KRANNERT SCHOOL OF MANAGEMENT

Purdue University
West Lafayette, Indiana

Corporate Finance and Monetary Policy

By

Cathy Zhang
Guillaume Rocheteau
Randall Wright

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Guillaume Rocheteau
University of California, Irvine

Randall Wright
University of Wisconsin, Madison; FRB Chicago and FRB Minneapolis

Cathy Zhang
Purdue University

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Abstract

We develop a model where entrepreneurs can finance random investment opportunities using trade credit, bank-issued assets, or money. They search for funding in an over-the-counter market where the terms of the contract, including the interest rate, loan size, and down payment, are negotiated subject to pledgeability constraints. The theory has implications for the cross-sectional distribution of corporate loans and interest rates, pass through from nominal to real rates, and the transmission of monetary policy, described by either changes in the money growth rate or open market operations. We also consider the effects of imposing different regulations on banks.

JEL Classification Nos: D83, E32, E51
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1 Introduction

This project integrates modern monetary theory and corporate finance in order to analyze the effects of policy on interest rates and investment. It is commonly thought, and taught, that the central bank influences economic activity through its impact on short-term nominal rates in the Fed Funds market which then gets passed through to the real rates at which individuals can borrow. While perhaps appealing heuristically, it is not easy to model this rigorously. We build on recent advances in the study of money, banking, and asset markets using methods from general equilibrium, search and game theory (see the literature review below). In this context, we analyze the channels through which monetary policy affects firms' demand for liquidity, corporate lending and investment.

An advantage of being more explicit about assets and their liquidity/regulatory roles is that our formulation generates a rich structure of returns, including real and nominal yields on overnight rates in the interbank market, on liquid bonds, on illiquid bonds, and on corporate lending. This is in sharp contrast with many models that have one interest rate, typically interpreted as both the rate set by policy and the rate relevant for investment. We are also explicit in distinguishing different types of policy interventions, including changes in money growth or inflation rates, unanticipated money injections in the Fed Funds market, and bond purchases in the open market. The goal is to show how monetary policy and regulation affect the endogenous yield structure and real investment.

1.1 Preview

In the basic model, entrepreneurs receive random opportunities to invest, but may not be able to get sufficient trade credit from suppliers due to explicit frictions. Hence they may use either retained earnings held in liquid assets (internal finance) or loans from banks that issue short-term liabilities (external finance). See Figure 1. Banks have a comparative advantage in monitoring and enforcing repayment, and in equilibrium their liabilities can serve as payment instruments. Realistically, our market for bank loans is an over-the-counter (OTC) market, featuring search and bargaining. Loan contracts are negotiated by entrepreneurs and banks, in terms of the interest rate, loan size, and down payment. Due to limited commitment/enforcement, only a fraction of investment returns are pledgeable. Additionally, finding someone to extend a loan is a time-consuming process and not always successful. Hence, we model the credit market as having both an intensive margin – the size of loans – and extensive margin – the ease of obtaining a loan.[1]

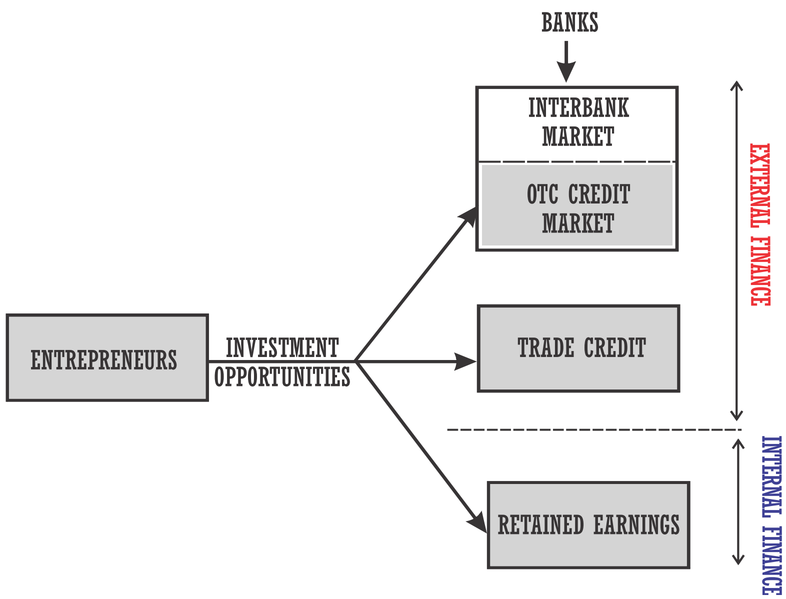

Figure 1: Sketch of the model

With only external finance, the efficient level of investment can be achieved if returns are sufficiently pledgeable. When this is not the case, loan contracts depend on pledgeability, bargaining power, and technology. With heterogeneity among entrepreneurs in terms of bargaining power, the model predicts a negative correlation between the corporate lending rate and loan size in the cross section; alternatively, with heterogeneity in terms of pledgeability, there is no correlation between these variables. Thus, our model makes precise when and how investment and lending rates are related. When entrepreneurs can obtain credit either directly

[1] To be clear, the concern here is not with firms' choice to issue equity or bonds in order to acquire new capital; we are instead interested in the choice between using retained earnings held in liquid assets, or credit, and in the latter case the choice between bank or trade credit.

from suppliers or from banks, some investment is financed by trade credit and some by bank credit, consistent with evidence (see Section 1.3). We also show how entry into banking affects lending conditions and the impact of policy.

To incorporate a tradeoff between external and internal finance, we let entrepreneurs accumulate outside money, the opportunity cost of which is the nominal interest rate on bonds. Money held by firms has two roles: an insurance function, allowing them to finance more investment internally; and a strategic function, allowing them to negotiate better loans. Consistent with evidence discussed below, firms' money demand increases with idiosyncratic risk and decreases with the pledgeability of output. By lowering the nominal rate, a central bank encourages the holding of liquidity, allowing firms to finance larger investments and get better deals on loans. However, low interest rates reduce banks' margins and their incentives to participate in the credit market, thereby reducing entrepreneurs' access to external funds. Moreover, the ability to self finance raises pledgeable output, and thus creates an amplification mechanism for policy.

The model predicts pass through from the nominal rate set by policy to real rates. An increase in the nominal rate – the opportunity cost of keeping retained earnings liquid – reduces down payments and raises real interest rates. We obtain closed-form expressions for pass through, and emphasize that it does not require nominal rigidities or regulatory restrictions. The extent of pass through depends on frictions in the credit market, bargaining power, and idiosyncratic risk. Real rates are more responsive to policy when banks have more bargaining power and entrepreneurs have better access to lending. The relationship between the policy rate and loan size is nonmonotone, but the fraction of investment financed internally is maximized when the nominal rate is zero. In addition, heterogeneity across entrepreneurs generates differential effects of policy. Consistent with the evidence, an increase in the nominal policy rate has a larger effect on firms that rely more on internal finance, are more capital intensive, and have lower bargaining power. The theory is also consistent with cross-country evidence on the effects of monetary policy on banks' interest margins.

To study open market operations (OMOs), we introduce short-term government bonds, regulatory requirements, and a competitive interbank market where banks trade reserves and bonds overnight. This generates a realistic structure of returns on interbank loans, corporate loans, liquid bonds, and illiquid bonds. Under a strict reserve requirement, an injection of cash in the interbank market raises reserves and promotes lending. Since money is injected in the interbank market, the resulting increase in the price level reduces firms' ability to self finance, which alters the composition of corporate finance and investment. Under a broader requirement satisfied by money or bonds, if the supply of bonds is reduced, their nominal yield can hit zero, and thus the economy can fall into a liquidity trap; if the bond supply is not too low, a permanent increase can lower the loan rate, and increase (decrease) external (internal) finance. We think all these results put monetary policy and its relation to corporate finance in new light, based on explicit microfoundations for the notion of liquidity.

1.2 Related theory literature

We build on the New Monetarist framework surveyed by Nosal and Rocheteau (2011) and Lagos et al. (2016), except we emphasize firms' financing investment, while that work emphasizes households' financing consumption.[2] As in most of those models, we have search frictions, but here they affect credit markets, not capital or goods markets. Recent search-based models of credit in goods markets include Gu et al. (2016) and Lotz and Zhang (2016); we differ by focusing on credit intermediated by banks.[3] Also related is work by Duffie et al. (2005) and Lagos and Rocheteau (2009), who study intermediation in OTC financial markets. Our credit market is more similar to Wasmer and Weil (2004), except we are relatively more explicit

[2] Silveira and Wright (2011) and Chiu et al. (2015) provide a model where firms trade ideas and technologies in decentralized markets. Those environments are quite different, however, even though Chiu et al. (2015) discuss the role of banking as a way to reallocate liquidity, along the lines of Berentsen et al. (2007).

[3] Other work similar in spirit includes Cavalcanti and Wallace (1999), Gu et al. (2013), Donaldson et al. (2016), and Huang (2016), all of which model banking as an endogenous arrangement arising from explicit frictions, and have bank liabilities facilitating third-party transactions.

about frictions, formalize the role of money, have both internal and external finance, and endogenize loan size. Of course, the overall approach is related to the literature following Kiyotaki and Moore (1997), Holmstrom and Tirole (1998, 2011), and Tirole (2006) who similarly emphasize pledgeability.[4]

Bolton and Freixas (2006) also provide a setting for analyzing monetary policy and corporate finance but do not model money – they simply take the interest rate on Treasury bills as a policy instrument. In contrast, we model monetary exchange and credit frictions explicitly to provide foundations for a novel theory of corporate lending and the role of banking. We also generate regulatory motives for banks to hold money and/or bonds, and incorporate an interbank market; this is relevant because we can implement OMOs in the interbank market, which is realistic, and important for certain results.[5] Bernanke et al. (1999) survey the literature on credit frictions and monetary policy in New Keynesian models with nominal rigidities and credit frictions, emphasizing the effects of policy on the cost of borrowing and its amplification through balance-sheet effects.

While we also highlight credit frictions, arising here from limited enforcement and/or commitment, an important difference is our description of an OTC credit market with bilateral meetings and bargaining. This description is realistic, captures credit rationing along both the intensive and extensive margins, allows us to consider the impact of bargaining power, and formalizes the negotiation of loan contracts where outside options depend on monetary policy. Using this approach, we can delve further into the details of the transmission mechanism and show how market structure, search frictions, and bargaining power impact firms' demand for cash, loan contracts, and pass through. Importantly, our results do not require nominal

[4] New Monetarist papers that feature pledgeability considerations include Lagos (2010), Williamson (2012, 2015), Venkateswaran and Wright (2013), Rocheteau and Rodriguez-Lopez (2014), He et al. (2015), and Rocheteau et al. (2015). Relatedly, in finance, see DeMarzo and Fishman (2007) and Biais et al. (2007). Also, while Bernanke et al. (1999) and Holmstrom and Tirole (1998) rationalize limited pledgeability using moral hazard, in a Supplemental Appendix we provide alternative foundations using limited commitment, as in Kehoe and Levine (1993).

[5] Some of these results are similar to restricted participation models, e.g., Alvarez et al. (2002) or Khan and Thomas (2015). However, while we also feature market segmentation, our approach using OTC credit is very different and provides distinct insights on the role of policy.

rigidities or bank regulation, although we also consider the latter in Section 6, and in principle could consider the former, too. Moreover, many effects discussed below are operative even when borrowing constraints are slack and there are no search frictions in the credit market.

1.3 Related empirical literature

Campello (2015) provides a general discussion of the importance of corporate liquidity management. Firms' demand for money is modeled here as a response to idiosyncratic opportunities and limited pledgeability. This is consistent with the findings in, e.g., Sánchez and Yurdagul (2013), who document that in 2011 firms held \$1.6 trillion in liquid assets, defined as short-term investments easily transferable into cash, and explain this by uncertainty over investment opportunities and credit constraints (see also Bates et al. 2009). Our firms use both cash and credit, consistent with the ample evidence in Mach and Wolken (2006). Some businesses also use credit cards, which we (loosely) model by allowing firms to use both bank and trade credit.[6] Our firms use more trade credit when lending at financial institutions tightens, as documented in the data by Petersen and Rajan (1997).

On bank credit in particular, again, we have an intensive margin, capturing loan size, and an extensive margin, capturing the ease of getting a loan. Having both is consistent with evidence discussed in the Joint Small Business Credit Survey (2014).[7] Also, actual credit markets feature price dispersion. Mora (2014), e.g., documents considerable dispersion in loan rates and argues it can be explained by bargaining power. Generally, we think the facts are best understood in the context of information and commitment frictions in models with explicit bilateral interactions between lenders and borrowers. There is also evidence on differential effects of monetary policy across industries. Dedola and Lippi (2005) find the impact of policy is stronger in industries that are more capital intensive (in the model, a

[6] Trade credit was used by 60% of small businesses in 2003, and 40% of all firms use both bank and trade credit (SBA, 2010).

