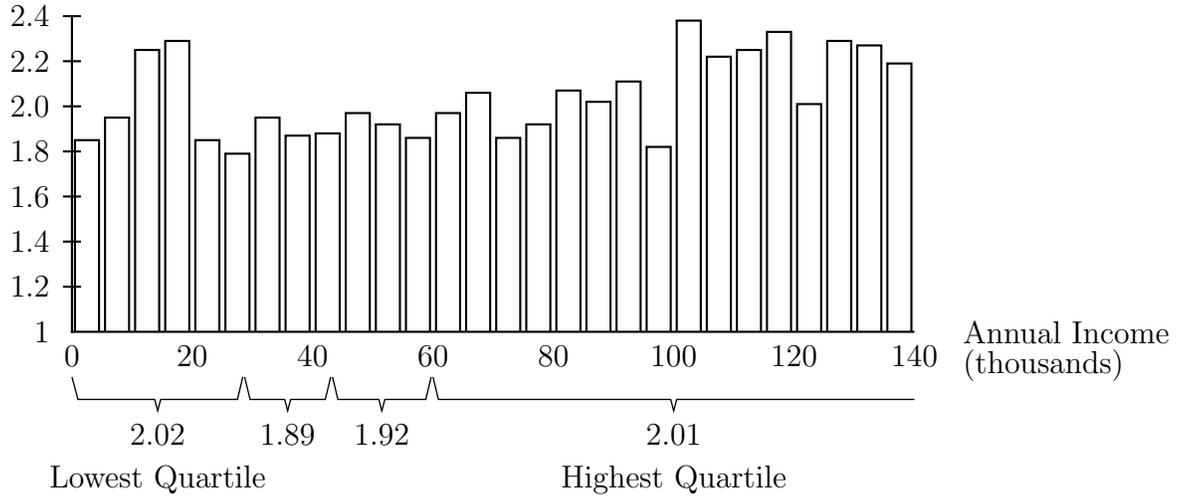
The results reported in Table 5 are consistent with those of Table 4 (page 22). A one dollar increase in the life time average hourly wage (evaluated at the mean wage) is associated with a decrease in the number of children between 0.014 and 0.031. The estimated income effect is positive, meaning that children are a normal good. Thus, the negative total effect implies that the substitution effect is larger in magnitude than the income effect.

For some, a positive value for the estimated income effect is perhaps surprising. It is sometimes claimed that high-income countries have lower fertility rates due to a negative income effect.²⁷ These results are contrary to that claim. In fact, it is apparent from an examination of the unconditional average number of children by average annual real family income level that the number of children increases with income (see Figure 3). These results support the claim that higher female wages are an important factor in explaining fertility decline (Butz and Ward 1979; Schultz 1985; Heckman and Walker 1990).²⁸

²⁷See Jones, Schoonbroodt, and Tertilt (2008) for a discussion of the relationship between income and fertility.

²⁸While it is argued that rising female wages in the last half of the 20th Century is an important explanation for the decline in fertility rates, it should be noted that fertility rates in the United States began to decline in the late 19th Century, long before any sizable increase in female wages.

Figure 3: Average Number of Children by Real Average Family Income



3.3 Efficiency Cost Computation

The sign of the estimated cross-price substitution effect for leisure and children implies that they are complements. This, in turn, implies that an optimal tax policy would involve a child tax rather than a subsidy. This section calculates the excess burden associated with the U.S. child tax benefits, given the estimated substitution effect for leisure and children and derives the optimal child tax.

We start by calculating the value of the child subsidy or tax, θ , that minimizes the excess burden. The addition of nonwage income to the model implies only a minor change to the denominator of the optimal child tax treatment equation as derived in Section 2:

$$\theta^* = \frac{R(S_{LL}N + S_{LN}H)}{S_{NN}(H^2 + HM) + S_{LN}(2NH + NM) + S_{LL}N^2}. \quad (23)$$

This expression is derived under the assumption that male and other income is subject to the linear income tax but does not respond to it. If male and other income was assumed to responded to the tax policy, the expression above would need additional terms representing the cross and own price substitution effects. Again, this is a fairly common assumption justified by empirical results suggesting the labor supply elasticity for men is approximately

zero (MaCurdy, Green, and Paarsch, 1990).

