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Sunk Cost and Entry

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Abstract. The usual mechanisms by which sunk costs are said to affect entry are through raising the expected average cost of an entrant, relative to that of incumbents. I show that in standard models and in the absence of risk premia imposed by financial markets on an entrant's cost of capital, sunk costs may make entry unprofitable because of their effect on the post-entry unit costs of incumbents.

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JEL Classifications: L40, O31, O38.

I. Introduction

Sunk costs are typically thought to constitute a barrier to entry because of the impact of sunk investments on the decisions of a potential entrant. Thus Baumol et al. (1982, p. 291) write that "The need to sink costs can be a barrier to entry" and say of sunk costs that "Their role as barriers to entry depends on the risk to which they subject the entrant." In one formalization (1982, p. 299) it appears that the risk in question affects an entrant's expected rental rate of capital services, and that "Any such difference in rental rates must be attributed to the possibility that the entrant may find himself forced to depreciate his capital fully during the disequilibrium period."

Baumol et al. (1983, p. 494)[1] similarly highlight the impact of cost sunkenness on the unit cost of entrants (emphasis added):[2]

> Suppose that a unit of capital purchased at a price of β per unit could be sold or utilized elsewhere after T for a salvage value of $\alpha \leq \beta$. ... If $\alpha < \beta$, the entrant can offer a price only as low as its average cost over T, and this will be somewhat higher than the true economic average cost because

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[1] Discussing Schwartz and Reynolds (1983).

[2] MacLeod (1987, p. 143) distinguishes between M- or malleability-sunk costs (resale price of capital goods less than purchase price) and T-sunk costs (possibility of exit available only at the end of time intervals of length T).

some of the economic value of its plant may be lost as a result of exit or of oligopolistic rivalry that takes place after the initial contracts expire at time T. Then, incumbents can charge a price above true average cost without inducing entry, because of the sunk economic costs *required of entrants*.

Here I show that in standard models sunk costs may make entry unprofitable because of the impact of entry, in the presence of sunk investment, on the unit cost of incumbents, not on the unit cost of entrants. In most oligopoly models, the equilibrium output of an incumbent or incumbents is less in the post-entry market than in the pre-entry market. In such cases, incumbents will need a smaller stock of physical capital after entry than before entry. If no part of investment is sunk, an incumbent can sell off unneeded capital. If the extent to which investment is sunk is sufficiently great – in a sense that will be made precise below – then after entry, incumbents will carry excess capacity until it is eliminated by physical depreciation. During this period the rental cost of capital services, for an incumbent, is zero. This lowers incumbents' unit cost, reduces an entrant's expected profit and may, for some values of fixed cost, make entry unprofitable where it would be profitable if investments were not sunk.

Section II derives results for a market in which capital does not depreciate physically and cannot be resold. Section III allows for physical depreciation of capital, and Section IV allows for both physical depreciation and resale. Section V concludes.

II. No Depreciation, No Resale

A simple example will bring out the intuition behind the result. Consider a market with a linear inverse demand curve

$$p = a - Q. \tag{1}$$

The market is initially supplied by a single firm, using technology described by the fixed-coefficient Stone–Geary production function:[3]

$$q = \begin{cases} \min\left(\dfrac{K - \overline{K}}{a_K}, \dfrac{L - \overline{L}}{a_L}\right) & K \geq \overline{K}, L \geq \overline{L} \\ 0 & \text{otherwise} \end{cases} . \tag{2}$$

Suppose also that capital is not subject to physical depreciation and cannot be resold. For simplicity, assume that the purchase price of capital and the wage rate of labor are constant over time.