[7] Among survey participants who applied for loans, 33% received what they requested, 21% received less, and 44% were denied.

higher capital share) and have smaller borrowing capacities (in the model, lower pledgeability). See Barth and Ramey (2001) for additional discussion.

There is much empirical work on the importance of the money and credit channels, including Romer and Romer (1990), Ramey (1993), and Bernanke and Gertler (1995). Kashyap et al. (1993) find evidence that tighter monetary policy leads to a shift in firms' mix of external and internal financing, as predicted by the theory presented here. Illes and Lombardi (2013) and Mora (2014) discuss related facts concerning the monetary transmission mechanism. In addition, our model has predictions about banks' net interest margins and how they are affected by policy. Claessens et al. (2016) find interest rate margins (in the model, bank profits) are low when short-term interest rates are low. This is explained in both the data and our theory by borrowers' alternative financing options and modeling their choice explicitly.

2 Environment

Similar to many papers in the New Monetarist literature, each period $t = 1, 2, ...$ is divided into two stages. In the first, there is a competitive market for capital and an OTC market for banking services. In the second, there is a frictionless market where agents settle debts and trade final goods and assets. This background environment is ideal for our purposes because at its core is an asynchronicity between expenditures and receipts, crucial for any analysis of money or credit. To address the issues at hand, we introduce three types of agents, $j = e, s, b$. Type e agents are *entrepreneurs* in need of capital; type s agents are *suppliers* that produce capital; and type b agents are *banks* whose role is discussed below. The measure of entrepreneurs is 1. Given CRS in the production of k, the measure of suppliers is irrelevant. The measure of banks is captured by matching probabilities, as explained below. All agents have linear period utility over a numéraire good c and discount across periods according to $\beta = 1/(1+\rho)$, $\rho > 0$.[8]

[8]All the results go through if agents have period utility $U(c, h)$, where h is labor, as long as U satisfies certain restrictions, e.g., quasi-linearity or homogeneity of degree 1 (again see the

In stage 1, capital is produced by suppliers at unit cost. In stage 2, entrepreneurs transform k acquired in stage 1 into $f(k)$ units of c, where $f(0) = 0$, $f'(0) = \infty$, $f'(\infty) = 0$, and $f'(k) > 0 > f''(k)$ $\forall k > 0$. For simplicity, k fully depreciates at the end of a period. Entrepreneurs face two types of idiosyncratic uncertainty: one related to investment opportunities, as in Kiyotaki and Moore (1997); the other related to financing opportunities, as in Wasmer and Weil (2004). Specifically, in the first stage, each entrepreneur has an investment opportunity with probability λ, in which case he can operate the technology f. Given such an opportunity, he accesses an OTC market where he meets a banker with probability α. (It is a simple extension to also let entrepreneurs meet capital suppliers probabilistically, but it adds little except notation.) Investment opportunities and meeting probabilities are independent. Hence, $\alpha\lambda$ is the probability an entrepreneur has an investment opportunity and access to banking services, while $\lambda(1-\alpha)$ is the probability he has an investment opportunity but not access to banking.

A key ingredient concerns the enforcement of debt. Consider an entrepreneur with k units of capital, and liabilities $\ell_b \geq 0$ and $\ell_s \geq 0$ owed to banks and suppliers. Post production, he can renege and abscond with part of the output, but creditors have partial recourse. In general, suppose banks can recover $\chi_b f(k)$ and suppliers $\chi_s f(k)$, with $\chi = \chi_b + \chi_s \leq 1$ representing the fraction of output that is pledgeable. Here χ_j is a primitive capturing properties of output and capital, like portability and tangibility, plus institutions including the legal system.[9] Limited pledgeability generates a demand for outside liquidity, modeled as fiat money, or inside liquidity, modeled as short-term bank liabilities. The money supply evolves according to $A_{m,t+1} = (1+\pi) A_{m,t}$, where π is the rate of monetary expansion (contraction if $\pi < 0$) implemented by lump sum transfers (taxes). The price of money in terms of numéraire is $q_{m,t}$, and in stationary equilibrium $q_{m,t} = (1+\pi)q_{m,t+1}$, so π is inflation.

above-mentioned surveys). We choose to not include labor, and make capital the only factor of production, so it is clear how firms accumulate assets out of retained earnings. One reason to have h in some contexts is to slacken the constraint $c \geq 0$, but here that never binds in steady state.

[9] However, we can also derive it from information and commitment frictions. Under public monitoring, we show in the Supplemental Appendix that an entrepreneur can borrow up to the expected discounted value of his future profit stream.

As standard, we impose $\pi > \beta - 1$.

Banks issue intraperiod liabilities in stage 1 and redeem them in stage 2. We exogenously impose commitment to redemption, but it can also arise endogenously: as in Gu et al. (2013), e.g., if banks are patient, connected and monitored enough, they do not default lest they lose their charter. Also, we emphasize that although bank liabilities are called notes, because they constitute a promise to pay the bearer, the theory applies equally well to demand deposits, where a check constitutes instructions to a banker to pay the party indicated. Under either interpretation, it is convenient to assume bank liabilities are perfectly recognizable within a period, but can be counterfeited in subsequent periods. This assumption merely precludes liabilities circulating across periods, which is not crucial, but slightly eases the presentation.[10]

There is also a fixed supply A_g of one-period government bonds that in stage 2 pay to the bearer 1 unit of numéraire, although nothing important changes if they instead pay off in dollars. These bonds are not pledgeable and cannot be used as a medium of exchange: they are book-keeping entries, not tangible assets that can pass between agents (although we can make bonds partially pledgeable, as in Williamson 2012 or Rocheteau et al. 2015, we want to emphasize instead a regulatory motive for holding them). The price of a newly-issued bond in stage 2 is q_g, its real return is $r_g = 1/q_g - 1$, and its nominal return is $i_g = (1+\pi)/q_g - 1$. Banks can trade money and bonds in a competitive interbank market, where $\hat{q}_g$ is the price of bonds and $\hat{q}_m$ the price of cash. Trades in this market are financed with intraperiod credit, as with overnight loans in the Fed Funds market. The interbank market plays no role, however, until regulatory requirements are introduced in Section 6.

3 Preliminaries

We now derive some general properties of agents' decision problems. Consider an entrepreneur at the beginning of stage 2 with k units of capital and financial wealth

[10] For detailed analyses of counterfeiting, recognizability, and liquidity, see Nosal and Wallace (2007), Lester et al. (2012) and Li et al. (2012).

ω denominated in numéraire. Financial wealth includes real balances, a_m, plus government bonds, a_g, minus debts, ℓ_b and ℓ_s. His value function satisfies

$$W^e(k,\omega) = \max_{c,\hat{a}_m,\hat{a}_g} \{c + \beta V^e(\hat{a}_m, \hat{a}_g)\} \text{ st } c = f(k) + \omega + T - (1+\pi)\hat{a}_m - q_g\hat{a}_g,$$

where T denotes transfers minus taxes and $V^e(\hat{a}_m, \hat{a}_g)$ is the continuation value in stage 1 with a new portfolio $(\hat{a}_m, \hat{a}_g)$. The constraint indicates the change in financial wealth, $(1+\pi)\hat{a}_m + q_g\hat{a}_g - \omega$, is covered by retained earnings, $f(k)+T-c$. Eliminating c using the constraint, we get

$$W^e(k,\omega) = f(k) + \omega + T + \max_{\hat{a}_m,\hat{a}_g \geq 0} \{-(1+\pi)\hat{a}_m - q_g\hat{a}_g + \beta V^e(\hat{a}_m, \hat{a}_g)\}.$$

Hence, W^e is linear in wealth, and the choice of $(\hat{a}_m, \hat{a}_g)$ is independent of (k,ω). Similarly, W^j is linear in wealth and $(\hat{a}_m, \hat{a}_g)$ is independent of ω for $j = s, b$.

Consider next the problem of a supplier in the capital market,

$$V^s(\hat{a}_m, \hat{a}_g) = \max_{k \geq 0} \{-k + W^s(\hat{a}_m + \hat{a}_g + q_k k)\},$$

where q_k is the price of k. Thus, he produces k units of capital at a unit cost and sells them at price q_k so that his wealth increases by $q_k k$. Using the linearity of W^s, if the capital market is active then $q_k = 1$ and $V^s(\hat{a}_m, \hat{a}_g) = W^s(\hat{a}_m + \hat{a}_g)$. Moreover, his portfolio problem is

$$\max_{\hat{a}_m,\hat{a}_g \geq 0} \{-(1+\pi)\hat{a}_m - q_g\hat{a}_g + \beta(\hat{a}_m + \hat{a}_g)\}.$$

Given $\pi > \beta - 1$ we have $\hat{a}_m = 0$ (suppliers hold no cash since they do not need liquidity). Additionally, they hold bonds only if $q_g = \beta$.

For an entrepreneur in stage 1,

$$V^e(\hat{a}_m, \hat{a}_g) = \mathbb{E}W^e(k, \hat{a}_m + \hat{a}_g - q_k k - \phi).$$

Thus, he purchases k at cost $\psi = q_k k$, pays ϕ for banking services, and $\psi + \phi$ is subtracted from his stage 2 wealth. Expectations are with respect to (k, ψ, ϕ) and depend on whether he has an investment opportunity (if not, $k = \psi = \phi = 0$) and whether he has access to bank lending (if not, $\phi = 0$). His choice of real balances reduces to

$$\max_{\hat{a}_m \geq 0} \left\{ -i\hat{a}_m + \mathbb{E}\left[f(k) - k - \phi \right] \right\},$$

where $i \equiv (1+\pi)(1+\rho) - 1$ and (k, ϕ) is a function of $\hat{a}_m$. As usual, i can be interpreted as the nominal rate on illiquid bonds (i.e., the dollars one would require tomorrow to give up a dollar today). Since government bonds provide no liquidity services to an entrepreneur, he holds them only if $q_g = \beta$.

Finally, for a bank in the interbank market,

$$V^b(\hat{a}_m, \hat{a}_g) = \max_{a_m, a_g \geq 0} \mathbb{E}W^b(\omega) \text{ st } \omega = a_m + a_g - \frac{\hat{q}_m}{q_m}(a_m - \hat{a}_m) - \hat{q}_g(a_g - \hat{a}_g) + \Pi,$$

where Π is profit from loans in stage 1, their net interest margin. Without regulatory requirements, $\Pi = \phi$; with regulation, as discussed below, Π depends on a_m and a_g. Thus, a bank with $(\hat{a}_m, \hat{a}_g)$ maximizes its financial wealth by choosing (a_m, a_g) and promises to repay $\hat{q}_m(a_m - \hat{a}_m)/q_m$ and $\hat{q}_g(a_g - \hat{a}_g)$ in stage 2, where $\hat{q}_m/q_m$ and $\hat{q}_g$ are the prices of real balances and bonds in the interbank market. Accordingly, $-q_g + \beta\hat{q}_g \leq 0$, with equality if $\hat{a}_g > 0$. Banks purchase bonds in stage 2 to carry into the interbank market only if the capital gain, $(\hat{q}_g - q_g)/q_g$, is equal to the discount rate, ρ. Similarly, $-q_{m,t-1} + \beta\hat{q}_{m,t} \leq 0$, with equality if $\hat{a}_m > 0$. Banks bring money into the interbank market only if its return, $(\hat{q}_{m,t} - q_{m,t-1})/q_{m,t-1}$, is equal to ρ.

It is easy to verify that banks do not hold money absent regulatory requirements. The cost of holding bonds in the interbank market, denoted τ_g, is the spread between their stage 1 and stage 2 prices, $\tau_g \equiv \hat{q}_g - 1$. If $\hat{a}_g > 0$, then $\hat{q}_g = q_g/\beta = (1+\pi)/\beta(1+i_g)$ and

$$\tau_g = (i - i_g) / (1 + i_g).$$

The cost of holding government bonds in the interbank market is the spread between the return on an illiquid bond, i, and on a government bond, i_g. Without regulation, $\hat{q}_g = 1$ and $q_g = \beta$, in which case $i_g = (1+\pi)(1+\rho) - 1 = i$ and $\tau_g = 0$. The cost of holding a unit of real balances in the interbank market is $\tau_m \equiv (\hat{q}_{m,t} - q_{m,t})/q_{m,t}$. If $\hat{a}_m > 0$, then

$$\tau_m = (1+\pi)(1+\rho) - 1 = i.$$

4 External finance

Here we study nonmonetary equilibrium, with only external finance. We first consider trade credit, then bank credit, then both.