We need estimates of each of the parameters in order to evaluate the expression. Sample averages for annual female earnings (\$18,362), nonwage and male income (\$28,001), and children (1.944) as reported in Table 3 (page 18) provide values for H and M . We also need an estimate of the cost of raising a child, P_N . There are various estimates of P_N in the literature and rather than make a judgment on which estimate is best, we will proceed by selecting two values: one high and the other low. For the high value, the USDA calculates that the average annual child-related expenditure for a middle-income married couple is about \$11,000 (Lino, 2007). The USDA method measure education and clothing expenditure on children and also attributes a portion of food, housing, utilities, and transportation to children. For the low value, the U.S. poverty thresholds increase with the number in the household and this implies a cost of raising a child of about \$3,400. The high value implies $N = 21,384$; the low value implies $N = 6,610$. The Congressional Budget Office reports that taxpayers paid an average of \$6,100 in federal income tax in 2003, so I take this as the value for R .²⁹

Estimation of the own-price substitution effect for children, S_{NN} , is not possible with this data because there is no observed cross-sectional variation in the cost of raising children. Gauthier and Hatzius (1997) find evidence of a small fertility response in a panel of 22 countries. Huang (2002) also found a small fertility response using times series data from Taiwan. Baughman and Dickert-Conlin (2003) find no fertility response to EITC increases for unmarried women and only a small positive response for married women. For the calculation, I use an estimate from Laroque and Salanié (2005) which uses a structural model of labor supply and fertility to explain the fertility response of families in France. They estimate the uncompensated cost elasticity of the demand for children to be about 0.2. This value implies that doubling the cost of children would reduce the total number of children born over a

²⁹See the CBO document “Historical Effective Federal Tax Rates: 1979 to 2003” located on the web at: <http://www.cbo.gov/ftpdocs/70xx/doc7000/12-29-FedTaxRates.pdf>. The CBO reports that taxpayers in 2003 paid an average of \$14,200 in federal taxes, 43 percent of which was due to the individual income tax.

woman's life time by 0.39 children on average. This uncompensated effect is the sum of the substitution effect, S_{NN} , and the income effect. The own-price substitution effect implied by the Laroque and Salanié (2005) estimate is -0.377.

An estimate for the own-price substitution effect for leisure, S_{LL} can be obtained by the same method as was used to estimate of the cross-price substitution effect for leisure and children. Using the same NLSY sample of women, I estimate the effect of wages and non-wage income on labor supply, assuming that labor supply is given by a linear function as in equation (16). The results indicate that a dollar increase in a woman's average hourly wage leads to an increase of approximately 0.48 hours per week of market work. The female labor supply elasticities implied by this result seem reasonable, the compensated wage elasticity is 0.344 and the uncompensated wage elasticity is 0.224, well within the range of estimated elasticities in the female labor supply literature (Killingsworth and Heckman, 1986).

Using the high or the low estimate for the cost of raising a child makes a large difference in the size of the optimal child tax. Using the low value for P_N of \$3,400 results in an optimal child tax of about \$100 per child. Using the high value for P_N of \$11,000 results in an optimal child tax of about \$800 per child. The reason for the large difference is that if children are quite expensive, then a child tax has the ability to raise a large amount of revenue and thus allow for lower income tax rates while at the same time causing very little distortion in the demand for children. If children are much less expensive, then a large tax would be much more distortive.

The optimal linear income tax rate derived from this method is quite low, 12.7 percent under low child costs and 9.4 percent under high child costs. The linear income tax in this model has no standard deduction or exemption so this is both the marginal and the average tax rate. For comparison, in order to provide child tax benefits of \$2,000 per child and still raise the \$6,100 revenue, a tax rate of 21.5 percent would be required.

The excess burden of the tax policy is a measure of the loss of welfare greater than would

have occurred if the tax revenue had been collected as a lump sum tax:

$$EB = -\frac{1}{2} (\tau^2 S_{LL} + \theta^2 S_{NN} - 2\tau\theta S_{LN}). \quad (24)$$

Under a lump sum tax this excess burden measure would have a value of zero. Under the high child cost assumption, the optimal tax policy of a \$800 per child tax and a 9.4 percent flat income tax rate produces an excess burden of \$25. If the government is not able to impose a child tax and instead sets θ to zero, then the flat income tax rate would be 13.2 percent and the excess burden would be \$52. In order for the government to provide a \$2,000 per child subsidy, the excess burden would be \$350. Under the low child cost assumption, the excess burden of providing a \$2,000 per child subsidy is \$496. Thus, when considering economic efficiency, the true cost of providing child tax benefits in the U.S. federal income tax is in the \$190 to \$200 billion range.