The present-discounted value of a monopoly supplier in a discrete time model (who, facing unchanging demand conditions, will produce the same output in each period) is

[3] None of the results obtained here depend on the assumption of a fixed coefficient production function, which is made for simplicity.

$$V_m = -p^k\left(\overline{K} + a_K q\right) + \left[(a-q)\,q - w\left(\overline{L} + a_L q\right)\right]$$
$$\times\left[\frac{1}{1+r} + \frac{1}{(1+r)^2} + \dots\right]$$
$$= \frac{\left(a - wa_L - rp^k a_K - q\right)q - \left(rp^k\overline{K} + w\overline{L}\right)}{r}$$
$$= \frac{(a - c_H - q)\,q - F_H}{r}. \tag{3}$$

The monopolist makes an initial purchase of capital stock and hires labor period by period. rp^k is the rental cost of capital services. Fixed cost is

$$F_H = rp^k\overline{K} + w\overline{L}, \tag{4}$$

and marginal cost/average variable cost per unit of output is

$$c_H = rp^k a_K + wa_L. \tag{5}$$

Monopoly output is

$$q_m = \frac{1}{2}(a - c_H) = \frac{1}{2}\left(a - wa_L - rp^k a_K\right), \tag{6}$$

and the incumbent purchases capital stock

$$K_m = \overline{K} + \frac{1}{2}a_K(a - c_H). \tag{7}$$

We now determine conditions under which it will be unprofitable for a second firm with access to the same technology to come into the market, and relate that condition to the sunk nature of investment.[4]

Suppose that if entry occurs, the entrant and the incumbent compete as Cournot oligopolists. If entry occurs, the entrant and the incumbent face unchanging conditions in each post-entry period. Value-maximizing output levels will not change over time. Let the subscript 1 denote the incumbent, the subscript 2 the entrant. The entrant picks q_2 to maximize its present-discounted value

$$V_2 = \frac{(a - c_H - q_1 - q_2)\,q_2 - F_H}{r}, \tag{8}$$

which is derived in the same way as (3).

[4] By extension, one can find conditions on fixed cost that make positive output by two firms profitable, with entry of a third firm rendered unprofitable by the impact entry would have on the rental cost of capital for the two incumbents, and so on for larger numbers of incumbents.

In general, and in particular for Cournot markets, the equilibrium duopoly output of a single firm is less than monopoly output. In the post-entry market, the incumbent has surplus and non-depreciating capital that cannot be resold and bears a shadow value of zero. In the post-entry market, the incumbent picks q_1 to maximize

$$V_1 = \frac{(a - c_L - q_1 - q_2)\, q_1 - F_L}{r}, \tag{9}$$

for marginal and fixed cost

$$c_L = wa_L \tag{10}$$

and

$$F_L = w\overline{L} \tag{11}$$

respectively.

It is a straightforward exercise to find Cournot equilibrium outputs and present-discounted values in the post-entry market. Provided the entrant's equilibrium output is nonnegative,[5] the entrant's equilibrium value is

$$V_2 = \frac{1}{r}\left[\frac{1}{9}\left(a - wa_L - 2rp^k a_K\right)^2 - F_H\right], \tag{12}$$

and if (*No-entry condition 1*)

$$\frac{1}{9}\left(a - wa_L - 2rp^k a_K\right)^2 < F_H = rp^k\overline{K} + w\overline{L}, \tag{13}$$

entry will be unprofitable.

If (13) holds, it is a subgame perfect equilibrium in a simple entry game[6] for the incumbent to purchase capital stock (7) and for potential entrants to stay out of the market. Further, if

$$\frac{1}{9}\left(a - rp^k a_K - wa_L\right)^2 \geq F_H > \frac{1}{9}\left(a - 2rp^k a_K - wa_L\right)^2 \tag{14}$$

(where the term on the left is gross duopoly profit if capital can be resold at the full purchase price), entry that would be profitable if investments were not sunk is

[5] If $a - wa_L - 2rp^k a_K \leq 0$, the entrant's equilibrium output is zero. By analogy with the innovation literature, one might refer to this as the case of drastic sunk costs.