4.1 Trade credit

In an economy without banks, entrepreneurs must rely on trade credit, as shown in the left panel of Figure 2. Such credit is subject to $\psi = k \leq \chi_s f(k)$, since an entrepreneur cannot credibly pledge more than a fraction χ_s of his output. Hence, an entrepreneur with financial wealth ω solves

$$\max_{k,\psi} W^e(k, \omega - \psi) \text{ st } \psi = k \leq \chi_s f(k). \tag{1}$$

By the linearity of W^e, this reduces to

$$\Delta(\chi_s) \equiv \max_{k \geq 0} \{f(k) - k\} \text{ st } k \leq \chi_s f(k). \tag{2}$$

There are two cases. If $k \leq \chi_s f(k)$ is slack, then $\psi = k = k^*$, where $f'(k^*) = 1$. This first-best outcome obtains when $\chi_s \geq \chi_s^* = k^*/f(k^*)$. If $\psi \leq \chi_s f(k)$ binds, then $\psi = k$ where k is the largest nonnegative solution to $\chi_s f(k) = k$. This second-best outcome obtains when $\chi_s < \chi_s^*$, and implies k is increasing in χ_s.

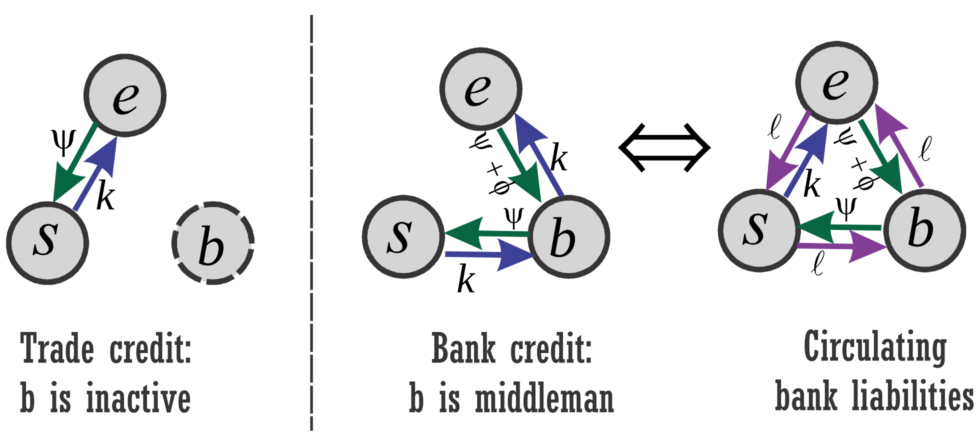

Figure 2: Transaction patterns

4.2 Bank credit

Now suppose trade credit is not viable – say, $\chi_s = 0$ – and consider banking. If an entrepreneur with an investment opportunity meets a bank, there are gains from trade, since banks can credibly promise payment to the supplier, and enforce payment from the entrepreneur up to the limit implied by χ_b. For this service, the bank charges the entrepreneur a fee, ϕ. Figure 2 shows two ways to achieve the same outcome. In the middle panel, the bank gets k from the supplier in exchange for a promise ψ, then gives k to the entrepreneur in exchange for a promise $\psi + \phi$. In the right panel, the bank extends a loan to the entrepreneur by crediting his deposit account the amount ℓ. Then the entrepreneur transfers his deposit claim to the supplier, who redeems it for ψ in stage 2, while the entrepreneur settles by returning $\psi+\phi$ to the bank. This arrangement uses deposit claims as inside money.[11]

A loan contract is a pair (ψ, ϕ), where $\psi = q_k k$. The terms are negotiated bilaterally, and if an agreement is reached, the entrepreneur's payoff is $W^e(k, \omega^e - \psi - \phi)$ while the bank's is $W^b(\omega^b + \phi)$. This implies individual surpluses

$$\begin{aligned} S^e &\equiv W^e(k, \omega^e - \psi - \phi) - W^e(0, \omega^e) = f(k) - \psi - \phi \\ S^b &\equiv W^b(\omega^b + \phi) - W^b(\omega^b) = \phi, \end{aligned}$$

and total surplus $S^e + S^b = f(k) - k$. Figure 3 depicts the frontier in utility space (right panel) and contract space (left panel). One can check the maximum surplus a bank can get is $\chi_b f(\hat{k}) - \hat{k} \leq f(\hat{k}) - \hat{k}$, where $\hat{k}$ solves $\chi_b f'(\hat{k}) = 1$. Notice that k cannot be below $\hat{k}$, as then we could raise the surplus of both parties.[12]

The Nash bargaining solution, where $\theta \in (0,1)$ is bank's share, is given by

$$(k, \phi) \in \arg\max \left[f(k) - k - \phi\right]^{1-\theta} \phi^{\theta} \quad \text{st} \quad k + \phi \leq \chi_b f(k). \tag{3}$$

[11] For some issues, the difference between the middle panel and right panel is not important, but there are scenarios where it might matter – e.g., if physical transfers of k are spatially or temporally separated, having a transferable asset can be essential.

[12] Also notice the bargaining set is not convex, but that actually does not matter for generalized Nash bargaining in this context. Moreover, in a Supplemental Appendix we provide strategic foundations for Nash bargaining using an alternating offer game.

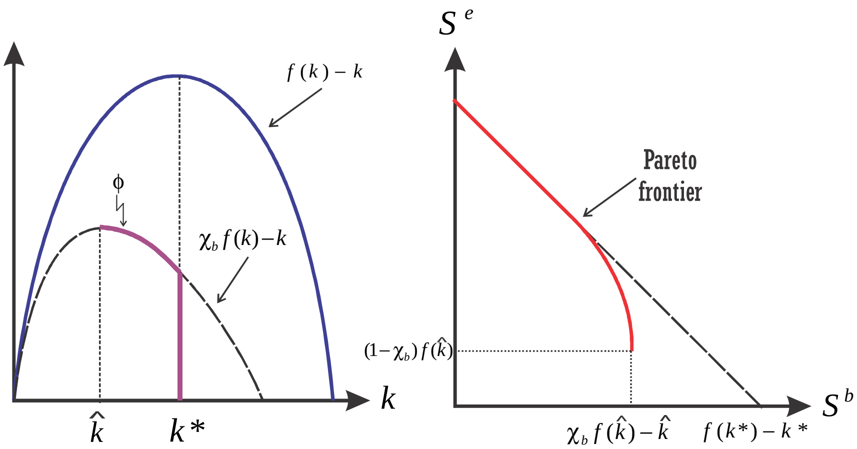

Figure 3: Pareto frontier for bank loans

If the pledgeability constraint does not bind, then $k = k^*$ and

$$\phi = \theta \left[f(k^*) - k^* \right]. \tag{4}$$

According to (4), ϕ is independent of χ_b, but increases with θ and $f(k^*) - k^*$. The lending rate is

$$r = \frac{\phi}{\psi} = \frac{\theta \left[f(k^*) - k^* \right]}{k^*}. \tag{5}$$

From (5), the lending rate is proportional to the average return $f(k^*)/k^* - 1$. The threshold for χ_b below which the pledgeability constraint binds is

$$\chi_b^* \equiv \frac{(1-\theta)k^* + \theta f(k^*)}{f(k^*)}.$$

If $\chi_b < \chi_b^*$ then the pledgeability constraint binds and

$$\phi = \chi_b f(k) - k \tag{6}$$

$$\frac{k}{f(k)} = \frac{\chi_b f'(k) - \theta}{(1-\theta) f'(k)}. \tag{7}$$

There is a unique solution $k \in \left(\hat{k}, k^*\right)$ to (7).[13] It is increasing in χ_b, with $k(0) = 0$ and $k(\chi_b^*) = k^*$. Also, $\partial k/\partial\theta < 0$ and $\partial\phi/\partial\theta > 0$, so banks with more bargaining

[13]The LHS of (7) is increasing in k from 0 to ∞, where the limits are obtained by L'Hopital's rule. The RHS is decreasing for all k such that the numerator is positive, and the RHS evaluated at k^*, $(\chi_b - \theta)/(1-\theta)$, is smaller than the LHS provided $\chi_b < \chi_b^*$. Moreover, at $k = \hat{k}$ the RHS is $1/f'(\hat{k}) = 1/\chi_b$, which exceeds the LHS.

power charge higher fees and finance less investment. The lending rate is

$$r = \frac{\chi_b f(k)}{k} - 1 = \frac{\theta\left[1 - \chi_b f'(k)\right]}{\chi_b f'(k) - \theta}. \tag{8}$$

One can check $\partial r/\partial\theta > 0$. However, $\partial r/\partial\chi_b$ is ambiguous, and in the special case $f(k) = zk^{\gamma}$, we have $r = \theta(1-\gamma)/\gamma$ independent of χ_b.

Although the above results are mainly a stepping stone to the case with both internal and external finance, they may also be of independent interest, with several predictions about how the interest rate and loan size depend on parameters. For instance, if bargaining power varies across entrepreneurs there is a negative correlation between k and r, while if pledgeability varies there is no correlation. In any case, before introducing internal finance, we show how to combine bank and trade credit and derive addition implications.

4.3 Trade and bank credit

Suppose $\chi_b > 0$ and $\chi_s > 0$. Without a bank, an entrepreneur can pledge a fraction χ_s to a supplier; with a bank, he can pledge an additional fraction χ_b; and his total obligation cannot exceed $\chi f(k)$ where $\chi = \chi_s + \chi_b$. Bank credit is essential if $\chi_s < \chi_s^* = f(k^*)/k^*$, since then trade credit alone cannot implement the first best. In this case, a measure $\lambda(1-\alpha)$ of investment projects are financed with trade credit while $\lambda\alpha$ are financed with bank and trade credit.

A loan contract now involves investment financed with trade credit k_s, investment financed with bank credit k_b, and the fee ϕ. The bargaining problem is

$$\max_{k_b,k_s,\phi} \left[f(k) - k - \phi - \Delta(\chi_s)\right]^{1-\theta}\phi^{\theta} \text{ st } \; k_b + \phi \le \chi_b f(k), \; k_s \le \chi_s f(k),$$

where $k = k_b + k_s$ and $\Delta(\chi_s)$ is the entrepreneur's threat point. The solution is $k = k^*$ and $\phi = \theta\left[f(k^*) - k^* - \Delta(\chi_s)\right]$ if $\chi_b \ge \chi_b^*(\chi_s)$ where

$$\chi_b^*(\chi_s) \equiv \frac{(1-\theta)k^* + \theta\left[f(k^*) - \Delta(\chi_s)\right] - \chi_s f(k^*)}{f(k^*)}.$$

Notice $\partial\chi_b^*/\partial\chi_s < 0$. Also, the loan rate is

$$r = \phi/k^* = \frac{\theta\left[f(k^*) - k^* - \Delta(\chi_s)\right]}{k^*}.$$

Notice $\partial r/\partial \chi_s < 0$, and $r \to 0$ as $\chi_s \to \chi_s^*$. Intuitively, the outside option of trade credit lets firms negotiate better terms and reduces χ_b^*.

If $\chi_b < \chi_b^*(\chi_s)$, then (k, k_s, ϕ) solve

$$\frac{(1-\chi)f'(k)}{(1-\chi)f(k)-\Delta(\chi_s)} = \frac{\theta}{1-\theta}\frac{1-\chi f'(k)}{\chi f(k)-k} \tag{9}$$

$$\phi = \chi f(k) - k \tag{10}$$

$$k_s = \chi_s f(k). \tag{11}$$

There is a unique k solving (9), and it increases with χ_b and χ_s. Notice higher χ_s increases output and hence an entrepreneur's bank credit, while higher χ_b increases his trade credit. In other words, the two types of credit interact. Other implications can be derived,[14] but the time has come to consider internal finance.

5 Internal and external finance

We now allow entrepreneurs to accumulate cash in stage 2 to finance investments in stage 1 next period. This is internal finance, defined as a firm's use of retained earnings to pay for new capital, with the following features emphasized by Bernanke et al. (1996): it constitutes an immediate funding source; it has no explicit interest payments; and it sidesteps the need for third parties. To ease the exposition, here we set $\chi_s = 0$. Also, we consider both a fixed set of banks, and then allow entry by banks to make the arrival rate α in the OTC market endogenous.

5.1 Exogenous set of banks

Consider an entrepreneur in stage 1 with an investment opportunity but no access to banking. Then $k \leq a_m^e$ and

$$\Delta^m(a_m^e) = f(k^m) - k^m \text{ where } k^m = \min\{a_m^e, k^*\}. \tag{12}$$

Notice $\Delta^m(a_m^e)$ is increasing and strictly concave for all $a_m^e < k^*$.

[14] Suppose, e.g., we hold total pledgeability constant but raise χ_s/χ, say because the seniority of suppliers' debt increases; then investment increases and the interest payment falls.