4 Numerical Simulation with Time Costs

Consider a model in which there is a necessary time and monetary cost associated with children. For example, if agents have Stone-Geary utility where at least one of the basic needs parameters increases in the number of children, then providing child tax benefits could increase social welfare. This is represented by the following utility function:

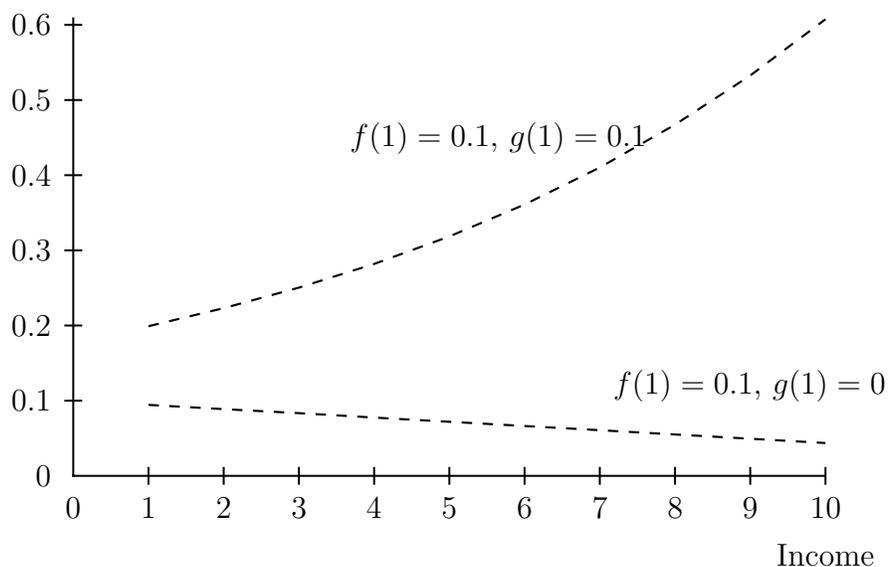
$$U(C, L, N) = \beta \ln (C - f(N)) + (1 - \beta) \ln (L - g(N)) + V(N) \quad (25)$$

where $f(N)$ is some necessary cost of raising a child which the agent does not consider consumption and $g(N)$ is some necessary time cost of raising a child which the agent does not consider leisure. The function $V(N)$ captures the direct utility from children. Expenditure on children, either in time or money, above the necessary levels of $f(N)$ and $g(N)$ is respectively

considered consumption or leisure.

In this example, children are assigned exogenously and so there is no efficiency cost of providing child tax benefits. With a utilitarian social welfare function, the motivation for child tax benefits is that for $f(1) > 0$ and $f'(N) > 0$ the marginal utility of consumption is greater for those with more children. The same is true for the marginal utility of leisure under equivalent conditions on $g(N)$. Thus, for any function $V(N)$, the optimal tax policy will redistribute to those with more children from those with less children. I show this numerically assuming five ability types $\{1, 2, 3, 4, 5\}$. Figure 4 shows the optimal child subsidy under two alternative assumptions for $f(1)$ and $g(1)$. In each case, $f(0) = 0$ and $g(0) = 0$.

Figure 4: Optimal Child Subsidy with no Efficiency Costs



With monetary costs alone, this exercise shows that the optimal child subsidy should decline as income rises. The intuition is that the difference in marginal utility of income for agents with different number of children declines as income rises and thus so does the social welfare gains from redistribution. There is still redistribution from high-income to low-income agents, but less redistribution from those without children to those with children at higher income levels.

Time costs of children lead to the opposite result. The intuition is that the difference

in the marginal utility of income for those with different numbers of children increases in the agents' ability level and thus income level. An agent with a high ability level or wage has a higher opportunity cost of the time spent raising children than an agent with a lower ability level. Because the difference in the marginal utility of income increases in income, the optimal child subsidy increases in income in models with only time costs.

In models with both monetary and time costs of raising children, the shape of the curve depends on the relative importance of the two costs. In Figure 4, the top curve is increasing in income because the time costs dominate the monetary costs. In this example, the time required to raise a child is fixed, representing the amount of supervision, care, and adult interaction that children need. However, in reality, there is no requirement that parents spend all of that time. For high-ability agents, some time costs can be avoided by paying for child care services. Thus, the monetary costs of raising a child may be more important than the time costs.