[6] That is, at the first decision node the incumbent buys capital, at the second decision node the potential entrant decides whether or not to come into the market. If entry occurs, firms compete as Cournot duopolists. If entry does not occur, the potential entrant's payoff is zero, the incumbent is a monopolist.

unprofitable because of the sunk nature of the incumbent's investment. For some ranges of fixed cost, sunk investments block entry because when costs are completely and irrevocably sunk, as in this example, entry makes the shadow value of the incumbent's capital stock equal to zero.[7]

III. Depreciation, No Resale

In the example of the previous section, suppose now that capital physically depreciates at a constant rate δ, $0 \leq \delta \leq 1$, keeping all other assumptions unchanged.[8]

A monopoly supplier maximizes its present-discounted value, now

$$V_m = -p^k\left(\overline{K} + a_K q\right) + \frac{1}{1+r}\left[(a-q)\,q - w\left(\overline{L} + a_L q\right)\right]$$
$$+ \frac{1}{1+r}\left\{-\delta p^k\left(\overline{K} + a_K q\right) + \frac{1}{1+r}\left[(a-q)\,q - w\left(\overline{L} + a_L q\right)\right]\right\} + \ldots$$
$$= \frac{\left[a - wa_L - (r+\delta)\,p^k a_K - q\right]q - \left[w\overline{L} + (r+\delta)\,p^k\overline{K}\right]}{r}$$
$$= \frac{(a - c_H - q)\,q - F_H}{r}, \qquad (15)$$

redefining marginal and fixed cost as

$$c_H = wa_L + (r+\delta)\,p^k a_K \qquad (16)$$

and

$$F_H = w\overline{L} + (r+\delta)\,p^k\overline{K} \qquad (17)$$

respectively.

Monopoly output per period in this extended model is

$$q_m = \frac{1}{2}(a - c_H) = \frac{1}{2}\left[a - wa_L - (r+\delta)\,p^k a_K\right] \qquad (18)$$

and the incumbent purchases capital stock

$$K_m = \overline{K} + \frac{1}{2}a_K(a - c_H) \qquad (19)$$

[7] We exclude strategic capacity choice on the part of the incumbent (a factor emphasized by Eaton and Lipsey, 1980) by assumption.

[8] Eaton and Lipsey (1981) make the depreciation rate a choice variable.

at the start of the first period.

Once again, we seek a condition under which entry will be unprofitable. The derivation is a tedious evaluation of Kuhn–Tucker conditions, and is outlined in Appendix A.

It is necessary to distinguish up to three time periods in the post-entry market:

- The first n_A periods, when incumbent's equilibrium output per period is

$$q_H = \frac{1}{3}\left[a - wa_L + (r+\delta)\, p^k a_K\right] \tag{20}$$

  and the incumbent can produce q_H without purchasing capital;
- periods n_B and after, when the incumbent's equilibrium output per period is

$$q_D = \frac{1}{3}\left[a - wa_L - (r+\delta)\, p^k a_K\right] \tag{21}$$

  and the incumbent must make positive investment each period to maintain the capital stock necessary to produce output q_D; and,
- if $n_B > n_A + 1$, periods $n_A + 1, n_A + 2, \ldots, n_B - 1$, during which the incumbent produces just enough output to fully utilize the existing capital stock, but does not purchase capital.

During periods of the third type, if they occur, the shadow value of a unit of capital stock is positive, but less than the purchase price of a unit of capital. How many such capital-constrained periods occur, if any, depends on the rate of depreciation δ.

We give results for the case $n_B = n_A + 1$.[9] Let q_L denote the entrant's equilibrium output per period during the first n_A periods, q_D the entrant's equilibrium per period output thereafter. ($q_D > q_L$; explicit expressions are given in Appendix A.) The entrant's equilibrium value is

$$V_2 = \frac{1}{r}\left\{q_D^2 - \left[1 - \frac{1}{(1+r)^{n_B}}\right]\left(q_D^2 - q_L^2\right) - F_H\right\}. \tag{22}$$