Consider next an entrepreneur in contact with a bank, where loan contracts now specify an investment level k, a down payment d, and the bank's fee ϕ. If the loan negotiations are unsuccessful, the entrepreneur can purchase k with cash and get $\Delta^m(a^e_m)$, so his surplus from the loan is $f(k) - k - \phi - \Delta^m(a^e_m)$.[15] Then the bargaining problem is

$$\max_{k,d,\phi} \left[f(k) - k - \phi - \Delta^m(a^e_m)\right]^{1-\theta} \phi^\theta \text{ st } k - d + \phi \leq \chi_b f(k) \text{ and } d \leq a^e_m. \tag{13}$$

With internal and external finance, what we previously called the pledgeability constraint is now called a liquidity constraint, reflecting credit plus cash. If this constraint does not bind, the solution to (13) is $k^c = k^*$ and $\phi^c = \theta\left[f(k^*) - k^* - \Delta^m(a^e_m)\right]$. Notice $\partial k^c/\partial a^e_m = 0$ and $\partial \phi^c/\partial a^e_m < 0$, so by having more cash in hand, the entrepreneur reduces payments to the bank and increases profit. Also, the constraint does not bind if $a^e_m > d^*$, where $d^* > 0$ if $\chi_b < \chi^*_b$.

If $a^e_m < d^*$ and the liquidity constraint binds, the bargaining solution is

$$\begin{aligned} \frac{a^e_m + \chi_b f(k^c) - k^c}{(1-\chi_b) f(k^c) - a^e_m - \Delta^m(a^e_m)} &= \frac{\theta}{1-\theta} \frac{1 - \chi_b f'(k^c)}{(1-\chi_b) f'(k^c)} && (14) \\ k^c + \phi^c &= a^e_m + \chi_b f(k^c). && (15) \end{aligned}$$

There is a unique solution for $k^c > \hat{k}$, and it implies $\partial k^c/\partial a^e_m > 0$. Hence, $\partial\left[a^e_m + \chi_b f(k^c)\right]/\partial a^e_m > 1$, which says that by accumulating a dollar, a firm raises its financing capacity by more than a dollar, since pledgeable output increases, which we consider an key implication of the theory.

The lending rate, $r \equiv \phi^c/(k^c - a^e_m)$, also depends on the entrepreneur's cash position,

$$r = \begin{cases} \frac{\theta[f(k^*) - k^* - \Delta^m(a^e_m)]}{k^* - a^e_m} & \text{if } a^e_m \in [d^*, k^*) \\ \frac{\chi_b f(k^c)}{k^c - a^e_m} - 1 & \text{if } a^e_m < d^* \end{cases}, \tag{16}$$

where we assume $d = a^e_m$ when $a^e_m \in [d^*, k^*)$. It is easy to check $\partial r/\partial a^e_m < 0$ and $\lim_{a^e_m \nearrow k^*} r = 0$. The fact that r decreases with a^e_m creates pass through from the nominal monetary policy rate to the real lending rate – another key contribution of the theory.

[15] We take $\Delta^m(z)$ as the threat point, but it could alternatively be considered an outside option, which affects the bargaining outcome only if it is credible (see, e.g., Osborne and Rubinstein 1990, Section 3.12). Using this alternative formulation, we get basically the same main results.

We now turn to an entrepreneur's endogenous choice of money balances,

$$\max_{a_m^e \geq 0} \left\{ -i a_m^e + \lambda \left(1-\alpha\right) \Delta^m(a_m^e) + \alpha \lambda \Delta^c(a_m^e) \right\}, \tag{17}$$

where $\Delta^c(a_m^e) \equiv f(k^c) - k^c - \phi^c$ can be written as follows:

$$\Delta^c(a_m^e) = \begin{cases} (1-\theta)\left[f(k^*) - k^*\right] + \theta \Delta^m(a_m^e) & \text{if } \; a_m^e \geq d^* \\ (1-\chi_b) f(k^c) - a_m^e & \text{otherwise.} \end{cases}$$

If $a_m^e \geq k^*$, the entrepreneur finances k^* without bank credit, so $\Delta^c(a_m^e) = f(k^*) - k^*$ is independent of a_m^e. If $a_m^e \in [d^*, k^*)$, the entrepreneur can still finance k^*, but only by using bank credit as well as cash, and the bank captures a fraction θ of the surplus. Now $\Delta^c(a_m^e)$ increases with a_m^e. Finally, if $a_m^e < d^*$, the liquidity constraint binds and the entrepreneur's surplus equals his nonpledgeable output net of real balances.

Given the above results, a monetary equilibrium with internal and external finance is defined as a list (k^m, k^c, a_m^e, r) solving (12), (13), (16), and (17). Notice this has a recursive structure. First, (17) determines $a_m^e \in [0, k^*]$, where a solution exists and is generically unique, even though the objective may not be concave, by an application of the argument in Gu and Wright (2016). Then (12) and (13) determine k^m and k^c. Finally, r comes from (16).

Consider $k^c = k^*$, which obtains if $\chi_b \geq \chi_b^*$ or if i is small enough that $a_m^e \geq d^*$. The FOC associated with (17) is

$$i = \lambda \left[1 - \alpha(1-\theta)\right] \left[f'(k^m) - 1\right].$$

Notice $\partial k^m / \partial i < 0$, $\partial k^m / \partial \lambda > 0$, $\partial k^m / \partial \alpha < 0$, and $\partial k^m / \partial \theta > 0$. In particular, as bank credit becomes less readily available, or more expensive because banks have better bargaining power, entrepreneurs reduce their reliance on it by holding more cash. We emphasize that entrepreneurs hold cash even if they have access to bank loans with certainty and the liquidity constraint is not binding, because access to more internal funds reduces the rent captured by banks as long as $\theta > 0$ (when $\theta = 0$ and $k^c = k^*$, money can only be valued if $\alpha < 1$). This strategic motive for holding cash is novel compared to other monetary models we know.

The real lending rate in this case, where $k^c = k^*$, is

$$r = \frac{\theta \{[f(k^*) - k^*] - [f(k^m) - k^m]\}}{k^* - k^m}. \tag{18}$$

One can check $\partial r/\partial i > 0$. This is another key implication of the theory: the nominal monetary policy rate i, as the opportunity cost of liquidity, affects firms' internal funds and hence the bargaining solution, including the real rate r they get from banks. When i is small, the pass through is approximated by

$$r \approx \frac{\theta i}{2\lambda [1 - \alpha(1-\theta)]}.$$

Some authors argue interest rate pass through has been significantly weaker since 2008 (e.g., Mora 2014). In the context of the model, this is consistent with new regulations that reduce banks' market power, tighter lending standards that lower the acceptance rate of loan applications, and more frequent investment opportunities.[16] It is also worth noting that there is interest rate pass through even with $\alpha = 1$ (no search friction in the credit market), as that implies $r \approx i/2\lambda$. Also, in this case, with $k^c = k^*$, although an increase in i raises r, it does not affect investment – it merely alters the corporate finance mix and transfers profit from firms to banks.

We now turn to the case $k^c < k^*$. Consider first $\theta = 0$, where k^c solves $a_m^e + \chi_b f(k^c) = k^c$. Then,

$$\frac{\partial \Delta^c(a_m^e)}{\partial a_m^e} = \frac{f'(k^c) - 1}{1 - \chi_b f'(k^c)}. \tag{19}$$

If the entrepreneur gets an additional unit of credit, he can purchase an additional unit of k, which raises his surplus by $f'(k^c) - 1$. The denominator in (19) is a *financing multiplier* that says one unit of k raises pledgeable output by $\chi_b f'(k^c)$, thereby increasing the entrepreneur's financing capacity. From (17), the choice of a_m^e is

$$(1-\alpha) f'(k^m) + \alpha \frac{(1-\chi_b) f'(k^c)}{1 - \chi_b f'(k^c)} = 1 + \frac{i}{\lambda}. \tag{20}$$

This has a unique solution, and $\partial a_m^e/\partial i < 0$, $\partial k^m/\partial i < 0$, and $\partial k^c/\partial i < 0$. Now an increase in the nominal policy rate reduces investment. If $\alpha = 1$, there is still a role

[16] Also, it should not be presumed that $r > i$, as r is an intraperiod rate, while i is an interperiod nominal rate. One can think of r as a pure premium over the rate that would prevail in a perfectly competitive loan market.

for money to relax the liquidity constraint by raising down payments. Moreover, $\partial k^c/\partial \chi_b > 0$. An increase in borrowing capacity does not reduce real balances one-for-one because it raises the nonpecuniary return on real balances through the financing multiplier.

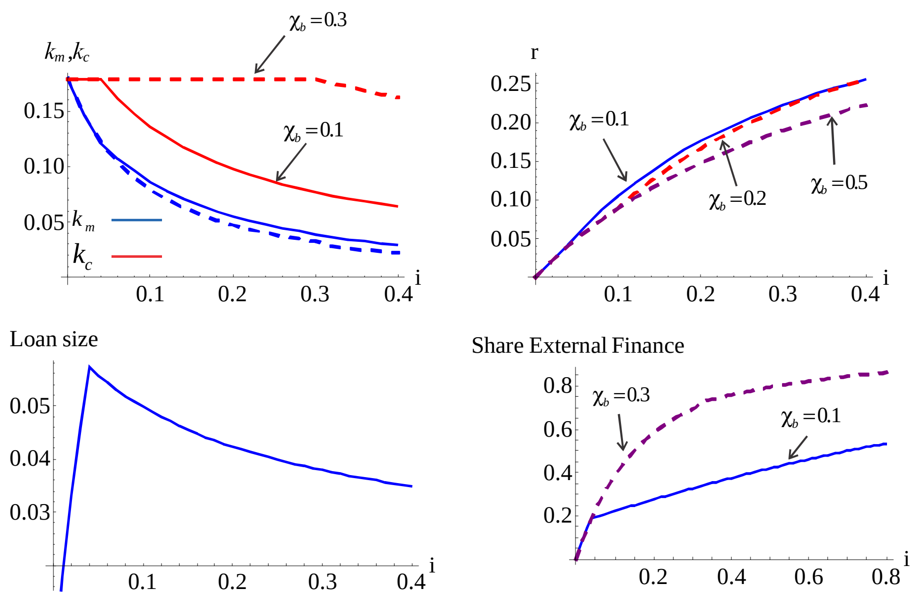

Figure 4: Internal and external finance

Figure 4 shows an example, where $f(k) = k^{0.3}$ with $\theta = 0.3$, $\lambda = 0.2$, and $\alpha = 0.5$ The top panels illustrate the transmission of monetary policy and interest rate pass through for different χ_b. The bottom left panel shows loan size is nonmonotone in i due to two opposing effects: a substitution effect, where an increase in i raises external finance and loan size; and a financing multiplier effect, where a reduction in a^e_m reduces pledgeable output and loan size. The former effect dominates for low i while the latter dominates for high i. The bottom right panel plots the share of external finance, $1 - k^m / [(1-\alpha)k^m + \alpha k^c]$, as a function of i. At the Friedman rule, $i = 0$, the share of external finance is zero since entrepreneurs finance all investment opportunities with cash. As i increases, so does the share of external finance as entrepreneurs with access to banks supplement real balances with loans.

The model can be used to study how different firms or industries respond to changes in policy, depending on their endogenous corporate finance structure. We

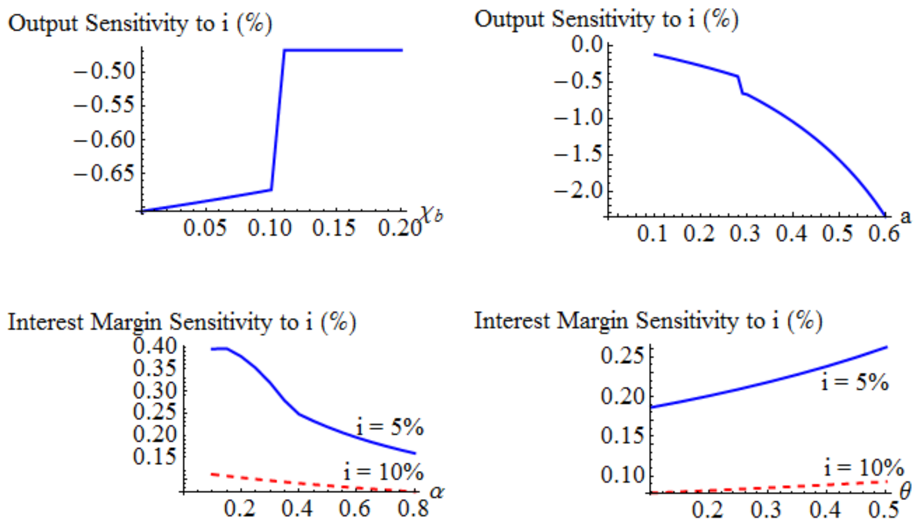

Figure 5: Output and interest margin growth following increase in i

can easily introduce heterogeneity across entrepreneurs since the distribution of entrepreneur characteristics – say, χ_b or f – affects the value of money but not its rate of return, so an individual entrepreneur's problem is still given by (17). The top panel of Figure 5 shows the growth of output, defined as $(1-\alpha)f(k^m)+\alpha f(k^c)$, following an increase in i from 10% to 11% (here we set $\alpha = \lambda = 0.5$). The horizontal axis is a firm's characteristic, the pledgeability of its output, or the input-elasticity of its technology. The top left panel shows firms with greater pledgeability rely more on external finance and hence are less sensitive to changes in i. The top right panel shows firms with greater capital intensities are more sensitive to changes in i. These results are consistent with Dedola and Lippi (2005), who find the impact of monetary policy is stronger in industries that are more capital intensive and have smaller borrowing capacities.