5 Conclusion

The budgetary cost of child tax benefits in the U.S. is estimated to be \$160 billion in 2008 and it will likely continue to grow. However, there has been little work analyzing these child subsidies. In this paper, I use a representative agent model to show that the efficiency costs depend on the cross-price substitution effect for children and leisure (non-market work) time. If children and leisure are complements then child subsidies are not optimal in terms of economic efficiency. Estimation of a child demand function using NLSY data indicated that children and leisure are complements over the life cycle. A back of the envelope style calculation of the optimal child tax implied by the estimates is in the range of \$100 to \$800 per child. However, this does not mean that it is optimal to tax children. These estimates imply that there is a large efficiency cost in subsidizing children, but it is possible that the benefits from these subsidies outweigh the costs.

There is a strong need for additional research on the optimal design of child subsidies. The child tax benefits in the United States are large and expensive and have the potential to grow even larger over the next few years. This is a research topic with far too little formal analysis on which to base policy recommendations. Careful thinking about the efficiency costs associated with child subsidies is only a first step. Further analysis of the consequences of child subsidies, especially for different ranges in the income distribution, presents an important area for future research.

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Appendix

Time Cost of Raising Children

Adding a time cost of raising children to the representative agent model does not change the result that it is optimal to tax children. The model is altered so that the agent now faces a time constraint that depends explicitly on children:

$$T = H + L + g(N). \tag{A-1}$$

The function $g(N)$ represents the time cost of raising children. The time cost likely increases in the number of children, $g'(N) > 0$, but due to economies of scale in families, $g''(N) < 0$. For convenience, we scale the units of each good so that the wage and the monetary costs of children are unity. The agent maximizes equation (1), subject to the following full income budget constraint:

$$T(1 - \tau) = L(1 - \tau) + g(N)(1 - \tau) + C + N(1 + \theta) \tag{A-2}$$

where M is non-labor income.

By totally differentiating the first order conditions from the optimal tax problem, we derive how the agent's leisure and child choices are affected by τ and θ .

$$\frac{\partial L}{\partial \tau} = -S_{LL} - g'(N)S_{LN} - Hi_L \tag{A-3}$$

$$\frac{\partial N}{\partial \tau} = -S_{LN} - g'(N)S_{NN} - Hi_N \tag{A-4}$$

$$\frac{\partial L}{\partial \theta} = S_{LN} - Ni_L \tag{A-5}$$

$$\frac{\partial N}{\partial \theta} = S_{NN} - Ni_N \tag{A-6}$$

If children and leisure are complements ($S_{LN} < 0$), then from equations (A-3) and (A-4) it is clear that increasing the income tax increases the compensated demand for both leisure and children. Equations (A-5) and (A-6) again show that a child tax would decrease the compensated demand for both leisure and children.³⁰

To formally derive the optimal tax policy, we again use the excess burden approximation given in equation (3). As before, the change in the price of children depends on the child tax treatment, but it now also depends on the income tax. Specifically, $\Delta P_N = (1 + \theta) + g'(N)(1 - \tau) - 1 - g'(N) = \theta - \tau g'(N)$. We then arrive at the following expression for the excess burden:

$$EB = -\frac{1}{2} [\tau^2 (S_{LL} + 2g'(N)S_{LN} + g'(N)^2 S_{NN}) - 2\theta\tau (S_{LN} + g'(N)S_{NN}) + \theta^2 S_{NN}]. \quad (\text{A-7})$$

By minimizing equation (A-7) subject to raising the required government revenue R we obtain the optimal tax expressions:

$$\tau = \frac{-\lambda(g'(N)S_{NN}N + S_{NN}H + S_{LN}N)}{S_{NN}S_{LL} - (S_{LN})^2} \quad (\text{A-8})$$

$$\theta = \frac{-\lambda(2g'(N)S_{LN}N + S_{LN}H + g'(N)^2 S_{NN}N + g'(N)S_{NN}H + NS_{LL})}{S_{NN}S_{LL} - (S_{LN})^2} \quad (\text{A-9})$$

For $g'(N) = 0$, equations (A-8) and (A-9) reduce to equations (8) and (9). Both τ and θ are unambiguously positive for $R > 0$ and $S_{LN} < 0$. Therefore, if leisure and children are complements ($S_{LN} < 0$) then it is not optimal to subsidize children. The introduction of time costs to the model does not alter the central result that child subsidies are not optimal if leisure and children are complements. If $S_{LN} > \frac{-(g'(N)^2 S_{NN}N + g'(N)S_{NN}H + NS_{LL})}{2g'(N)N + H}$ then it is optimal to subsidize children.