Then if (*No-entry condition 2*)

$$q_D^2 - \left[1 - \frac{1}{(1+r)^{n_B}}\right]\left(q_D^2 - q_L^2\right) < F_H, \tag{23}$$

entry is unprofitable. If (23) holds, it is a subgame perfect equilibrium of an entry game for the incumbent to install capital stock (19) and for potential entrants to stay out of the market. If

$$q_D^2 - \left[1 - \frac{1}{(1+r)^{n_B}}\right]\left(q_D^2 - q_L^2\right) < F_H \le q_D^2, \tag{24}$$

[9] Results for the case $n_B > n_A + 1$ are qualitatively similar to those obtained when $n_B > n_A + 1$, and are discussed in an appendix that is available on request from the author.

it is the sunk nature of the incumbent's investment that renders entry unprofitable. If entry occurs, the incumbent's rental cost of capital services is zero until the incumbent's inherited capital depreciates sufficiently, and during this depreciation period, the entrant's output and profit are smaller than they would be if investments were not sunk.

IV. Depreciation and Resale

Now extend the model of Section III to allow firms to resell capital, if they find it profitable to do so, at price αp^k per unit, with $0 \leq \alpha \leq 1$, keeping all other aspects of the specification unchanged. α describes the working of markets for used capital assets. One expects that α will be less than 1 when physical assets are highly specific to particular production processes. "Lemon problems" (Akerlof, 1970) may have the same effect.

There are two types of solutions (proofs are outlined in Appendix A).

No-entry condition 3: If $\alpha \geq (1 - \delta) / (1 + r)$, the incumbent sells unneeded capital at the moment of entry, and never carries excess capacity. The entrant's value is

$$V_2 = \frac{q_D^2 - F_H}{r} - \frac{1}{1+r}\left(q_D^2 - q_{21}^2\right), \tag{25}$$

for

$$q_{21} = q_D - \frac{1}{3}(1+r)(1-\alpha)\, p^k a_K. \tag{26}$$

If

$$\frac{q_D^2 - F_H}{r} - \frac{1}{1+r}\left(q_D^2 - q_{21}^2\right) < 0, \tag{27}$$

entry is blocked. If

$$0 \leq \frac{q_D^2 - F_H}{r} \leq \frac{1}{1+r}\left(q_D^2 - q_{21}^2\right) \tag{28}$$

entry is blocked that would be profitable if the incumbent's costs were not sunk.

To interpret the condition

$$\alpha \geq \frac{1-\delta}{1+r}, \tag{29}$$

note that αp^k is what a firm can sell a unit of excess capital for at the start of the period entry occurs, and after one period invested at interest rate r, this rises to $(1+r)\,\alpha p^k$. If the firm does not sell a unit of excess capital at the start of the period, the value remaining at the end of the period is $(1-\delta)\,p^k$. If $(1+r)\,\alpha p^k \geq (1-\delta)\,p^k$, the firm is at least as well off selling excess capital at the start of the period rather than holding it and allowing it to depreciate.

If $\alpha < (1-\delta)\,/\,(1+r)$, the incumbent does not resell capital in the event of entry, but lets its capital depreciate until positive investment is necessary to produce the value-maximizing output. This reduces to the case considered in Section III.

In industries where physical capital is relatively long-lived, the condition (29) will fail if α is only moderately less than 1. If $\delta = r = 0.1$, (29) fails for $\alpha < 9/11 \approx 0.82$, so that a firm with an excess supply of physical capital would hold excess capital rather than resell it even if the resale price were 80% of the purchase price.

This observation is not without policy implications. In industries where (29) does not hold, a post-entry price that is characterized as predatory because it is below an average or average variable cost figure that is derived valuing the rental cost of capital services at the level that is appropriate when capital is not in excess supply may well be above economic marginal or average cost if capital is in excess supply and the rental cost of capital services is set equal to its economic value of zero. In the same way, what are often interpreted as price wars during cyclical downturns in industries where sunk cost is an important part of total cost may instead be price-taking behavior down to a minimum economic value of average variable cost that is computed with the rental cost of capital services equal to zero.