There are also implications for the effects of policy on banks' net interest margins. We now interpret the comparative statics as comparing economies with different credit market structures, in terms of search frictions and bargaining power. The bottom panel of Figure 5 plots the growth rate of banks' interest margin, ϕ, following an increase in i from 5% to 6% (solid blue line) and 10% to 11% (dashed red line).

Consistent with cross-country evidence in Claessens et al. (2016), interest margins in the model respond positively to increases in i, with larger effects when i is lower. Moreover, interest margins responds more strongly to changes in policy when search frictions and banks' bargaining power are higher. As we show next, these effects have implications for banks' participation in the market and therefore entrepreneurs' access to credit.

5.2 Endogenous set of banks

To endogenize access to credit, consider allowing entry by bankers at flow cost $\varsigma \geq 0$. If the measure of entering banks is b, the matching probability for an entrepreneur is $\alpha(b)$, and for a bank is $\alpha(b)/b$. As standard, $\alpha(b)$ is increasing and concave, with $\alpha(0) = 0$, $\alpha(\infty) = 1$, $\alpha'(0) = \infty$, and $\alpha'(\infty) = 0$. The payoff of a bank that enters is $V^b = -\varsigma + \lambda\phi\alpha(b)/b + \beta \max\{V^b, 0\}$, and free entry means $V^b = 0$, or

$$\frac{b}{\alpha(b)} = \frac{\lambda\phi}{\varsigma}. \tag{21}$$

Banks enter as long as $\lambda\phi > \varsigma$, which requires $\theta > 0$. Given that ϕ decreases with a^e_m, (21) defines a negative relationship between b and a^e_m, shown as the BE curve in Figure 6. Notice $\phi \to 0$ and $b \to 0$ as $a^e_m \to k^*$.

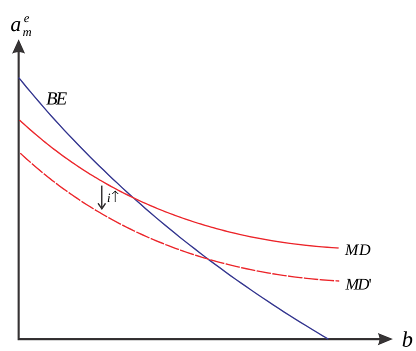

Figure 6: Equilibrium with entry of banks

If i is not too large, so that $k^c = k^*$, the entrepreneur's demand for money is

$$f'(a^e_m) = 1 + \frac{i}{\lambda\left[1 - \alpha(b)(1-\theta)\right]}. \tag{22}$$

Thus, a_m^e decreases with b, as shown by the MD curve in Figure 6. Evidently, multiplicity may be possible, but let us focus on natural equilibria where MD cuts BE from below. Notice that as $b \to 0$, a_m^e tends to its level in a pure monetary economy, satisfying $f'(a_m^e) = 1 + i/\lambda$. As $b \to \infty$, a_m^e approaches the solution to $f'(a_m^e) = 1 + i/\lambda\theta$. Hence there always exists a solution (b, a_m^e) to (21)-(22) where MD intersects BE from below. As i increases, MD shifts down, so a_m^e decreases while b increases. This means entry amplifies the effect of policy on real balances, since higher $\alpha(b)$ reduces a_m^e further. Also, notice that $a_m^e \to k^*$ and $b \to 0$ as $i \to 0$.[17]

5.3 Summary of results without regulation

The preceding analysis highlights several mechanisms through which monetary policy affects corporate finance and investment. The direct channel is through the opportunity cost of retaining earnings in liquid (instead of interest-bearing illiquid) assets. As i increases, firms reduce their cash balances and internally financed investment. If firms have access to banks and are not liquidity constrained, a small increase in i does not affect $k^c = k^*$. In this case, entrepreneurs reduce their down payment and increase loan size, and that raises r, but investment is the same. If entrepreneurs can obtain a bank loan with certainty, monetary policy has no effect on aggregate investment for low interest rates, but affects the lending rate and the composition of corporate finance.

If entrepreneurs are liquidity constrained, an increase in i reduces both the down payment and investment. Moreover, there is a financing multiplier since lower down payments reduce investment, pledgeable output and the loan size. Finally, monetary policy can also have an impact on the extensive margin of credit. As i increases, banks' net interest margins increase, which gives them a greater incentive to provide loans. As α increases, entrepreneurs with better access to external finance reduce

[17] Indeed, $i = 0$ (the Friedman rule) is optimal here, and it drives banks out of business. It is known how to overturn this kind of result – e.g., by making money only partially acceptable due to counterfeiting, having it subject to theft, or adding other frictions (see the surveys on New Monetarist economics cited in the Introduction).

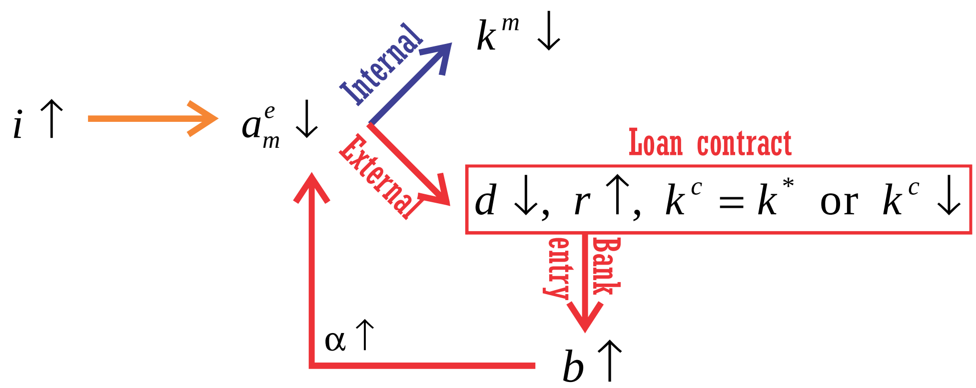

Figure 7: Transmission mechanism for anticipated change in i

their holdings of liquid assets, further reducing k^m. These different channels are summarized in Figure 7. The next step is to introduce regulation.

6 Reserve requirements

Suppose a fraction $\nu_g \in [0,1]$ of bank liabilities must be backed by liquidity in terms of government bonds or fiat money, and a fraction $\nu_m \leq \nu_g$ by money. We interpret ν_m as a strict reserve requirement and ν_g as a broad requirement. Given a loan $\ell = k - d$, the bank must hold $\nu_m \ell$ in real money balances and $(\nu_g - \nu_m)\ell$ in broad liquidity at the start of stage 2. The cost of this regulation on a bank is $\bar{\tau}\ell$, reducing its profit to $\Pi = \phi - \bar{\tau}\ell$ where $\bar{\tau} \equiv \tau_m \nu_m + \tau_g (\nu_g - \nu_m)$.[18]

Assuming $f'(a_m^e) \geq 1 + \bar{\tau}$, so there are gains from trade, a loan contract solves

$$\max_{k,\phi,d} [f(k) - k - \phi - \Delta^m(a_m^e)]^{1-\theta} [\phi - \bar{\tau}(k-d)]^{\theta} \tag{23}$$

$$\text{st } k + \phi \leq d + \chi_b f(k),\ d \leq \min\{k, a_m^e\}. \tag{24}$$

If the liquidity constraint does not bind, (k^c, ϕ^c) solves

$$f'(k^c) = 1 + \bar{\tau}, \tag{25}$$

$$\phi^c = (1-\theta)\bar{\tau}(k^c - a_m^e) + \theta [f(k^c) - k^c - \Delta^m(a_m^e)]. \tag{26}$$

[18] There is no claim such regulations are part of an optimal arrangement; we take them as given. They capture cash reserve ratios (Calomiris et al. 2012), liquidity coverage ratios (Basel Committee 2013), or the requirement that banks must purchase government bonds (Goodhart 1995). In terms of the literature, our formalization of these regulations is similar to, e.g., Romer (1985), Freeman (1987), Schreft and Smith (1997), Gomis-Porqueras (2002) or Bech and Monnet (2015).

There are two novelties. First, $\bar{\tau} > 0$ acts as a tax on investment, implying $k^c < k^*$ and $\partial k^c / \partial \bar{\tau} < 0$. Second, $\partial \phi^c / \partial \bar{\tau} > 0$ for $\theta < 1$. The constraint bind iff a_m^e is below a threshold d^* depending on $\bar{\tau}$ and χ_b. If it binds, (k^c, ϕ^c) solves

$$\begin{aligned} \frac{(1+\bar{\tau})(a_m^e - k^c) + \chi_b f(k^c)}{(1-\chi_b) f(k^c) - a_m^e - \Delta^m(a_m^e)} &= \frac{\theta}{1-\theta} \frac{(1+\bar{\tau}) - \chi_b f'(k^c)}{(1-\chi_b) f'(k^c)} \quad (27) \\ \phi^c &= a_m^e + \chi_b f(k^c) - k^c. \quad (28) \end{aligned}$$

One can check $\partial k^c / \partial \bar{\tau} < 0$ and $\partial k^c / \partial a_m^e > 0$.

The supply of bonds in the interbank market is A_g and the supply of real balances is $\hat{A}_m^b = \hat{a}_m^b$. The demands for bonds and real balances arise from regulatory policy: a measure $\alpha\lambda$ of banks demand $(\nu_g - \nu_m)\ell$ in broad liquidity and $\nu_m \ell$ in real balances, where $\ell = k^c - a_m^e$. Market clearing implies

$$\hat{A}_m^b \begin{cases} = \\ = \\ \geq \end{cases} \alpha\lambda\nu_m\ell \text{ and } A_g + \hat{A}_m^b \begin{cases} \geq \\ = \\ = \end{cases} \alpha\lambda\nu_g\ell \text{ if } \tau_g \begin{cases} = 0 < \tau_m \\ \in (0, \tau_m) \\ = \tau_m > 0. \end{cases} \quad (29)$$

If $\tau_g = 0$, banks can hold excess liquidity. If $\tau_g = \tau_m = i$, money and bonds are perfect substitutes for regulatory purposes. Finally, if $\tau_g \in (0, \tau_m)$, banks hold just enough real balances and bonds to satisfy requirements. Equilibrium is now a list $(k^m, k^c, a_m^e, \hat{a}_m^b, r, i_g)$ solving (13), (17), (23), (24), and (29).

6.1 Strict reserve requirements

From (29), $i_g = i$ when bonds do not satisfy regulatory requirements. If the liquidity constraint does not bind, $\partial k^c / \partial i < 0$ and

$$r = \nu_m i + \theta \left[\frac{f(k^c) - k^c - \Delta^m(a_m^e)}{k^c - a_m^e} - \nu_m i \right].$$

The first component of the lending rate is the cost due to the reserve requirement; the second reflects the bank's surplus. For small i,

$$r \approx \left\{ \nu_m + \frac{\theta (1 - \lambda\nu_m)}{2\lambda [1 - \alpha(1-\theta)]} \right\} i.$$

So a reserve ratio, ν_m, raises pass through. In the case of 100% required reserves (narrow banking), the difference between r and i is positive and increases with

θ. Responses of equilibrium to policy are similar to the model without reserve requirements, as illustrated by Figure 8 using the same parameters as Figure 4. The solid lines correspond to $\nu_m = 10\%$ and the dashed lines to $\nu_m = 100\%$.