³⁰The estimation of the cross-price substitution effect for leisure and children in Section 3 and for the own-price substitution effect for leisure in Section 3.3 were not estimates of S_{LN} and S_{LL} in this time-cost model. Instead, the estimate of what was called the cross-price substitution effect for leisure and children is, in fact, an estimate of the combined term $S_{LN} + g'(N)S_{NN}$. The estimate of the own-price substitution effect for leisure is instead an estimate of the combined term $S_{LL} + g'(N)S_{LN}$.

After scaling N and L, the agent's Lagrangean is:

$$\mathcal{L} = U(C, L, N) + \lambda \left[T(1 - \tau) + M - L(1 - \tau) - g(N)(1 - \tau) - C - N(1 + \theta) \right] \quad (\text{A-10})$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial C} = U_C - \lambda = 0 \quad (\text{A-11})$$

$$\frac{\partial \mathcal{L}}{\partial L} = U_L - \lambda(1 - \tau) = 0 \quad (\text{A-12})$$

$$\frac{\partial \mathcal{L}}{\partial N} = U_N - \lambda(1 + g'(N)(1 - \tau) + \theta) = 0 \quad (\text{A-13})$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = T(1 - \tau) + M - L(1 - \tau) - g(N)(1 - \tau) - C - N(1 + \theta) = 0 \quad (\text{A-14})$$

Totally differentiating (A-11) - (A-14) and placing in matrix form, the system of differential equations is:

$$\begin{bmatrix} U_{CC} & U_{CL} & U_{CN} & -1 \\ U_{LC} & U_{LL} & U_{LN} & (\tau - 1) \\ U_{NC} & U_{NL} & U_{NN} - \lambda g''(N)(1 - \tau) & g'(N)(\tau - 1) - (1 + \theta) \\ -1 & (\tau - 1) & g'(N)(\tau - 1) - (1 + \theta) & 0 \end{bmatrix} \begin{bmatrix} dC \\ dL \\ dN \\ d\lambda \end{bmatrix} \quad (\text{A-15})$$

$$= \begin{bmatrix} 0 \\ -\lambda d\tau \\ -\lambda g'(N)d\tau + \lambda d\theta \\ (T - L - g(N))d\tau + Nd\theta + (1 - \tau)dT + dM \end{bmatrix}$$

The matrix above is called the bordered Hessian and is symmetric by Young's Theorem. The

solution to the system is found by taking the inverse of the bordered Hessian matrix

$$\left(\text{Bordered Hessian} \right)^{-1} = \frac{1}{D} \begin{bmatrix} D_{11} & D_{21} & D_{31} & D_{41} \\ D_{12} & D_{22} & D_{32} & D_{42} \\ D_{13} & D_{23} & D_{33} & D_{43} \\ D_{14} & D_{24} & D_{34} & D_{44} \end{bmatrix} \quad (\text{A-16})$$

where D is the determinant of the bordered Hessian and D_{ij} is the cofactor found by taking the determinant of the bordered Hessian after deleting row i and column j . Note that the inverse of the bordered Hessian is also symmetric. The solution to the system of equations is given by the following expressions:

$$dC = - \left[\lambda \frac{D_{21}}{D} + \lambda g'(N) \frac{D_{31}}{D} - H \frac{D_{41}}{D} \right] d\tau + \left[\lambda \frac{D_{31}}{D} + N \frac{D_{41}}{D} \right] d\theta + \left[(1 - \tau) \frac{D_{41}}{D} \right] dT + \left[\frac{D_{41}}{D} \right] dM \quad (\text{A-17})$$

$$dL = - \left[\lambda \frac{D_{22}}{D} + \lambda g'(N) \frac{D_{32}}{D} - H \frac{D_{42}}{D} \right] d\tau + \left[\lambda \frac{D_{32}}{D} + N \frac{D_{42}}{D} \right] d\theta + \left[(1 - \tau) \frac{D_{42}}{D} \right] dT + \left[\frac{D_{42}}{D} \right] dM \quad (\text{A-18})$$