V. Conclusion

Sunk cost is a barrier to entry. It is not necessary to rely on the cost of financial capital to entrants with unknown ability, to risk aversion on the part of entrants, or to the possibility of strategic behavior by incumbents, to reach this conclusion. When some portion of incumbents' costs are sunk, entry renders some part of incumbents' capital stocks superfluous and reduces incumbents' rental cost of capital services, either to zero or to the resale value of capital. With lower economic costs, incumbents produce more, all else equal, than they would if costs were not sunk, for any output level of an entrant. The entrant's equilibrium output is less than would be the case if incumbents' costs were not sunk. For some ranges of fixed cost, entry that would be profitable if incumbents' costs were not sunk may be rendered unprofitable.

Appendix A[10]

ENTRY CONDITION 2

Let q_{1t} be the incumbent's output in period t, q_{2t} the entrant's output in period t, $I_{1t} \geq 0$ the incumbent's purchases of capital in period t, and λ_{1t} the Lagrangian multiplier associated with the incumbent's capital stock constraint in period t.

If entry occurs, then at the start of the first period after entry, the incumbent's problem is to maximize (with $K_{10} = K_m$)

$$V_1 = \sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}} \left\{ \mu_{1t} \left[(1-\delta) K_{1,t-1} + I_{1t} - K_{1t} \right] - p^k I_{1t} \right.$$

$$+ \frac{1}{1+r} \left[(a - q_{1t} - q_{2t}) q_{1t} - \left(w a_L q_{1t} + w \overline{L} \right) \right]$$

$$\left. + \lambda_{1t} \left(K_{1t} - \overline{K} - a_K q_{1t} \right) \right\}. \quad \text{(A1)}$$

λ_{1t} is the Lagrangian multiplier for the capital input constraint in period t. μ_{1t} is the Lagrangian multiplier for the identity that defines the capital stock in period t.

Kuhn–Tucker first-order conditions for this constrained optimization problem are

$$(1+r)^{t-1} \frac{\partial V_1}{\partial \mu_{1t}} = (1-\delta) K_{1,t-1} + I_{1t} - K_{1t} = 0 \quad \text{(A2)}$$

$$(1+r)^{t-1} \frac{\partial V_1}{\partial I_{12}} = \mu_{1t} - p^k \leq 0 \quad I_{1t} \left(\mu_{1t} - p^k \right) \equiv 0 \quad I_{1t} \geq 0 \quad \text{(A3)}$$

$$(1+r)^{t} \frac{\partial V_1}{\partial q_{1t}} = a - w a_L - \lambda_{1t} a_K - 2 q_{1t} - q_{2t} = 0 \quad \text{(A4)}$$

$$(1+r)^{t-1} \frac{\partial V_1}{\partial K_{1t}} = -\mu_{1t} + \frac{\lambda_{1t} + (1-\delta) \mu_{1,t+1}}{1+r} = 0 \quad \text{(A5)}$$

$$(1+r)^{t} \frac{\partial V_1}{\partial \lambda_{1t}} = K_{1t} - \overline{K} - a_K q_{1t} \geq 0 \quad \lambda_{1t} \left(K_{1t} - \overline{K} - a_K q_{1t} \right) \quad \lambda_{1t} \geq 0. \quad \text{(A6)}$$

The entrant maximizes (with $K_{20} = 0$)

$$V_2 = \sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}} \left\{ \mu_{2t} \left[(1-\delta) K_{2,t-1} + I_{2t} - K_{2t} \right] - p^k I_{2t} \right.$$

[10] A more detailed appendix is available on request from the author.

$$+ \frac{1}{1+r}\left[(a - q_{1t} - q_{2t})\, q_{2t} - \left(wa_L q_{2t} + w\overline{L}\right)\right]$$

$$+ \lambda_{2t}\left(K_{2t} - \overline{K} - a_K q_{2t}\right)\Bigg\}. \tag{A7}$$

μ_{2t} is the Lagrangian multiplier for the identity that defines the capital stock in period t. λ_{2t} is the Lagrangian multiplier for the capital input constraint in period t.