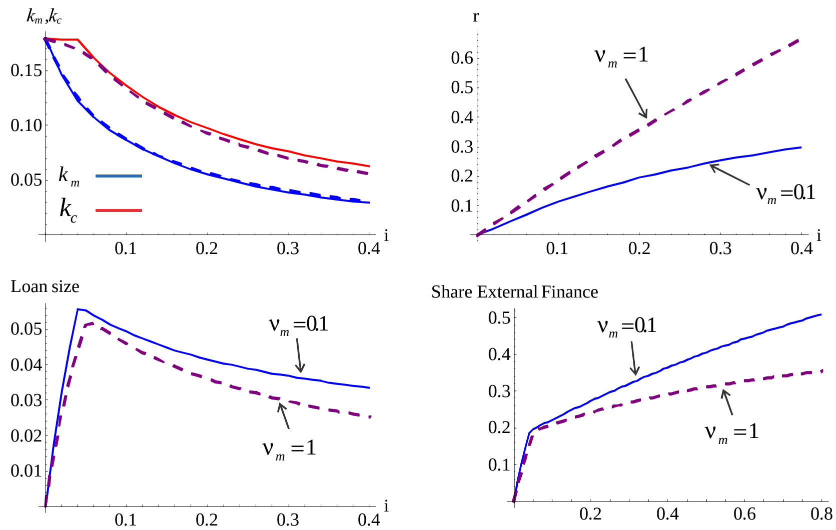

Figure 8: Monetary policy under strict reserve requirements

Now consider a one-time, unanticipated OMO in the interbank market, reducing A_g while raising the money supply by μA_m, where $\mu > 0$. Since bonds have no regulatory role, in this case, only the change in A_m is relevant. We focus on equilibria where the economy returns to steady state in stage 2 with q_m scaled down by $1+\mu$. As a result, $a_m^{e\prime} = a_m^e/(1+\mu)$, where prime denotes a variable at the time of the monetary injection. By classical neutrality, $a_m^{e\prime} + \hat{A}_m^{b\prime} = a_m^e + \hat{A}_m^b$, and hence

$$\hat{A}_m^{b\prime} = \frac{\mu}{1+\mu} a_m^e + \hat{A}_m^b. \tag{30}$$

The first term on the RHS corresponds to the increase in banks' real balances financed by the inflation tax on entrepreneurs' real balances. In equilibria where banks hold no excess reserves, $\hat{A}_m^{b\prime} = \alpha\lambda\nu_m \left(k^{c\prime} - a_m^{e\prime}\right)$. From (30),

$$k^{c\prime} - k^c = \frac{\left(1 - \alpha\lambda\nu_m\right)\mu}{\alpha\lambda\nu_m(1+\mu)} a_m^e.$$

Hence, if $\lambda\nu_m < 1$ then $\partial k^c/\partial\mu > 0$, $\partial k^m/\partial\mu < 0$ and $\partial\tau_m/\partial\mu < 0$. The OMO thus reduces the cost of borrowing reserves, so banks offer larger loans, but it also reduces $a_m^{e\prime}$ and $k^{m\prime}$ by redistributing liquidity from entrepreneurs to banks. The overall change in investment is

$$\alpha\lambda(k^{c\prime} - k^c) + \lambda(1-\alpha)(a_m^{e\prime} - a_m^e) = \left(\frac{1-\lambda\nu_m}{\nu_m}\right)\frac{\mu}{1+\mu}a_m^e.$$

If $\lambda\nu_m < 1$, this is positive. Intuitively, money is more effective at financing investment when held by banks, because they can *leverage liquid assets* by issuing liabilities, and, through the interbank market can *reallocate liquidity* towards banks with lending opportunities. These results also indicate that, if firms are heterogeneous, those relying on internal finance will benefit less from the OMO than the ones dependent on bank credit.

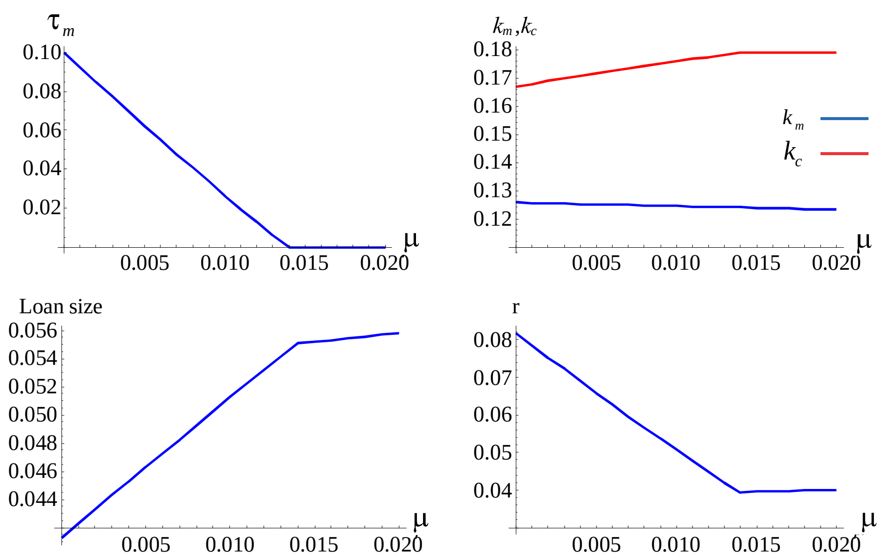

Figure 9: OMO in the interbank market

Figure 9 depicts the effects of an OMO in the interbank market for $f(k) = k^{0.3}$, $\theta = 0.3$, $\chi_b = 0.1$, $\lambda = 0.5$, $\alpha = 0.5$, $\nu_m = 0.5$, and $i = 0.1$. The top left panel shows such a money injection reduces the cost of borrowing reserves. The top right panel shows how k^c increase while k^m decreases with μ. Pass through to the lending rate is shown in the bottom right panel, where r decreases with μ so long as $\tau_m > 0$. A liquidity trap occurs when $\tau_m = 0$, in which case reserves are abundant and $k^c = k^*$. However, a_m^e and k^m keep falling with μ. Moreover, in the bottom left panel both

r and loan size increase. So OMOs have real effects in the liquidity trap, but they are opposite to the ones with $\tau_m > 0$.

6.2 Broad reserve requirements

Now consider broad liquidity requirements: $\nu_g > \nu_m = 0$. First suppose liquidity constraints are slack. Then the FOC for entrepreneurs' real balances is

$$\frac{i - \alpha\lambda\theta\nu_g\tau_g}{\lambda\left[1-\alpha(1-\theta)\right]} = f'(k^m) - 1. \tag{31}$$

Now $\tau_g > 0$ implies $\partial a_m^e/\partial\nu_g > 0$. We distinguish three regimes. Suppose first $\tau_g = 0$. Then the outcome is the same as without regulation, and changes in A_g are neutral. From (29), this regime obtains when $A_g \geq \bar{A}_g \equiv \alpha\lambda\nu_g\left(k^* - a_m^e\right)$.

Second, consider a liquidity trap regime where $i_g = 0$. From (25) and (31),

$$\begin{aligned} f'(k^c) &= 1 + \nu_g i && (32)\\ f'(k^m) &= 1 + \frac{1-\alpha\lambda\theta\nu_g}{\lambda\left[1-\alpha(1-\theta)\right]} i. && (33)\end{aligned}$$

Hence, $\partial k^c/\partial i < 0$, $\partial k^m/\partial i < 0$ but $\partial k^c/\partial A_g = \partial k^m/\partial A_g = 0$. Also, from (26),

$$r = \theta\nu_g i + \theta\left[\frac{f(k^c) - f(k^m)}{k^c - k^m} - 1\right]. \tag{34}$$

One can check $\partial r/\partial i > 0$. Also, from (29), this regime obtains when $A_g \leq \underline{A}_g \equiv \alpha\lambda\nu_g\left(k^c - k^m\right)$ where $\underline{A}_g < \bar{A}_g$ and $\underline{A}_g > 0$ if $\left[1-\alpha(1-2\theta)\right]\lambda\nu_g < 1$.

Third, consider a regime with $i_g \in (0, i)$. From (25), (29) and (31):

$$\begin{aligned} k^c - k^m &= \frac{A_g}{\alpha\lambda\nu_g} && (35)\\ f'(k^m) &= 1 + \frac{i - \alpha\lambda\theta\nu_g\tau_g}{\lambda\left[1-\alpha(1-\theta)\right]} && (36)\\ f'(k^c) &= 1 + \nu_g\tau_g && (37)\end{aligned}$$

One can show $\partial k^m/\partial\tau_g > 0$, $\partial k^c/\partial\tau_g < 0$, $\partial\tau_g/\partial A_g < 0$ and $\partial i_g/\partial A_g > 0$. Consistent with the evidence in Krishnamurthy and Vissing-Jorgensen (2012), e.g., τ_g decreases with A_g. Moreover, $\partial k^m/\partial i < 0$, $\partial k^c/\partial i < 0$ and $\partial\tau_g/\partial i > 0$. The expression for r is given by (34) with i replaced by τ_g.

We now turn to the case where the liquidity constraint binds. If $\theta = 0$, k^c solves

$$a_m^e - k^c + \frac{\chi_b}{1+\tau_g \nu_g} f(k^c) = 0. \tag{38}$$

This shows the liquidity requirement is formally equivalent to a reduction in χ_b. The entrepreneur's choice of cash balances is now given by

$$(1-\alpha) f'(k^m) + \alpha \frac{\left(1 - \frac{\chi_b}{1+\tau_g\nu_g}\right) f'(k^c)}{1 - \frac{\chi_b}{1+\tau_g\nu_g} f'(k^c)} = 1 + \frac{i}{\lambda}. \tag{39}$$

In the regime $i_g \in (0, i)$, (k^m, k^c) solve (39) and $k^c - k^m = A_g/(\alpha\lambda\nu_g)$, and one can check $\partial k^c/\partial A_g > 0$, $\partial k^m/\partial A_g < 0$, and $\partial i_g/\partial A_g > 0$.

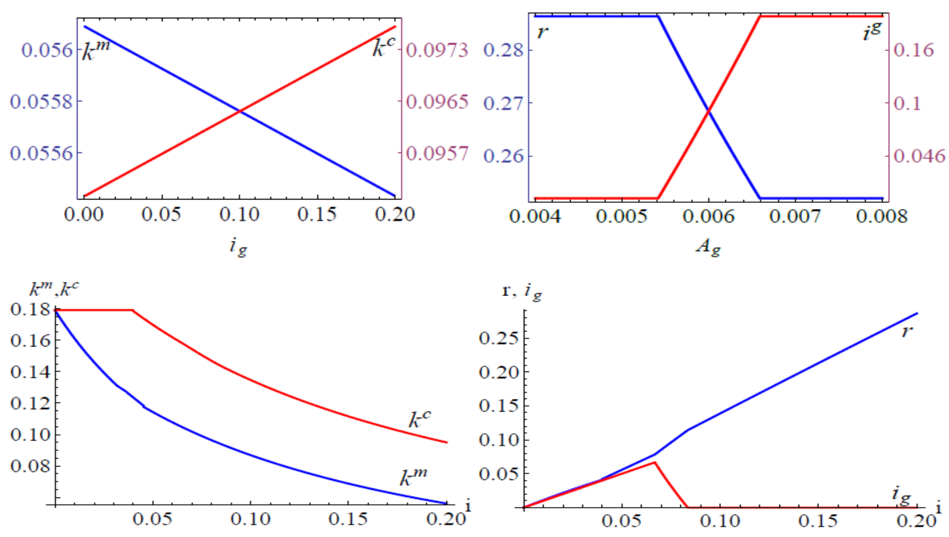

Figure 10: Monetary policy under broad liquidity requirements

Figure 10 depicts an example like the previous one except now $\nu_g = 0.5$, $i = 0.2$ (top panel) and $A_g = 0.004$ (bottom panel). The top right panel has a one-to-one relationship between i_g and A_g, because as A_g increases, the regulatory premium on bonds falls. The lending rate decreases with A_g, because as liquidity becomes more abundant, the cost of lending in the interbank market falls. There is negative pass through between i_g and r but positive pass through between τ_g and r. In the top left panel, an increase in i_g reduces τ_g and changes the composition of investment, with k^m falling and k^c rising. In the bottom left panel, k^c and k^m decrease with i.

The bottom right panel shows a nonmonotone relationship between i_g and i. For low i, entrepreneurs hold lots of cash, the demand for loans is low, and $i_g = i$. For intermediate i, they hold less cash and demand more loans. Banks compete for scarce liquidity, which generates a regulatory premium on bonds, and i_g decreases with i. When i is high, the demand for loans is so large that banks start competing with entrepreneurs to hold money to complement their bonds, $i_g = 0$.