$$dN = - \left[\lambda \frac{D_{23}}{D} + \lambda g'(N) \frac{D_{33}}{D} - H \frac{D_{43}}{D} \right] d\tau + \left[\lambda \frac{D_{33}}{D} + N \frac{D_{43}}{D} \right] d\theta + \left[(1 - \tau) \frac{D_{43}}{D} \right] dT + \left[\frac{D_{43}}{D} \right] dM \quad (\text{A-19})$$

equations (A-3) - (A-6) in the text are derived by combining equations (A-18) and (A-19) with the definitions of the Slutsky matrix:

$$\begin{aligned} S_{LC} &= \lambda \frac{D_{21}}{D} & S_{NC} &= \lambda \frac{D_{31}}{D} & i_C &= -\frac{D_{41}}{D} \\ S_{LL} &= \lambda \frac{D_{22}}{D} & S_{NL} &= \lambda \frac{D_{32}}{D} & i_L &= -\frac{D_{42}}{D} \\ S_{LN} &= \lambda \frac{D_{23}}{D} & S_{NN} &= \lambda \frac{D_{33}}{D} & i_N &= -\frac{D_{43}}{D}. \end{aligned}$$

Note that the symmetry of the inverse of the bordered Hessian matrix means that the Slutsky matrix is also symmetric. This implies that the effect of a small increase in the price of good i on the compensated demand of good j is identical to the effect of an equivalent increase in the price of good j on the compensated demand of good i ($S_{ij} = S_{ji}$).

The excess burden approximation is given by:

$$EB = -\frac{1}{2} \left[\Delta P_L \left(\frac{\partial L^c}{\partial \tau} \tau + \frac{\partial L^c}{\partial \theta} \theta \right) + \Delta P_N \left(\frac{\partial N^c}{\partial \theta} \theta + \frac{\partial N^c}{\partial \tau} \tau \right) \right] \quad (\text{A-20})$$

Substituting in the compensated demand effects from equations (A-3) - (A-6) we obtain:

$$EB = -\frac{1}{2} \left[-\tau \left((-S_{LL} - g'(N)S_{LN})\tau + S_{LN}\theta \right) + (\theta - \tau g'(N)) \left(S_{NN}\theta + (-S_{LN} - g'(N)S_{NN})\tau \right) \right].$$

By simply rearranging terms, we arrive at equation (A-7) in the text. The Lagrangean for the optimal tax problem is:

$$\mathcal{L} = EB - \lambda(\tau H + \theta N - R) \quad (\text{A-21})$$

The first order conditions are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \tau} &= -\lambda H + \frac{1}{2} \left(S_{LN}\theta + \tau g'(N) \left(-S_{LN} - g'(N)S_{NN} \right) + g'(N)S_{NN}\theta \right) \\ &\quad - \frac{1}{2} \left(\theta - \tau g'(N) \right) \left(-S_{LN} - g'(N)S_{NN} \right) - \tau \left(S_{LL} + g'(N)S_{LN} \right) \\ \frac{\partial \mathcal{L}}{\partial \theta} &= -\frac{1}{2} \left(S_{NN}\theta - \tau S_{LN} + \tau \left(-S_{LN} - g'(N)S_{NN} \right) - S_{NN} \left(\theta - \tau g'(N) \right) \right) - \lambda N \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= \tau H + \theta N - R \end{aligned}$$

Solving these first order conditions for τ and θ give the optimal tax policy expressions:

$$\tau = \frac{R(g'(N)S_{NN}N + S_{NN}H + S_{LN}N)}{N^2S_{LL} + 2N^2g'(N)S_{LN} + N^2g'(N)^2S_{NN} + 2NS_{LN}H + 2Ng'(N)S_{NN}H + S_{NN}H^2} \quad (\text{A-22})$$

$$\theta = \frac{R(2g'(N)S_{LN}N + S_{LN}H + g'(N)^2S_{NN}N + g'(N)S_{NN}H + NS_{LL})}{N^2S_{LL} + 2N^2g'(N)S_{LN} + N^2g'(N)^2S_{NN} + 2NS_{LN}H + 2Ng'(N)S_{NN}H + S_{NN}H^2} \quad (\text{A-23})$$