Kuhn–Tucker first-order conditions for the entrant's value maximization problem are

$$(1+r)^{t-1}\frac{\partial V_2}{\partial \mu_{22}} = (1-\delta)\, K_{2,t-1} + I_{2t} - K_{2t} = 0 \tag{A8}$$

$$(1+r)^{t-1}\frac{\partial V_2}{\partial I_{22}} = \mu_{2t} - p^k \leq 0 \quad I_{2t}\left(\mu_{2t} - p^k\right) = 0 \quad I_{2t} \geq 0 \tag{A9}$$

$$(1+r)^{t-1}\frac{\partial V_2}{\partial K_{2t}} = -\mu_{2t} + \frac{\lambda_{2t} + (1-\delta)\,\mu_{2,t+1}}{1+r} = 0 \tag{A10}$$

$$(1+r)^{t}\frac{\partial V_2}{\partial q_{2t}} = a - wa_L - \lambda_{2t}a_K - q_{1t} - 2q_{2t} = 0 \tag{A11}$$

$$(1+r)^{t}\frac{\partial V_2}{\partial \lambda_{2t}} = K_{2t} - \overline{K} - a_K q_{2t} \geq 0 \quad \lambda_{2t}\left(K_{2t} - \overline{K} - a_K q_{2t}\right) = 0 \quad \lambda_{2t} \geq 0. \tag{A12}$$

The nature of the solution is driven by the observation that while the incumbent's capital stock is in excess supply, its rental cost of capital services (λ_{1t}) is zero; the entrant never maintains excess capital and always has a positive rental cost of capital services.

During the first n_A periods, the incumbent's inherited capital stock permits it to produce $q_H = \frac{1}{3}\left[a - wa_L + (r+\delta)\, p^k a_K\right]$ per period without purchasing capital. During periods n_B and after, the incumbent must invest to maintain a capital stock sufficient to produce $q_D = \frac{1}{3}\left[a - wa_L - (r+\delta)\, p^k a_K\right]$.

n_A is the greatest integer in the value n which, ignoring integer constraints, identifies the last time at which the incumbent's capital stock is sufficient to produce its equilibrium output if its rental cost of capital services is zero:

$$(1-\delta)^n\left(\overline{K} + a_K q_m\right) = \overline{K} + \frac{1}{3}a_K q_H. \tag{A13}$$

n_B is the least integer that is greater than the value n which, ignoring integer constraints, identifies the moment at which the incumbent's capital stock is sufficient to product its equilibrium output if its rental cost of capital services takes the value that applies when it makes positive investment:

$$(1-\delta)^n \left(\overline{K} + a_K q_m\right) = \overline{K} + \frac{1}{3} a_K q_D. \tag{A14}$$

For the first n_A periods, equilibrium values are[11]

$$\begin{pmatrix} q_{1t} \\ q_{2t} \end{pmatrix} = \begin{pmatrix} q_H \\ q_L \end{pmatrix} = \frac{1}{3} \begin{pmatrix} a - wa_L + (r+\delta)\, p^k a_K \\ a - wa_L - 2(r+\delta)\, p^k a_K \end{pmatrix} \tag{A15}$$

$$\begin{pmatrix} \lambda_{1t} \\ \lambda_{2t} \end{pmatrix} = \begin{pmatrix} 0 \\ (r+\delta)\, p^k \end{pmatrix} \tag{A16}$$

$$\begin{pmatrix} \mu_{1t} \\ \mu_{2t} \end{pmatrix} = \begin{pmatrix} \left(\frac{1-\delta}{1+r}\right)^{n_A+1-t} \\ 1 \end{pmatrix} p^k \tag{A17}$$

$$\begin{pmatrix} I_{11} \\ I_{21} \end{pmatrix} = \begin{pmatrix} 0 \\ \overline{K} + a_K q_{20} \end{pmatrix} \tag{A18}$$