6.3 Summary of results with regulation

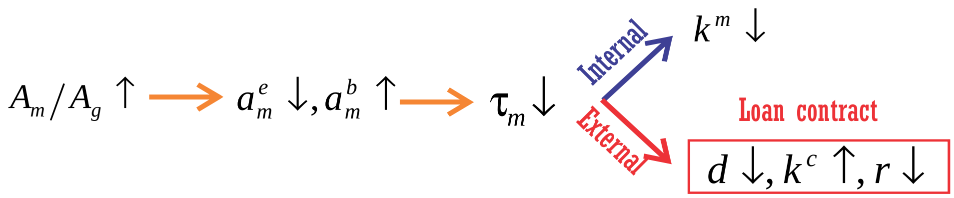

Figure 11: Transmission mechanism for OMO under strict reserve requirement

Once regulation is introduced, the theory has even richer policy implications, as summarized in Figure 11, under a strict reserve requirement. The effects of OMOs depend on whether government bonds serve as reserves, and on whether the operation takes place in the interbank market. If bonds have no regulatory role, OMOs matter only to the extent that they affect the money supply. A change in the money supply is neutral unless it takes place in the interbank market, where it has real effects. An increase in the money supply in the interbank market reduces entrepreneurs' real balances and internally financed investment. The borrowing cost of reserves falls while lending and aggregate investment increases. If bonds have a regulatory role, then OMOs have real effects whether or not they take place in the interbank market. A decrease in the supply of government bonds raises their regulatory premium, which raises banks' cost of extending loans and reduces investment. If the bond supply falls below a threshold, the bond yield is zero and changes in the bond supply are irrelevant.

7 Conclusion

This paper has developed a theory of corporate finance and its relation to monetary policy, formalized as either changes in the money growth rate or OMOs. The environment has entrepreneurs with random investment and financing opportunities, as in Kiyotaki and Moore (1997) and Wasmer and Weil (2004), respectively. Different from much of the related macro literature, loan contracts are bilaterally negotiated to determine the interest rate, loan size, and down payment. In this setting, there is a transmission mechanism where changes in the nominal policy rate affect real lending rates, which we consider a main contribution of the theory. Moreover, many macro models used in policy analysis have a single interest rate, while we deliver a richer structure of yields, including the overnight rates in the interbank market, the rate on government bonds, the rate on illiquid bonds, and the corporate lending rate. Policy affects this structure in interesting ways, depending on the details of policy and regulation.

Monetary policy affects corporate finance and investment because low nominal rates promote financing through retained earnings, although loan size is non-monotone in the policy rate due to a financing multiplier. The mechanism has different effects across firms depending on their borrowing capacity, among other things, and across countries, with larger effects at low inflation. There are various empirical implications that can be related to existing evidence. The impact of OMOs is consistent with conventional wisdom, but we provide search-and-bargaining foundations for credit market frictions that matter for these effects. Purchases of bonds in the interbank market reduce interest on overnight trades and, due to reserve requirements, this implies lower lending rates and larger loans. Bank financing rises and internal financing falls due to a redistribution of real balances, but on net investment increases. The economy can also fall into a liquidity trap, where the interbank rate is zero; then increases in the money supply raise the corporate lending rate and reduce investment. We think these results help us understand monetary policy and corporate finance better.

We end by remarking on our overall objective as monetary economists. In any model with a choice between debt and equity, one has to break the classic Modigliani-Miller irrelevance result to say something interesting about that decision. As discussed in Tirole (2006) and Holmstrom and Tirole (2011), the literature has approached the problem in different ways – e.g., by adding institutional features that favor one or the other, such as taxes, bankruptcy laws, and limited liability, or by considering moral hazard and other information problems. Monetary theory has similar irrelevance results in terms alternative payment instruments, including money and credit, as well as money and public or private bonds, domestic and foreign currencies, etc. Gu et al. (2016) and Lotz and Zhang (2016) are recent papers explicitly providing irrelevance theorems, and exceptions, in terms of money and credit. The exceptions use approaches similar to those used to break Modigliani-Miller – e.g., legal restrictions, counterfeiting and other information frictions, etc. This paper has been precise about specifying an environments where money and credit are not pefect substitutes, which is obviously important for analyzing this aspect of corporate finance, and especially for understanding the impact of monetary policy.

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Supplemental Appendices (not for publication)

A1. The bargaining set

In a match between an entrepreneur and a bank, the surpluses are $S^e = f(k) - k - \phi$ and $S^b = \phi$. If the pledgeability constraint is slack, the surplus is maximized at $f(k^*) - k^*$. Then the frontier is linear, $S^e + S^b = f(k^*) - k^*$, as in the right panel of Figure 3. The constraint is slack if $\phi \leq \chi_b f(k^*) - k^*$. Hence, the frontier has a linear portion iff $\chi_b \geq k^*/f(k^*)$, and is entirely linear if $f(k^*) - k^* \leq \chi_b f(k^*) - k^*$, which only occurs when $\chi_b = 1$.

If the pledgeability constraint binds then $\phi = \chi_b f(k) - k$, as in the left panel of Figure 3. Take a pair (ϕ, k) below the curve $\chi_b f(k) - k$ such that $k < k^*$. By raising k, S^e increases. Moreover, $k \geq \hat{k} = \arg\max\left[\chi_b f(k) - k\right]$, since otherwise one could raise $\phi = \chi_b f(k) - k$ and increase both surpluses. Hence, the frontier when the constraint binds is

$$\left\{(S^e, S^b) \in \mathbb{R}_{2+} : S^e = (1 - \chi_b) f(k),\ S^b = \chi_b f(k) - k,\ k \in \left[\hat{k}, \bar{k}\right]\right\},$$

where $\bar{k} = k^*$ if $\chi_b f(k^*) \geq k^*$, and $\bar{k}$ is the largest solution to $\chi_b f(\bar{k}) - \bar{k} = 0$ otherwise. It is easy to check the frontier is downward sloping, $\partial S^e/\partial S^b < 0$, and $\partial S^e/\partial S^b \to -\infty$ as $k \to \hat{k}$. If $\chi_b f(k^*) \geq k^*$ then $\partial S^e/\partial S^b \to -1$ as $k \to k^*$. The bargaining set is not convex since the point on the frontier that maximizes S^b, $\chi_b f(\hat{k}) - \hat{k}$, is above the horizontal axis. Hence, the entrepreneur enjoys a positive surplus, $(1- \chi_b) f(\hat{k})$, due to limited pledgeability.

A2. Alternative bargaining solution

As an alternative to the Nash, many recent models use Kalai's proportional bargaining solution, which is this context is given by:

$$\max_{\phi,k} S^b = \phi \text{ st } S^e \geq \frac{1-\theta}{\theta} S^b \text{ and } k + \phi \leq \chi_b f(k).$$

Thus, a bank chooses (ϕ, k) to maximize S^b subject to the entrepreneur getting at least a fraction $1- \theta$ of the total surplus. In fact, the strict proportional solution requires a strict equality in the first constraint; we use an inequality to guarantee existence despite nonconvexity of the bargaining set, which formally corresponds to the lexicographic proportional solution. Provided $\chi_b \geq \chi_b^*$, the pledgeability constraint is slack and Kalai coincides with Nash. If the constraint binds, k solves

$$(\chi_b - \theta) f(k) = (1-\theta)k \text{ if } \ \chi_b > \theta \text{ and } k \geq \hat{k};\ k = \hat{k} \text{ otherwise.}$$

Thus, the solution $k \geq \hat{k}$ splits the surplus so the bank gets a share θ of the surplus and satisfies the constraint. If $k < \hat{k}$, the solution is not Pareto optimal: by increasing k to $\hat{k}$, S^b reaches its maximum, while S^e increases. In that case, we select $k = \hat{k}$, in accordance with the lexicographic proportional solution. Interest on the loan when the constraint binds is

$$r = \frac{\theta(1-\chi_b)}{\chi_b - \theta} \text{ if } \theta \leq \hat{\theta} \equiv \frac{\chi_b f(\hat{k}) - \hat{k}}{f(\hat{k}) - \hat{k}};\ r = \frac{\hat{\theta}(1-\chi_b)}{\chi_b - \hat{\theta}} \text{ otherwise.}$$

Provided θ is not too large, r decreasing with χ_b. If $f(k) = zk^\gamma$, e.g., One can check r and k are given by:

$$r = \begin{array}{c} \frac{\theta(1-\gamma)}{\gamma} \\ \frac{\theta(1-\chi_b)}{\chi_b-\theta} \\ \frac{1-\gamma}{\gamma} \end{array} \text{ and } k = \begin{array}{c} (\gamma z)^{\frac{1}{1-\gamma}} \\ \left[\frac{(\chi_b-\theta)z}{1-\theta}\right]^{\frac{1}{1-\gamma}} \\ (\chi_b z \gamma)^{\frac{1}{1-\gamma}} \end{array} \text{ if } \chi_b \in \begin{array}{c} \geq (1-\theta)\gamma + \theta \\ \left[\frac{\theta}{1-\gamma(1-\theta)}, (1-\theta)\gamma + \theta\right) \\ < \frac{\theta}{1-\gamma(1-\theta)} \end{array}$$

For low χ_b, r is maximized and independent of χ_b and θ; in this case the constraint binds and k maximizes S^b. For intermediate χ_b, r is decreasing in χ_b and increasing in θ. For high χ_b, the constraint is slack, so k and r are independent of χ_b.

A3. Limited commitment

In the text the entrepreneur's borrowing limit is a fraction χ_b of $f(k)$. This can be motivated by, instead of moral hazard, limited commitment. Assume banks can no longer seize output: entrepreneurs can abscond with it all and default on the loan. However, banks have a record of repayment histories, and can punish defaulters by exclusion from future credit. An endogenous debt constraint ensures entrepreneurs repays debts, which depends on $\bar{W}^e = W^e(0,0) = \beta\{\alpha\lambda[f(k) - k - \phi] + \bar{W}^e\}$. An entrepreneur in stage 2 with no wealth has an investment opportunity in the next period with probability $\alpha\lambda$, in which case he gets surplus $f(k) - k - \phi$. Solving for $\bar{W}^e$, we obtain

$$\bar{W}^e = \frac{\alpha\lambda\,[f(k) - k - \phi]}{\rho}. \tag{40}$$

Thus, the value of being an entrepreneur is the discounted sum of profits, net of fees. By defaulting, an entrepreneur is banished to autarky and loses $\bar{W}^e$, making the borrowing constraint $\psi + \phi \leq \bar{W}^e$.

Under Nash bargaining the loan contract solves

$$(k, \phi) \in \arg\max \left[f(k) - k - \phi\right]^{1-\theta} \phi^{\theta} \text{ st } k + \phi \leq \bar{W}^e. \tag{41}$$

The problem is convex, since $\bar{W}^e$ is independent of k. The frontier is

$$S^e + S^b = f(k^*) - k^* \text{ if } S^b \leq \bar{W}^e - k^*;\ \Delta^{-1}(S^e + S^b) + S^b = \bar{W}^e \text{ otherwise,}$$

where $\Delta(k) \equiv f(k) - k$ is the total surplus when the constraint binds. Relative to Figure 3, the frontier now intersects the horizontal axis at $S^e = 0$. Notice $k = k^*$ and $\phi = \theta \left[f(k^*) - k^*\right]$ if $\bar{W}^e \geq k^* + \phi$. Using this, the value of an entrepreneur who is not constrained is $\bar{W}^e = \alpha\lambda \left(1 - \theta\right) \left[f(k^*) - k^*\right] / \rho$. Accordingly, entrepreneurs are not constrained if

$$\rho \leq \rho^* \equiv \frac{\alpha\lambda \left(1 - \theta\right) \left[f(k^*) - k^*\right]}{(1 - \theta)k^* + \theta f(k^*)}.$$

Next suppose the constraint binds. The solution to (41) is

$$\bar{W}^e = \frac{\theta f(k) + (1 - \theta) f'(k) k}{(1 - \theta) f'(k) + \theta}. \tag{42}$$

Now the borrowing limit $\bar{W}^e$ is a weighted average of $f(k)$ and the supplier's cost, k. In this case,

$$\bar{W}^e = \frac{\alpha\lambda}{\rho + \alpha\lambda} f(k). \tag{43}$$

The limit from (43) is analogous to the pledgeability constraint in Section 4 where $\chi_b = \alpha\lambda/(\rho + \alpha\lambda)$. Here pledgeability depends on ρ, λ and α. A difference however is that the RHS of (43) use future output.

Substituting $\bar{W}^e$ from (43) into (42), k solves

$$\frac{k}{f(k)} = \frac{\alpha\lambda(1 - \theta) f'(k) - \rho\theta}{(\rho + \alpha\lambda) \left(1 - \theta\right) f'(k)}. \tag{44}$$

Notice $k = 0$ always solves (44), as is standard. In addition, there is solution $k > 0$ uniquely determined, since the LHS (44) is increasing in k while the RHS is decreasing for all k such that $\alpha\lambda(1- \theta) f'(k) > \rho\theta$. The positive solution increases with α and λ and decreases with ρ and θ. The lending rate is

$$r = \frac{\bar{W}^e - k}{k} = \frac{\alpha\lambda}{\rho + \alpha\lambda} \frac{f(k)}{k} - 1,$$

which increases with θ. Notice r depends on ρ, since the debt limit is determined by future surpluses, as well as λ and α.

Given $f(k) = zk^{\gamma}$, when the borrowing constraint is slack, $k^* = (\gamma z)^{\frac{1}{1-\gamma}}$ and $r = \theta(1-\gamma)/\theta$, identical to Section 4. When it binds,

$$k = \left[\frac{\chi_b(1-\theta)z\gamma}{(1-\theta)\gamma + (1-\chi_b)\theta}\right]^{\frac{1}{1-\gamma}} \text{ and } r = \frac{(1-\chi_b)\theta}{(1-\theta)\gamma},$$

where $\chi_b \equiv \alpha\lambda/(\rho + \alpha\lambda)$. Now k increases while r decreases with pledgeability.

A4. Strategic foundations for bargaining

While the strategic foundations of Nash bargaining are very well known, there are some nuances here, like commitment issues and nonconvexities; therefore we provide the details. Consider a game with alternating offers between the entrepreneur and bank. There is no discounting, but an exogenous risk of breakdown. At the initial stage, the entrepreneur makes an offer (k^e, d^e, ϕ^e), and the bank can say either yes or no. If it says yes, the offer is implemented. If it say no, the game continues. With probability δ^e negotiations end with no loan; with probability $1-\delta^e$ the bank makes an offer (k^b, d^b, ϕ^b), and the entrepreneur can either say yes or no. If he says yes, the offer is implemented. If he say no, the game continues. With probability δ^b negotiations end; with probability $1-\delta^b$ the games continues as in the initial stage. See the game tree in Figure 12. A node with two players corresponds to a simultaneous move and the risk of breakdown is a move by Nature.

Consider stationary equilibria with offers, (k^e, d^e, ϕ^e) and (k^b, d^b, ϕ^b). We restrict attention to acceptance rules in the form of reservation surpluses, R^e and R^b, that specify a minimum surplus required for an agent to accept. Entrepreneurs accept an offer if $f(k) - \psi - \phi \geq R^e$, and banks accept if $\phi \geq R^b$. When it is the entrepreneur turn to make an offer,

$$S^e(R^b) = \max_{k,\phi} \left\{ [f(k) - k - \phi]\, \mathbb{I}_{\{\phi \geq R^b\}} \right\} \text{ st } k + \phi \leq \chi_b f(k) + a^e_m,$$

where $\mathbb{I}_{\{\phi \geq R^b\}}$ is an indicator function that equals one if $\phi \geq R^b$ (we ignore the down payment d, because the entrepreneur uses his real balances before requesting a loan). The solution is:

$$\begin{aligned} S^e(R^b) &= f(k^*) - k^* - R^b \text{ if } R^b \leq \chi_b f(k^*) - k^* + a^e_m && (45)\\ &= f(k) - k - R^b \text{ if } R^b \in (\chi_b f(k^*) - k^* + a^e_m, \chi_b f(\hat{k}) - \hat{k} + a^e_m] && (46) \end{aligned}$$

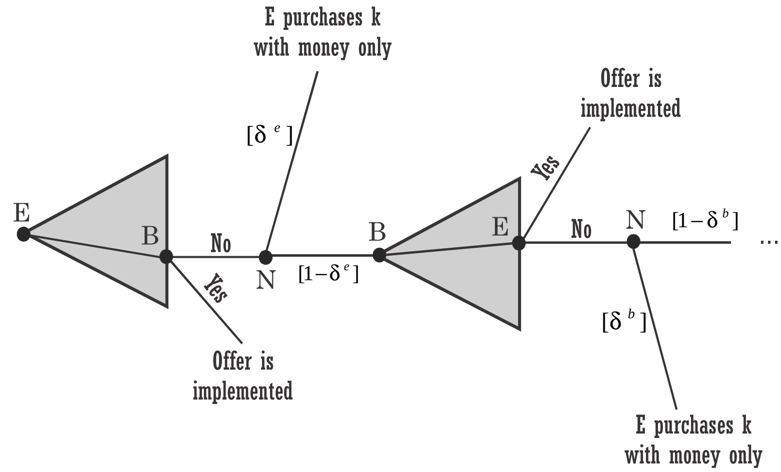

Figure 12: Game tree

where k is the largest solution to $\chi_b f(k) - k = R^b - a^e_m$. If the reservation surplus of the bank is sufficiently low the entrepreneur can finance k^* and $\phi = R$; if is larger but not too large, the entrepreneur asks for the largest loan satisfying the liquidity constraint; if it is too large the entrepreneur cannot satisfy $\phi \geq R^b$ and et a surplus. It can be checked that $S^e(R^b)$ is decreasing and concave with $S^e(0) > 0$.

Similarly, the bank's surplus when it is his turn to make an offer is

$$S^b(R^e) = \max_{k,\phi} \left\{ \phi \mathbb{I}_{\{f(k)-k-\phi \geq R^e\}} \right\} \text{ st } k + \phi \leq \chi_b f(k) + a^e_m.$$

The bank maximizing his payoff subject to the acceptance rule and liquidity constraint. The solution is

$$\begin{aligned} S^b(R^e) &= f(k^*) - k^* - R^e \text{ if } R^e \in [(1-\chi_b)f(k^*) - a^e_m, f(k^*) - k^*] && (47)\\ &= \chi_b f(\hat{k}) - \hat{k} + a^e_m \quad \text{ if } \quad R^e \leq (1-\chi_b) f(\hat{k}) - a^e_m && (48)\\ &= f(k) - k - R^e \qquad \text{otherwise,} && (49) \end{aligned}$$

where k solves $(1-\chi_b)f(k) = R^e + a^e_m$. If the entrepreneur's reservation surplus is large but not so large the bank would not participate, the bank offers to finance k^*; if is low the bank asks for a payment such that the constraint binds; and below a threshold for R^e k maximizes $\chi_b f(k) - k$. It can be checked that $S^b(R^e)$ is nondecreasing, concave, and $S^b(R^e) > 0$.

The endogenous reservations surpluses solve

$$\begin{aligned} R^e &= (1-\delta^b) S^e(R^b) + \delta^b \Delta^m(a^e_m) && (50)\\ R^b &= (1-\delta^e) S^b(R^e). && (51) \end{aligned}$$

Thus, R^e is the surplus that makes the entrepreneur indifferent between accepting or rejecting, and similarly for (51). Note that after a breakdown the bank receives no surplus.

Figure 13 shows (50) in blue and (51) in red; both are downward sloping and concave. To establish existence, let $\bar{R}^e > 0$ be the R^e such that $S^b(R^e) = 0$. By the duality of the entrepreneur and bank problems, $\bar{R}^e = S^e(0)$. Moreover, provided that $a^e_m < k^*$ then $\Delta^m(a^e_m) < S^e(0)$. Hence, the blue curve is below the red curve at $R^b = 0$. The red curve has a maximum $(1 - \delta^e)S^b(0) < \chi_b f(\hat{k}) - \hat{k} + a^e_m$. So at $R^b = \chi_b f(\hat{k}) - \hat{k} + a^e_m$ the blue curve is to the right of the red curve. Hence, they intersect, so a solution exists. Uniqueness follow from concavity of the relationships and the fact that when they are linear they have different slopes.

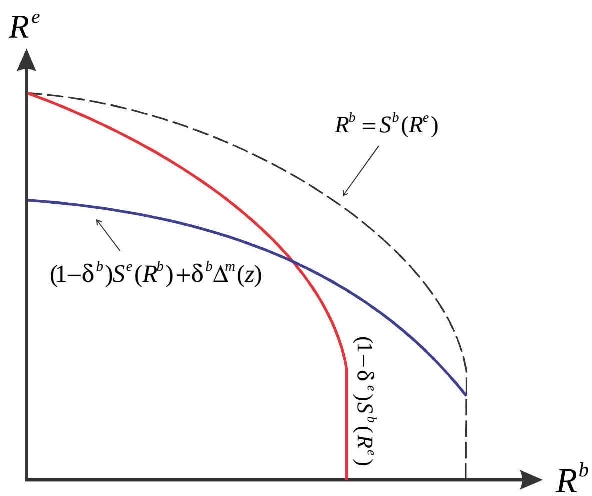

Figure 13: Determination of (R^b, R^e)

A stationary, subgame perfect equilibrium is composed of two offers, (k^e, d^e, ϕ^e) and (k^b, d^b, ϕ^b), and two reservation surpluses, R^e and R^b, solving the above conditions, as is completely standard. Existence and uniqueness here follow from the above discussion. Now consider letting the risk of breakdown get small by rewriting $\delta^e = \varepsilon\bar{\delta}^e$ and $\delta^b = \varepsilon\bar{\delta}^b$. As $\varepsilon \to 0$, $S^e(R^b) - R^e \to 0$ and $S^b(R^e) - R^b \to 0$ (i.e., when the breakdown risk gets small, first-mover advantage vanishes). Graphically, the reservation values at the intersection of the curves in Figure 13 converge to a point on the dashed curve.

Suppose first the borrowing constraint does not bind. Then $S^e(R^b) = f(k^*) - k^* - R^b$ and $S^b(R^e) = f(k^*) - k^* - R^e$. Thus, both the entrepreneur and bank offer

k^*, and use ϕ to satisfy the acceptance rules. Taking the limit as $\varepsilon \to 0$,

$$R^e \;\to\; \frac{\bar{\delta}^e\left[f(k^*) - k^*\right] + \bar{\delta}^b \Delta^m(a_m^e)}{\bar{\delta}^e + \bar{\delta}^b} \tag{52}$$

$$R^b \;\to\; \frac{\bar{\delta}^b}{\bar{\delta}^e + \bar{\delta}^b}\left[f(k^*) - k^* - \Delta^m(a_m^e)\right]. \tag{53}$$

The banks' surplus approaches a fraction $\bar{\delta}^b/(\bar{\delta}^e + \bar{\delta}^b)$ of the total surplus, coinciding with the Nash solution with $\theta = \bar{\delta}^b/\left(\bar{\delta}^e + \bar{\delta}^b\right)$.

Now suppose the liquidity constraint binds. Then $S^e(R^b) = f(k^e) - k^e - R^b$ where k^e is the highest solution to $k^e + R^b = \chi_b f(k^e) + a_m^e$, and $S^b(R^e) = f(k^b) - k^b - R^e$ where k^b is the solution to $(1 - \chi_b) f(k^b) = R^e + a_m^e$. Now (k^e, k^b, R^e, R^b) solves

$$R^e = (1 - \bar{\delta}^b \varepsilon)\left[f(k^e) - k^e - R^b\right] + \bar{\delta}^b \varepsilon \Delta^m(a_m^e) \tag{54}$$

$$R^f = (1 - \bar{\delta}^e \varepsilon)\left[f(k^b) - k^b - R^e\right] \tag{55}$$

$$R^b = \chi_b f(k^e) - k^e + a_m^e \tag{56}$$

$$R^e = (1 - \chi_b) f(k^b) - a_m^e. \tag{57}$$

Rearranging (54)-(55) we obtain

$$R^e = \frac{(1 - \bar{\delta}^b \varepsilon)\left\{f(k^e) - k^e - (1 - \bar{\delta}^e \varepsilon)\left[f(k^b) - k^b\right]\right\} + \bar{\delta}^b \varepsilon \Delta^m(a_m^e)}{1 - (1 - \bar{\delta}^b \varepsilon)(1 - \bar{\delta}^e \varepsilon)}.$$

Letting $\varepsilon \to 0$ and using L'Hopital's rule, we get

$$R^e = \frac{\bar{\delta}^e\left[f(k) - k\right] + \left[f'(k) - 1\right]\left(\frac{dk^e}{d\varepsilon} - \frac{dk^b}{d\varepsilon}\right) + \bar{\delta}^b \Delta^m(a_m^e)}{\bar{\delta}^b + \bar{\delta}^e}. \tag{58}$$

The terms $dk^e/d\varepsilon$ and $dk^b/d\varepsilon$ are obtained by differentiating (54)-(57) in the neighborhood of $\varepsilon = 0$,

$$\frac{dk^e}{d\varepsilon} - \frac{dk^b}{d\varepsilon} = \frac{\bar{\delta}^e\left[f(k) - k - R^e\right]}{1 - \chi_b f'(k)}. \tag{59}$$

Substituting (59) into (58) and replacing R^e by $(1 - \chi_b) f(k) - a_m^e$, we get

$$\left(\frac{\bar{\delta}^b}{\bar{\delta}^e}\right) \frac{1 - \chi_b f'(k)}{(1 - \chi_b) f'(k)} = \frac{\chi_b f(k) - k + a_m^e}{(1 - \chi_b) f(k) - a_m^e - \Delta^m(a_m^e)}. \tag{60}$$

This corresponds to the FOC from Nash bargaining with $\theta = \bar{\delta}^b/(\bar{\delta}^e + \bar{\delta}^b)$. As usual, subgame perfect equilibrium in the game generates the same outcome as Nash bargaining.