$$\begin{pmatrix} I_{1t} \\ I_{2t} \end{pmatrix} = \begin{pmatrix} 0 \\ \delta\left(\overline{K} + a_K q_{2t}\right) \end{pmatrix}, t = 2, 3, ..., n_A, \tag{A19}$$

after which

$$q_{1t} = q_{2t} = q_D = \frac{1}{3}(a - c_H) \tag{A20}$$

$$\lambda_{1t} = \lambda_{2t} = (r+\delta)\, p^k \tag{A21}$$

$$\mu_{1t} = \mu_{2t} = p^k \tag{A22}$$

$$\begin{pmatrix} I_{1,n_A+1} \\ I_{2,n_A+1} \end{pmatrix} = \begin{pmatrix} \overline{K} + a_K q_D - (1-\delta)^{n_A+1} K_m \\ \overline{K} + a_K q_D - (1-\delta)\left(\overline{K} + a_K q_L\right) \end{pmatrix} \tag{A23}$$

$$I_{1t} = I_{2t} = \delta\left(\overline{K} + a_K q_{2t}\right),\ t = n_A + 2, n + 3, ... \tag{A24}$$

The entrant's equilibrium value for the case $n_B = n_A + 1$ is found by substituting equilibrium values in the entrant's maximand (noting that the expressions multiplied by Lagrangian multipliers drop out by the Kuhn–Tucker conditions and

[11] Assuming $a - wa_L - 2(r+\delta)\, p^k a_K > 0$; see footnote 5.

using the first-order conditions for output to express the entrant's gross profit per period as the square of its output). This leads to (22).

ENTRY CONDITION 3

Let $I_{it} \geq 0$ be investment by firm i in period t, $J_{it} \geq 0$ sales of capital by firm i in period t. The Lagrangian that describes the incumbent's constrained optimization problem, from the moment of entry, is, with $K_{10} = K_m$ (see (19)),

$$V_1 = \sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}} \Big\{ \mu_{1t} \left[(1-\delta) K_{1,t-1} + I_{1t} - J_{1t} - K_{1t} \right] - p^k I_{1t} + \alpha p^k J_{1t} + \frac{1}{1+r} \left[(a - q_{1t} - q_{2t}) q_{1t} - \left(w a_L q_{1t} + w\overline{L} \right) \right] + \lambda_{1t} \left(K_{1t} - \overline{K} - a_K q_{1t} \right) \Big\}. \tag{A25}$$

μ_{1t} is the Lagrangian multiplier for the identity that defines the capital stock in period t. λ_{1t} is the Lagrangian multiplier for the capital input constraint in period t.

The Kuhn–Tucker first-order conditions for period t are:

$$(1+r)^{t-1} \frac{\partial V_1}{\partial \mu_{1t}} = (1-\delta) K_{1,t-1} + I_{1t} - J_{1t} - K_{1t} = 0. \tag{A26}$$

$$(1+r)^{t-1} \frac{\partial V_1}{\partial I_{1t}} = \mu_{1t} - p^k \leq 0 \quad I_{1t} \left(\mu_{1t} - p^k \right) = 0 \quad I_{1t} \geq 0 \tag{A27}$$

$$(1+r)^{t-1} \frac{\partial V_1}{\partial J_{1t}} = -\mu_{1t} + \alpha p^k \leq 0 \quad J_{1t} \left(-\mu_{1t} + \alpha p^k \right) = 0 \quad J_{1t} \geq 0 \tag{A28}$$

$$(1+r)^{t} \frac{\partial V_1}{\partial q_{1t}} = a - 2q_{11} - q_{21} - w a_L - \lambda_{1t} a_K = 0 \tag{A29}$$

$$(1+r)^{t} \frac{\partial V_1}{\partial K_{1t}} = -\mu_{1t} + \frac{\lambda_{1t} + (1-\delta) \mu_{1,t+1}}{1+r} = 0 \tag{A30}$$

$$(1+r)^{t} \frac{\partial V_1}{\partial \lambda_{1t}} = K_{1t} - \overline{K} - a_K q_{1t} \geq 0 \quad \lambda_{1t} \left(K_{1t} - \overline{K} - a_K q_{1t} \right) = 0 \quad \lambda_{1t} \geq 0. \tag{A31}$$

The entrant maximizes, with $K_{20} = 0$ and $J_{21} = 0$,

$$V_2 = \sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}} \Big\{ \mu_{2t} \left[(1-\delta) K_{2,t-1} + I_{2t} - J_{2t} - K_{2t} \right] - p^k I_{2t} + \alpha p^k J_{2t}$$

$$+ \frac{1}{1+r} \left[(a - q_{1t} - q_{2t}) q_{2t} - \left(w a_L q_{2t} + w\overline{L} \right) \right] + \lambda_{2t} \left(K_{2t} - \overline{K} - a_K q_{2t} \right) \Big\}.$$

μ_{2t} is the Lagrangian multiplier for the identify that defines the capital stock in period t. λ_{2t} is the Lagrangian multiplier for the capital input constraint in period t.

The Kuhn–Tucker first-order conditions for period t are

$$(1+r)^{t-1} \frac{\partial V_2}{\partial \mu_{2t}} = (1-\delta) K_{2,t-1} + I_{2t} - J_{2t} - K_{2t} = 0 \tag{A32}$$

$$(1+r)^{t-1} \frac{\partial V_2}{\partial I_{2t}} = \mu_{2t} - p^k \leq 0 \quad I_{2t} \left(\mu_{2t} - p^k \right) = 0 \quad I_{2t} \geq 0 \tag{A33}$$

$$(1+r)^{t-1} \frac{\partial V_2}{\partial J_{2t}} = \alpha p^k - \mu_{2t} \leq 0 \quad J_{2t} \left(\alpha p^k - \mu_{2t} \right) = 0 \quad J_{2t} \geq 0 \tag{A34}$$

$$(1+r)^{t} \frac{\partial V_2}{\partial q_{2t}} = a - 2q_{2t} - q_{2t} - w a_L - \lambda_{2t} a_K = 0 \tag{A35}$$

$$(1+r)^{t-1} \frac{\partial V_2}{\partial K_{2t}} = -\mu_{2t} + \frac{\lambda_{2t} + (1-\delta) \mu_{2,t+1}}{1+r} = 0 \tag{A36}$$

$$(1+r)^{t} \frac{\partial V_2}{\partial \lambda_{2t}} = K_{2t} - \overline{K} - a_K q_{2t} \geq 0$$

$$\lambda_{2t} \left[K_{2t} - \overline{K} - a_K q_{2t} \right] = 0 \qquad \lambda_{2t} \geq 0. \tag{A37}$$

We sketch the steps to find equilibrium values. The entrant buys capital in every period; its rental cost of capital services is $(r + \delta)\, p^k$.

If the incumbent sells capital in the first period, and thereafter buys,

$$J_{11} > 0 \Rightarrow \mu_{11} = \alpha p^k \qquad I_{12} > 0 \Rightarrow \mu_{12} = p^k. \tag{A38}$$

Then from

$$-\mu_{11} + \frac{\lambda_{11} + (1-\delta) \mu_{12}}{1+r} = 0 \tag{A39}$$

we find

$$\lambda_{11} = [(1+r)\,\alpha - (1-\delta)]\,p^k = [(r+\delta) - (1+r)\,(1-\alpha)]\,p^k < (r+\delta)\,p^k. \tag{A40}$$

The incumbent's rental cost of capital in the first period is less than the entrant's rental cost of capital in the first period.

A consistency condition for this solution to apply is $\lambda_{11} \geq 0$; this leads to (29). Knowing the rental costs of capital in the first period and that both firms purchase capital in subsequent periods permits working out equilibrium characteristics in all periods.

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